LECTURE NOTES

ON

THEORY OF MACHINES

4TH SEMESTER

DIPLOMA IN MECHANICAL ENGINEERING

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Introduction

Theory of machine :→

It is ~~tha~~ defined as the branch of engineering science which deals with the study of relative motion between the various parts of a machine & forces which act on them.

Machine :-

(i) It is a device which receives energy in some available form & ultilises it to do some particular type of work.

(ii) It is device which reduces human effort.

Sub-Division of Theory of Machine :-

Theory of machines may be sub-divided into four branches.

(a) Kinematics :→ It is the branch of theory of machine which deals with the relative motion betn the various parts of the machines.

(b) Kinetics :→ It is the branch of theory of machines which deals with the study of inertia forces.

(c) Dynamics :→ It is the branch of theory of machine which deals with the forces & their effect, when the machine parts are in motion.

(d) Statics :→ It is the branch of theory of machine which deals with the study of forces & their effect, when the machine parts are at rest.

Fundamental units

Every quantity is measured by internationally accepted units, called fundmental units. All physical quantities are expressed interms of ~~3~~ three fundamental quantities i.e

Length (L)
mass (m)
Time (t)

Derived units :- Some units are expressed interms of fundamental units known as derived units.

Exp :- The unit of velocity, Accelaration, Pressure etc.

Systems of units

There are four system of units, i.e

(1) M.K.S (metre, Kilogram, Second)
(2) C.G.S (Centimeter, Gram, Second)
(3) F.P.S (Foot, Pound, Second)
(4) S.I (Standard International unit)

Various units

Density → Kg/m^3
Force → N (Newton)
pressure → Pa (pascal) or N/m^2
Work, energy → J, N-m
Power → J/s, Watt
Absolute viscosity → Kg/m-s
Kinematic viscosity → m^2/s

Velocity → m/s
Accelaration → m/s^2
Angular Accelaration → Rad/s^2
Frequency → HZ
Amount of substance → mole
Electric current → Ampere
Distance → meter
Time → second
Mass → Kg

Force :→ It may be defined as an agent, which produces or tends to produce, destroy or tends to destroy motion.
$F = m \cdot a$ Newton

Plane motion :→ When the motion of the body is confined to only one plane, the motion is called plane motion.

Translatory motion :→ When the motion is along straight line path, the motion is known as translatory motion.

Curvilinear motion :→ When the motion is along a curved path confined to one plane, the motion is known as curvilinear motion.

plane Rotational motion :→ When all the particles of a body travel in circular path of constant radius. The motion is known as plane rotational motion.

Exp:- Pulley rotate about a fixed shaft
Shaft rotating about its own axis.

Linear Displacement :- It may be defined as the distance moved by a body with respect to a certain fixed point.
It is a vector quantity & has both magnitude & direction.
It is represent graphically by a straight line.

Linear Velocity :→ It may be defined as the rate of change of linear displacement of a body with respect to time.
It is a vector quantity.
Mathematically $V = \frac{ds}{dt}$

Linear Accelaration :→ It may be defined as the rate of change of linear velocity of a body with respect to time.
It is a vector quantity.
mathematically $a = \frac{dv}{dt}$

The negative accelaration is called deceleration or retardation.

Angular Displacement :→ It may be defined as the angle described by a particle from one point to another, with respect to time.
It is a vector quantity because it has both magnitude & direction.

Angular Velocity :-

It may be defined as the rate of change of angular displacement with respect to time.

Mathematically,

Angular velocity $\boxed{\omega = \frac{d\theta}{dt}}$

It is a Vector quantity because it has both magnitude & direction.

Angular Acceleration :-

It may be defined as the rate of change of angular velocity with respect to time.

Mathematically,

Angular acceleration $\boxed{\alpha = \frac{d\omega}{dt}}$

It is a vector quantity, but its direction may not same as angular displacement & Angular velocity.

Difference between mass & Weight

Mass	Weight
(i) It is the amount of matter contained in a body.	(i) It is the amount of pull which the earth exerts upon a given body.
(ii) It does not vary with the change in its position on earth surface.	(ii) It varies with distance of the body from the centre of the earth.
(iii) Mass is not a force	(iii) Weight is a force
(iv) $\boxed{m = \frac{W}{g}}$ Kg	(iv) $\boxed{W = m \cdot g}$ Newton

Momentum

It is the total motion possesses by a body.

mathematically, $\boxed{\text{momentum} = \text{mass} \times \text{velocity}}$

Rate of change of momentum $= \frac{m \cdot v - m \cdot u}{t} = \frac{m(v-u)}{t}$

$= m \cdot a \quad \because \boxed{\frac{v-u}{t} = a}$

Where m = Mass of the body

u = Intial velocity of body

v = final velocity of body

t = Time to change velocity from u to v in second.

Moment of Force

It is the turning effect produced by a force, on the body on which it acts.

$\boxed{\text{moment of force} = F \times l}$

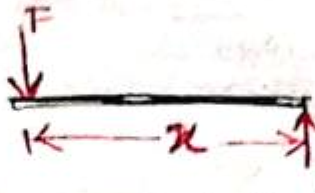
Where F = Force acting on the body

l = ⊥ar distance of the point & line of action of the force

(moment of Force)

Couple

The two equal & opposite parallel forces, whose line of action are different, is known as couple.



The moment of couple is the product of one of the force & the arm of the couple(x).

Mathematically

$$\text{moment of couple} = F \times x$$

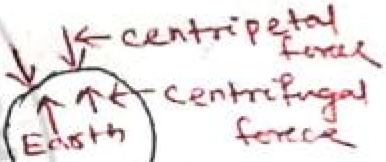
The couple produces a motion of rotation of the body, on which it acts.

Centripetal & Centrifugal Force

↳ The centripetal force acts radially inwards.

$$F_c = m\omega^2 r$$

r = Radius
m = mass
ω = Ang. velocity

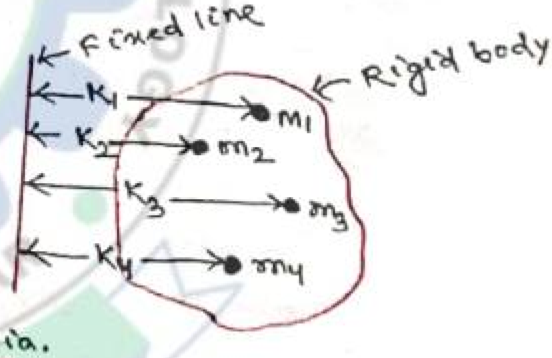


↳ The centrifugal force acts radially outwards.

$$F_c = m\omega^2 r$$

Mass moment of Inertia :→

The rigid body is composed of small particles. If the mass of every particle of a body is multiplyed by the square of its far distance from a fixed line, then the sum of these quantities is known as mass moment of inertia.



$$I = m_1(K_1)^2 + m_2(K_2)^2 + m_3(K_3)^2 + m_4(K_4)^2 + \cdots$$

$$I = mK^2$$

K = Radius of gyration.

* It may be defined as the distance, from a given reference, where the whole mass of the body is assumed to be concentrated.

Torque

It may be defined as the product of force & far distance of its line of action from the given point or axis.

↳ It is equivalent to a couple acting upon a body.

↳ It is directly proportional to the rate of change of angular momentum.

$$T = I \cdot \alpha$$

Work:-

Whenever a force acts on a body & the body undergoes a displacement in the direction of the force, then work is said to be done.

Workdone = Force × displacement

$$W = F \times x$$

F = Force
x = displacement

When a couple or torque is acting on the body

$$W = T \cdot \theta$$

T = Torque
θ = Angular displacement

Unit:- N-m or Joule (J)

Power:->

It may be defined as the rate of doing work or workdone per unit time.

Mathematically
$$\text{power} = \frac{\text{Workdone}}{\text{Time taken}}$$ watt

If Torque is considered, then

$$P = T \cdot \omega$$

T = Torque transmitted
ω = angular speed
$= \frac{2\pi N}{60}$ rad/s
N = Speed in r.p.m

Efficiency

The ratio of power output to power input, is known as the efficiency of the machine. It is represented in percentage.

$$\eta = \frac{\text{Output}}{\text{Input}}$$

Energy:->

It may be defined as the capacity to do work.

The various forms of energy are:- mechanical, Electrical, Chemical, Light, Heat etc.

The mechanic energies are of 3 types:-

(a) Kinetic energy:-> The energy possesses by a body, by virtue of its velocity of motion is known as kinetic energy.

$$K.E = \frac{1}{2} \cdot m \cdot v^2$$

m = mass
v = velocity

(b) Potential energy:-> The energy possesses by the body, by virtue of its position, is known as potential energy.

$$P.E = m \cdot g \cdot h$$

m = mass of body
g = Acceleration due to gravity
h = Height of the body

(c) Strain energy:->

It is the potential energy stored by an elastic body when deformed. A compressed spring possesses this type of energy

$$\text{Strain energy} = \frac{1}{2} \cdot W \cdot x$$
$$= \frac{1}{2} S x^2$$

W = Load
x = distance
S = stiffness of spring

If Torsional spring is considered,

$$\text{strain energy} = \frac{1}{2} \times q \, \theta^2$$

q = Torsional stiffness of spring
θ = Angle of twist in radian.

Principle of Conservation of Energy :→

It state that " The energy can neither be created nor be destroyed, it can only transform from one form to another form.

Impulse or Impulsive Force

The impulse is the product of force & time.

mathematically, $\boxed{Impulse = F \times t}$ F = Force, t = Time

If the force acts for a very short time, then it is known as impulsive force. or blow.

↳ Exp:- Hammer blow

Objective Type

(1) The unit of linear accelaration is

(a) kg-m (b) m/s (c) m/s^2 (d) rad/s^2 Ans:- c

(2) The angular velocity (in rad/s) of a body rotating N r.pm is

(a) $\pi N/60$ (b) $2\pi N/60$ (c) $\pi N/120$ (d) $\pi N/180$

Ans:- b

(3) The linear velocity of a body rotating ω rad/s along a circular path has

(a) $\omega . r$ (b) ω / r (c) $\omega^2 . r$ (d) ω^2 / r

Ans:- a

(4) When a particle moves along a circular path, then the particle has

(a) Tangential accelaration only (b) Centripetal accelaration only (c) Both a & b (d) None of the above

Ans:- a

(5) When a particle moves with a uniform velocity along a circular path then the particle has:-

(a) Tangential accelaration only (b) Centripetal accelaration only (c) Both a & b (d) None of the above

Ans:- b

(6) When a body of mass moment of inertia 'I' is rotated about that axis with an angular velocity, then Kinetic energy of rotation is

(a) $0.5 I \omega$ (b) $I . \omega$ (c) $0.5 I \omega^2$ (d) $I . \omega^2$

Ans:- c

(7) The coefficient of ~~restriction~~ restitution for inelastic body is —

(a) zero (b) Between zero & one (c) one (d) more than one

Ans:- a

(8) A body of mass m moving with a contant velocity v strikes another body of same mass m ~~moving~~ moving with same velocity but in opposite direction. The common velocity of both the bodies after collision is ____

(a) v (b) 2v (c) 4v (d) 8v

Ans:- b

Simple Mechanisms

Link or Kinematic Link or Element

- Each part of machine which moves relative to some other part is know as Kinematic link.
- A link consist of several parts, which are rigidly fastened together.
- A body is said to be resistant body if it is capable of transmitting the required force without deformation.
- The link have two characteristics i.e
  - (i) It should have relative motion.
  - (ii) It must be resistant body.

Types of Link

(a) Rigid Link:- The link which does not undergo any deformation while transmitting motion is known as rigid link. The rigid link do not exist.

Exp:- Connecting rod, crank etc. of reciprocating steam engine (using steam to pressure piston of engine)

(b) Flexible Link:- The link which is partly deformed in a manner not to affect the transmission of motion, is known as flexible link. It transmit tensile force only.

Exp. - Belt, Rope, chain & wires

(c) Fluid Link:- The link which is formed by having a fluid in a container & the motion is transmitted through the fluid by pressure or compression only, is known as fluid link.

Structure

It is an assemblage of number of resistant bodies or members having no relative motion between them.

Exp:- Railway Bridge, machine frame etc.

Difference between Machine & Structure

Machine	Structure
(i) The part of a machine move relative to one another.	(i) The member of a Structure do not move relative to one another.
(ii) A machine transform the avaible energy into some useful work.	(ii) In a structure no energy is transformed into useful work.
(iii) The links of a machine may transmit both power & motion.	(iii) The member of a Structure transmit force only.

Kinematic pair

When the two links contact with each other, then it forms a pair. If the relative motion between them is in definite direction, the pair is known as Kinematic pair.

Classification of kinematic pair

(1) According to type of relative motion betn the elements :-

(a) Sliding pair :-> When the two elements of a pair are connected in such a way that one can only slide relative to other, the pair is known as sliding pair.

Exp :- Piston & cylinder, Ram & it guides in shaper, Tailstock on the lathe bed.

(b) Turning pair :-> When the two elements of a pair are connected in such a way that one can only turn about a fixed axis of another link, is known as turning pair.

Exp :- Lathe spindle support in head stock
Cycle wheels turning over their axle
Crankshaft in a Journal bearing in an engine

(c) Rolling pair :-> When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair.

Exp :- Ball & Roller bearing

(d) Screw pair :-> When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, is known as screw pair.

Exp :- Bolt with a nut

(e) Spherical pair :-> When the two elements of a pair are connected in such a way that one element with spherical shape turned about the other fixed element, is known as spherical pair.

Exp :- Attachment of car mirror, pen stand etc.

(2) According to the type of contact betn the elements :->

(a) Lower pair :-> When two elements of a pair have surface contact when relative motion takes place & surface of one element slides over the other, the pair formed is known as Lower pair.

Exp :- Sliding pair, turning pair, screw pair

(b) Higher pair :-> When the two elements of a pair have line or point contact, when relative motion takes place & the motion between the two elements is partly turning & partly sliding, then the pair formed is known as Higher pair.

Exp :- Toothed gearing, Belt & Rope drives, Ball & roller bearings, Cam & Follower

Kinematic chain

When the Kinematic pairs are coupled in a such a way that the last link is joined to the first link to transmit definite motion, is known as Kinematic chain.

If each link is assumed to form two pairs with two links, then the relative motion betn the number of pairs forming a Kinematic Chain & the number of links is

$$\boxed{l = 2P - 4} \quad \text{——— (i)}$$

l = No. of links
P = No. of pairs

Another relation between the number of links & no. of joints which constitute a Kinematic chain is

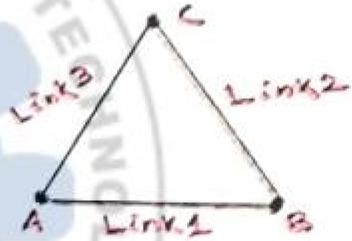
$$\boxed{j = \frac{3}{2}l - 2} \quad \text{— (ii)}$$

j = No. of joints
l = No. of links

The eqn's (i) & (ii) applicable only to Kinematic chains.

(i) Consider a arrangement of 3 links :→

Number of links $l = 3$
Number of pairs $P = 3$
Number of joints $j = 3$



From eqn (i) :→ $l = 2P - 4 = 2\times3 - 4$
$\Rightarrow l = 6 - 4$
$\Rightarrow 3 = 2$
L.H.S > R.H.S

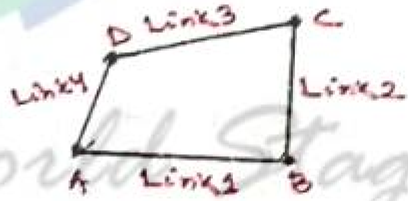
From eqn (ii) → $j = \frac{3}{2}l - 2$
$\Rightarrow 3 = \frac{3}{2}\times 3 - 2$
$\Rightarrow 3 = 2.5$ L.H.S > R.H.S

Since the arrangement of three links does not satisfied the eqn (i) & (ii). So it is not a Kinematic chain.

(ii) Consider a arrangement of 4 links :-

$l = 4, P = 4$ & $j = 4$

From eqn (i) → $l = 2P - 4$
$\Rightarrow 4 = 2\times4 - 4$
$\Rightarrow 4 = 4$
L.H.S = R.H.S



From eqn (ii) → $j = \frac{3}{2}l - 2$
$\Rightarrow 4 = \frac{3}{2}\times4 - 2$
$\Rightarrow 4 = 4$
L.H.S = R.H.S

Since the arrangement of four links satisfied eqn (i) & (ii), so it is a Kinematic chain.

Note

A chain having more than 4 links, known as Compound Kinematic chain.

Types of joints in a chain

(a) **Binary joint** :→ When two links are joined at the same connection, the joint is known as binary joints.

Inorder to determine nature of chain wheather it is locked chain (structure) or Kinematic chain, a relation is given by A.W.Klein :-

$$j + \frac{h}{2} = \frac{3}{2}l - 2$$

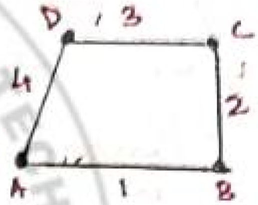
j = No. of binary joints
h = No. of higher pairs
l = No. of links

If $h = 0$, then

$$\boxed{j = \frac{3}{2}l - 2}$$

Consider a arrangement of 4 links :-

$l = 4$, $j = 4$



From eqn $j = \frac{3}{2}l - 2$

$\Rightarrow 4 = \frac{3}{2} \times 4 - 2$

$\Rightarrow 4 = 4$ L.H.S = R.H.S

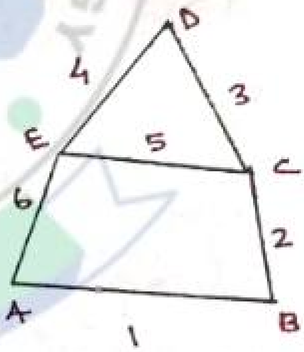
So, it is a Kinematic chain.

(b) **Ternary joint** :→ When the three links are joined at the same connection, the joint is known as ternary joint.

It has 3 Binary joints at A, B, & D.
& two ternary joints at C & E.

No. of link $l = 6$

No. of Binary joint $j = 3 + (2 \times 2)$
$= 7$



From eqn → $j = \frac{3}{2}l - 2$

$\Rightarrow 7 = \frac{3}{2} \times 6 - 2$

$\Rightarrow 7 = 7$
L.H.S = R.H.S

So, it is a Kinematic chain.

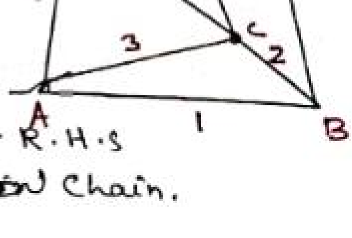
(C) **Quaternary joint** :→ When four links are jointed at the same connection, the joint is called quaternary joint.

(i) Consider a arrangement of 11 links

It has one binary joint at D.
Four ternary joints at A, B, E, F
Two quaternary joints at C & G.

No. of links $l = 11$

No. of Binary joints $= 1 + (4 \times 2) + (2 \times 3)$
$= 15$



From eqn → $j = \frac{3}{2}l - 2$

$\Rightarrow 15 = \frac{3}{2} \times 11 - 2$

$\Rightarrow 15 = 14.5$ L.H.S > R.H.S

So, it is not a Kinematic chain.

(ii) Consider a arrangement of 10 links :→

No. of links $l = 10$

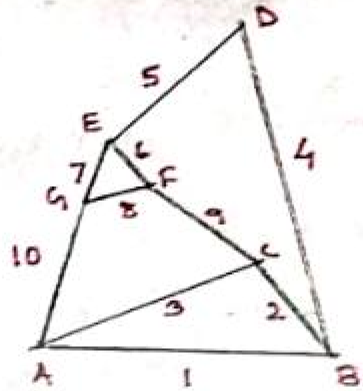
No. of Binary joints $j = 1 + (2 \times 6) = 13$

From eqn → $j = \frac{3}{2}l - 2$

$\Rightarrow 13 = \frac{3}{2} \times 10 - 2$

$\Rightarrow 13 = 13$ L.H.S. = R.H.S

So, it is a Kinematic chain.



Mechanism

When one links of the Kinematic Chain is fixed, the chain is known as mechanism.

Exp :- Engine indicator, Typewriter etc.

↳ A mechanism with four links is known as Simple mechanism.

↳ A mechanism with more than four links is known as compound mechanism.

↳ When a mechanism is required to transmit power or to do some particular type of work, then it becomes a Machine.

Number of Degrees of Freedom

It is defined as the number of input parameters i.e pair variables, which must be independently controlled inorder to bring the mechanism into a useful engineering purpose.

Kutzbach Criterion for a plane mechanism :→

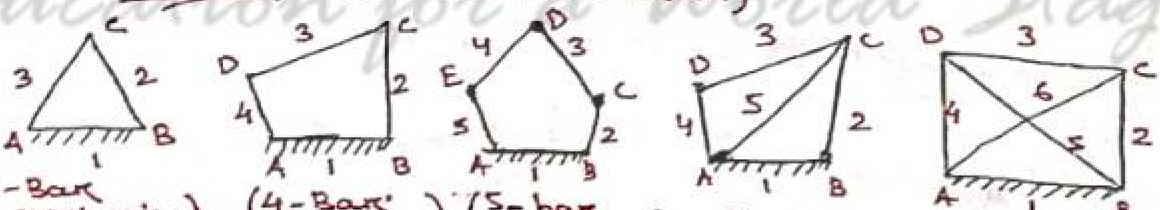
$$n = 3(l-1) - 2j - h$$

n = No. of degree of freedom
h = No. of higher pairs
l = No. of links
j = No. of Binary joints.

If $h = 0$, then

$$n = 3(l-1) - 2j$$

Applications (Kutzbach Criterion)



(3-Bar mechanism) a

(4-Bar mechanism) b

(5-bar Mechanism) c

(5-bar Mechanism) d

(6-Bar Mechanism) e

(a) $l = 3, j = 3$

$$n = 3(l-1) - 2j = 3(3-1) - 2 \times 3 = 0$$

As $n = 0$, This mechanism forms a structure.

(b) $l=4$, $j=4$, $n = 3(l-1) - 2j = 3(4-1) - 2\times4 = 1$

When $n=1$, mechanism can be driven by a single input motion.

(c) $l=5$, $j=5$, $n = 3(l-1) - 2j = 3(5-1) - 2\times5 = 2$

When $n=2$, mechanism can be driven by two input motion.

(d) $l=5$, $j=6$, $n = 3(l-1) - 2j = 3(5-1) - 2\times6 = 0$

(e) $l=6$, $j=8$, $n = 3(l-1) - 2j = 3(6-1) - 2\times8 = -1$,

When $n=-1$, it forms a statically indeterminate structure

Grubler's criterion for plane mechanisms

$$\boxed{3l - 2j - 4 = 0}$$ $\boxed{n=1 \ \& \ n=0}$ in Kutzbach criterion

This equ^n is known as Grubler's criterion for plane mechanisms.

Inversion of Mechanism

The method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as inversion of the mechanism.

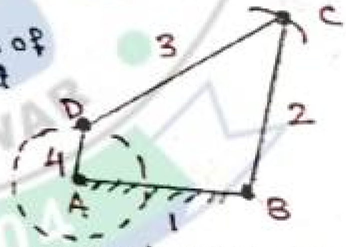
We can obtain many mechanism by fixing different links.

Types of Kinematic Chain

3 types :- (a) Four bar chain or quardric cyclic chain
(b) Single slider crank chain
(c) Double slider crank chain

(a) Four bar chain :→

It is consist of 4 links, each of them formed a turning pair at A, B, C & D.



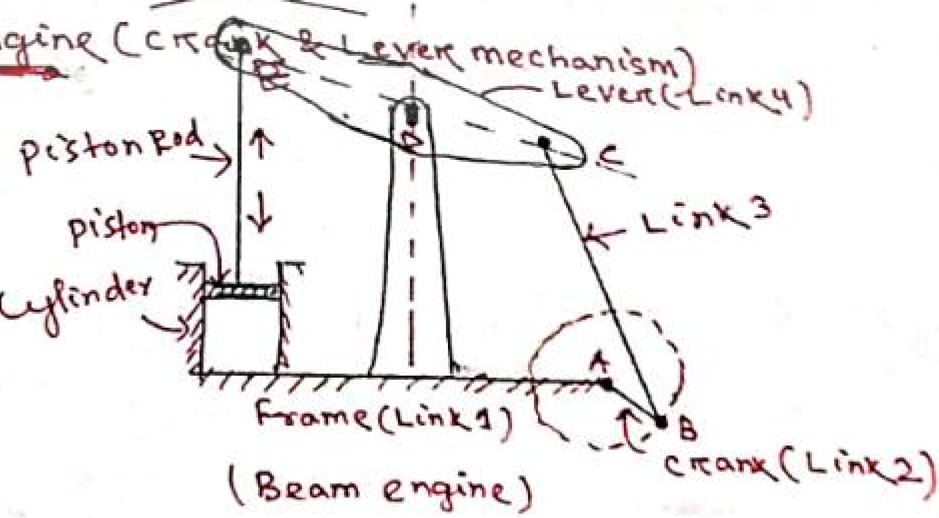
According to Grashof's law for four bar mechanism,

the sum of shortest & longest link lengths should not be greater than the sum of the remaining two link lengths if there is continuos relative motion between them.

Inversion of Four bar chain

(i) Beam Engine (Crank & Lever mechanism)

used in pump to water

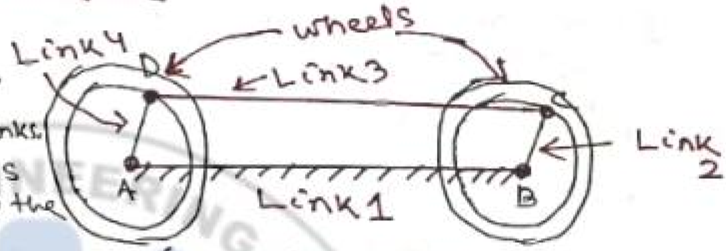


(Beam engine)

- It is also known as crank lever mechanism.
- When crank rotates about the fixed centre 'A', the lever oscillates about a fixed centre 'D'.
- The end E of the lever CDE is connected to the piston rod which reciprocates due to the rotation of crank.
- The purpose of this mechanism is to convert rotary motion into reciprocating motion.

(ii) Coupling rod of a Locomotive

- It is also known as double crank mechanism.
- It is consist of four links. The links AD & BC act as crank & are connected to the respective wheels

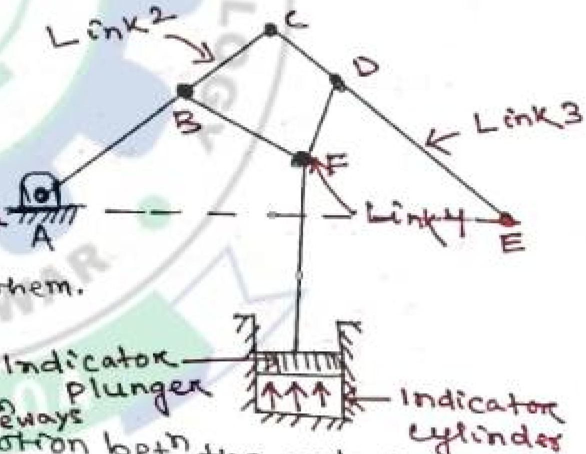


(Coupling rod of Locomotive)

- The link AB is fixed & the link CD act as a coupling rod.
- The purpose of this mechanism is to convert rotary motion of one wheel to the other wheel.

(iii) Watt's indicator mechanism

- It is also known as Watt's straight line mechanism or double lever mechanism.
- The four links are:-
  - Fixed link A
  - Link AC
  - Link CE
  - & Link BFD
- BF & FD form one link because these two parts have no relative motion between them.



Uses :- Rear axle of some car suspensions

~~It provide relative motion~~

* It prevent relative sideways motion betn the axle & body of the car.

Objective Type

(1) In a reciprocating steam engine, which of the following form a kinematic chain?

(a) Cylinder & piston
(b) Piston & connecting rod
(c) Crankshaft & flywheel
(d) Flywheel & engine frame

Ans:- c

(2) The motion of the piston in the cylinder of a steam engine is an example of :-

(a) Completly constrained motion
(b) Incompletly constrained motion
(c) Successfully constrained motion
(d) None of these

Ans:- a

(3) The motion transmitted between the teeth of gears in mesh is
(a) sliding (c) may be rolling or sliding depends upon the type of teeth
(b) Rolling (d) partly sliding & partly rolling

Ans:- d

(4) The Cam & follower without a spring forms a
(a) Lower pair (c) Self Closed pair
(b) Higher pair (d) Force closed pair
Ans - C

(5) A ball & socket joint forms a
(a) Turning pair (c) Sliding pair
(b) Rolling pair (d) Spherical pair
Ans:- d

(6) The lead screw of a Lathe with nut forms a
(a) sliding pair (c) Screw pair
(b) Rolling pair (d) Turning pair
Ans:- C

(7) When the elements of the pair are kept in contact by the action of external forces, the pair is said to be ___
(a) Lower pair (c) Self Closed pair
(b) Higher pair (d) Force Closed pair
Ans:- d

(8) Which of the following is a turning pair?
(a) piston & cylinder of a reciprocating steam engine
(b) Shaft with collars at both ends fitted in a circular hole.
(c) Lead screw of a lathe with nut.
(d) Ball & socket joint.
Ans:- b

(9) The relation between the number of pairs (P) forming a kinematic chain & the number of links (l) is
(a) $l = 2P-2$ (c) $l = 2P-4$
(b) $l = 2P-3$ (d) $l = 2P-5$
Ans:- c

(10) The relation betn the number of links (l) & the numbers of binary joints (j) for a kinematic chain having constrained motion is given by $j = \frac{3}{2}l - 2$. If the left hand side of this equation is greater than the right hand side, the chain is ___
(a) Locked chain (c) Successfully constrained chain
(b) Completly constrained chain (d) Incompletly constrained chain
Ans:- a

(11) In a kinematic chain, a quaternary joint is equivalent to ___
(a) One binary joint (c) Three binary joints
(b) Two binary joints (d) Four binary joints
Ans:- C

(12) If n links are connected at the same joint, the joint is equivalent to ___
(a) (n-1) binary joints (c) (2n-1) binary joints
(b) (n-2) binary joints (d) None of these
Ans:- a

(13) In a 4-bar linkage, if the lengths of the shortest, longest & other two links are denoted by s, l, p & q, then it would result in Grashof's linkage provided that ____

(a) $l+p < s+q$ (c) $l+p = s+q$

(b) $l+s < p+q$ (d) None of the above

Ans:- b

(14) The mechanism forms a structure, when the number of degrees of freedom(n) is equal to ____

(a) 0 (c) 2

(b) 1 (d) -1

Ans:- a

(15) In a four bar chain or quadric cycle chain ___

(a) Each of four pair is turning pair

(b) one is turning pair & three are sliding pair

(c) Three are turning pairs & one is sliding pair

(d) Each of four pair is sliding pair.

Ans:- a

(16) Which of the following is an inversion of single slider crank chain?

(a) Beam engine (c) Elliptical trammels

(b) Watt's indicator mechanism (d) Whitworth quick return motion mechanism.

Ans:- d

(17) which of the following is an inversion of double slider crank chain?

(a) coupling rod of a locomotive (c) Elliptical trammels

(b) Pendulum pump (d) Oscillating cylinder engine

Ans:- c

(18) In reciprocating engine, which of the following restraining body does not exist?

(a) connecting rod (c) Slider

(b) crank (d) Lever

Ans:- d

(19) A kinematic pair consist of

(a) Two links (c) Four links

(b) Three links (d) Any number of links

Ans:- a

(20) A kinematic pair cannot be classified according to

(a) Nature of contact betn the links.

(b) Type of relative motion betn links

(c) Nature of mechanical constraints betn the links.

(d) Number of links connected.

Ans:- d

(21) which of the following forms a higher pair?

(a) Sliding pair (c) Rolling pair

(b) Turning pair (d) None of these

Ans:- c

(22) A lower pair has

(a) Surface contact
(b) Line contact
(c) point contact
(d) All of the above

Ans:- a

(23) A rigid body in space has ___ degrees of freedom.

(a) Two
(b) Three
(c) six
(d) Eight

Ans:- c

——— x ———

Cams & Followers

- A cam is a rotating machine element which gives reciprocating or oscillating motion to another element known as follower.
- It have a line contact & constitute a higher pair.
- The cams are usually rotated at uniform speed by a shaft & the follower motion is predetermined, according to the shape of the cam.
- The cams are widely used for operating inlet & exhaust valves of I.C engines, paper cutting machines, Feed mechanism of automatic Lathe, etc.

Friction

↳ The opposing force, which acts in the opposite direction of the movement, is called force of friction or friction.

↳ The friction between the wheels & the road is essential for the car to move forword.

Types of Friction:-

(a) Static friction:- It is the friction experienced by a body when at rest.

(b) Dynamic friction:→ It is the friction experienced by a body, when at motion. It is also called Kinetic friction

Types

(i) Sliding friction:→ It is the friction experienced by a body, when it slides over another body.

(ii) Rolling friction:→ It is the friction experienced betn the surfaces which has balls & rollers interposed between them.

(iii) Pivot friction:→ It is the friction experienced by a body, due to the motion of rotation. (Foot step bearing)

The friction may further classified as:→

(a) Friction betn unlubricated surfaces

(b) Friction betn Lubricated surface

(a) Friction betn Unlubricated Surface

↳ The friction experienced betn two dry & unlubricated surfaces in contact, is known as solid or dry friction.

↳ It includes rolling friction & sliding friction.

(b) Friction betn Lubricated Surface

When lubricant (oil or grease) is applied betn two surface in contact, the friction are of two types:→

(i) Boundry friction:→ It is also known as greasy friction or Non-viscous friction.

↳ It is the friction experienced betn the rubbing surfaces, when the surface have a very thin layer of lubricant.

↳ It follows the laws of solid friction.

(ii) Fluid friction:→ It is the friction experienced by the rubbing surfaces, when the surface have a thick layer of lubricant.

Limiting friction:→

The maxm value of frictional force, which comes into play when a body just begins to slide over the surface of the other body, is known as limiting force of friction or limiting friction.

Laws of Static Friction

(i) The force of friction always acts in a direction, opposite to that in which the body tends to move.

(ii) The magnitude of the force of friction is equal to the force, which tends the body to move.

(iii) The magnitude of limiting friction (F) bears a constant ratio to the normal reaction (R_N).

Mathematically $\boxed{\frac{F}{R_N} = \text{constant.}}$

(iv) The force of friction is independent of the area of contact betn the two surfaces.

(v) The force of friction depends upon the roughness of the surfaces.

Laws of Dynamic or Kinetic friction :>

(i) The force of friction always acts in a direction, opposite to that in which the body moving.

(ii) The magnitude of dynamic friction bears a constant ratio to the normal reaction betn the two surfaces.

(iii) For medium speeds, the force of friction remain constant. But it decreases slightly with the increase of speed.

Laws of Solid friction

(i) The force of friction is directly proportional to the normal load betn the surfaces.

(ii) The force of friction depends upon the material of which the contact surfaces are made.

(iii) The force of friction independent of the velocity of one body relative to the other body.

Laws of Fluid Friction

(i) The force of friction is almost independent of the load.

(ii) The force of friction reduces with the increase of the temp. of the lubricant.

(iii) The force of friction is different for different lubricant.

Coefficient of friction (μ)

It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is denoted by μ.

Mathematically $\boxed{\mu = \frac{F}{R_N}}$

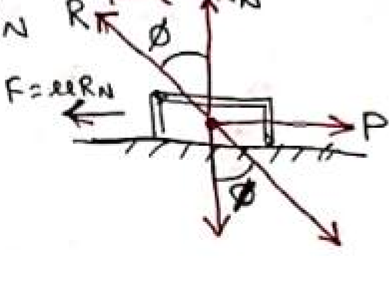
Limiting Angle of Friction or Friction Angle (ϕ)

It may be defined as the angle which the resultant reaction (R) makes with the normal reaction (R_N).

$$\tan\phi = \frac{F}{R_N} \qquad \because F = \mu R_N$$

$$\Rightarrow \tan\phi = \frac{\mu R_N}{R_N}$$

$$\Rightarrow \boxed{\tan\phi = \mu}$$

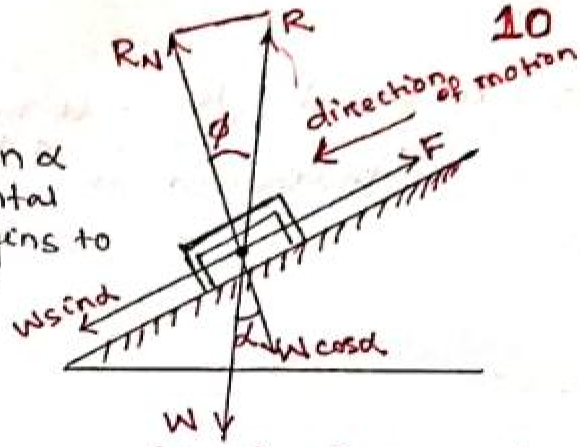


Angle of Repose(α)

If the angle of inclination α of the plane to the horizontal is such that the body begins to move down the plane, then the angle α is called the angle of repose.

$\tan\alpha = \mu = \tan\phi$

$\Rightarrow \boxed{\alpha = \phi}$



(Angle of Repose)

Screw Friction

↳ The various temporary fastening screws, nuts, bolts etc. have screw threads, which are made by cutting a continuos helical groove on the cylindrical surface.

↳ If the threads cut on the outer surface of the solid rod, it is known as external threads. If the threads cut in the internal surface of hollow rod, then it is known as interenal threads.

↳ The screw threads are of two types:-

(a) V-threads (b) screw threads

(i) Strongere & more frictional resistant to motion than screw threads.
EXP:- Bolts & nuts (Tightening purpose)

(ii) Square ~~the~~ threads are used in screw jack, vice screws etc.

Important terms in screw

(a) **Helix**:→ It is the curve traced by a particle while moving along a screw thread.

(b) **Pitch**:- It is the distance from a point of a screw to a corresponding point on the next thread, measured ||al to the axis of the screw.

(c) **Lead**:- It is the distance, a screw thread advances axially in one turn.

(d) **Depth of thread**:- It is the distance betⁿ the top & bottom surfaces of the thread. It is also known as root or crest of a thread.

(e) **Single-threaded screw**:- If the lead of a screw is equal to its pitch, it is known as single-threaded screw.

(f) **Multi-threaded screw**:- If more than one thread is cut in one lead distance of a screw, it is known as multi-threaded screw.

Mathematically

Lead = Pitch × Number of threads

(9) **Helix Angle** :- It is the slope or inclination of the thread with the horizontal.

Mathematically

$$\tan\alpha = \frac{\text{Lead of screw}}{\text{circumference of screw}}$$

$$= \frac{p}{\pi d} \quad \text{(in single-threaded screw)}$$

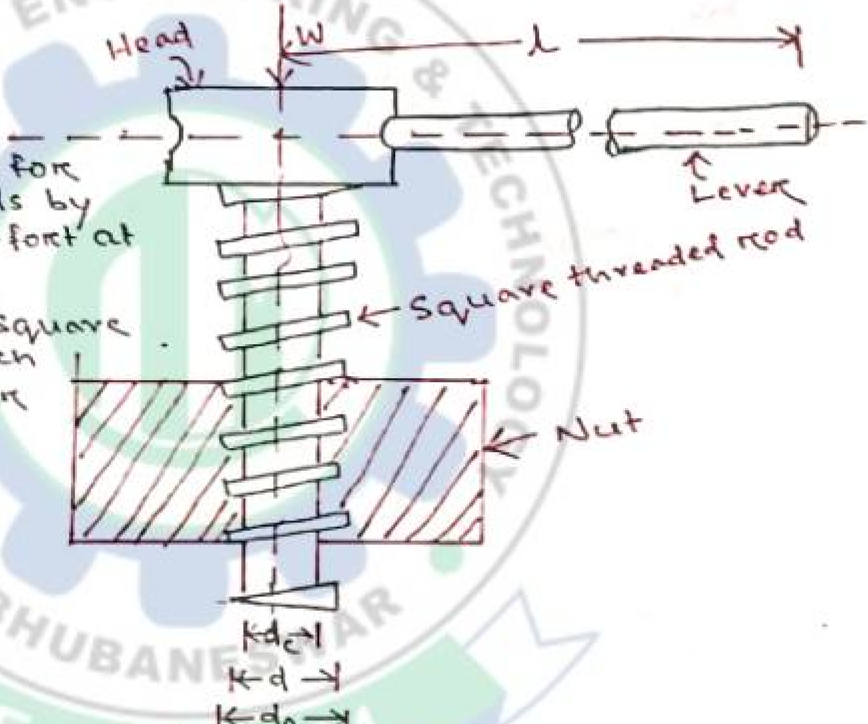
$$= \frac{n.p}{\pi d} \quad \text{(In muti-threaded screw)}$$

α = Helix angle
p = pitch of screw
d = mean diameter of screw
n = Number of thread in one lead

Screw Jack

It is a device used for lifting heavy loads by applying smaller effort at its handle.

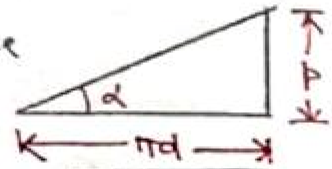
It is consist of a square threaded rod which is fits into the inner threads of nut



(Screw Jack)

Torque required to Lift the load by Screw jack

Let p = pitch of the screw
d = mean dia. of screw
α = Helix angle
P = Effort applied at the circumference to lift the load.
W = Load to be lifted
μ = coefficient of friction = $\tan\phi$
ϕ = friction angle



$$\tan\alpha = \frac{p}{\pi d}$$

* Effort or force required at the circumference of the screw

$$P = W \tan(\alpha+\phi) = W\left[\frac{\tan\alpha + \tan\phi}{1 - \tan\alpha \cdot \tan\phi}\right]$$

* Torque required to overcome friction betn Screw & nut

$$T_1 = P \times \frac{d}{2} = W \tan(\alpha + \phi) \times \frac{d}{2}$$

* Torque required to overcome friction at the collar

$$T_2 = \mu_1 W R$$

R_1 = outside radius of collar
R_2 = Inside radius of collar
R = mean radius of collar $= \frac{R_1 + R_2}{2}$
μ_1 = coefficient of friction for collar

* Total torque required to overcome friction

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1 W R$$

* If an effort P_1 is applied at the end of the lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever i.e

$$T = P \times \frac{d}{2} = P_1 \times l$$

l = Length of lever
P_1 = Effort at lever

Note

* mean diameter $d = \frac{d_0 + d_c}{2}$
$= d_0 - \frac{p}{2}$
$= d_c + \frac{p}{2}$

d_c = core or internal diameter
d_0 = outer or Nominal diameter
d = mean diameter

* Mechanical Advantage
It is the ratio of the load lifted (W) & effort applied (P_1).

$$M.A = \frac{W}{P_1}$$

* power $(P) = T \cdot \omega$

ω = Angular speed $= \frac{2\pi N}{60}$ rads/sec.
N = Speed in r.pm $= \frac{\text{Speed of the Nut}}{\text{Pitch of the Screw}}$

* Load (on valve) W = pressure × Area
$W = P_s \times \frac{\pi}{4} \times D^2$ D = dia. of valve

* Torque applied to the handwheel

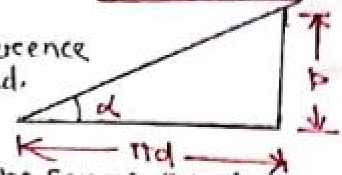
$$T = 2P_1 \times \frac{D_1}{2}$$

D_1 = diameter of handle

Torque required to Lower the load in Screw jack

Let p = Pitch of the screw
d = mean dia. of screw
α = Helix angle
P = Effort applied at the circumference of the screw to lower the load.
W = weight to be lowered
μ = coefficient of friction betn the screw & nut $= \tan\phi$

$$\tan\alpha = \frac{p}{\pi d}$$



Effort or Force required at the circumference of the screw

$$P = W\tan(\phi - \alpha) = W\left[\frac{\tan\phi - \tan\alpha}{1 - \tan\phi \cdot \tan\alpha}\right]$$

Torque required to overcome friction between the screw & nut

$$T = P \times \frac{d}{2} = W\tan(\phi - \alpha)\frac{d}{2}$$

Efficiency of a Screw jack

It is defined as the ratio between the ideal effort (P_0) to the actual effort (P).

Ideal effort :→ The effort required to move the load, neglecting friction.

Actual effort :→ The effort required to move the load considering friction.

mathematically

$$\eta = \frac{P_0}{P} = \frac{W\tan\alpha}{W\tan(\alpha + \phi)} = \frac{\tan\alpha}{\tan(\alpha + \phi)}$$

* If Screw friction & collar friction is taken into account, then

$$\eta = \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1 W R}$$

T_0 = Torque required to move the load, neglecting friction
T = Torque required to move the load in screw & collar friction.

* It also be defined as the ratio of mechanical advantage to the velocity ratio.

$$\eta = \frac{M.A}{V.R}$$

$$V.R = \frac{2l}{\tan\alpha \times d}$$

$$M.A = \frac{W}{P_1}$$

Max^m efficiency of a Screw jack

$$\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}$$

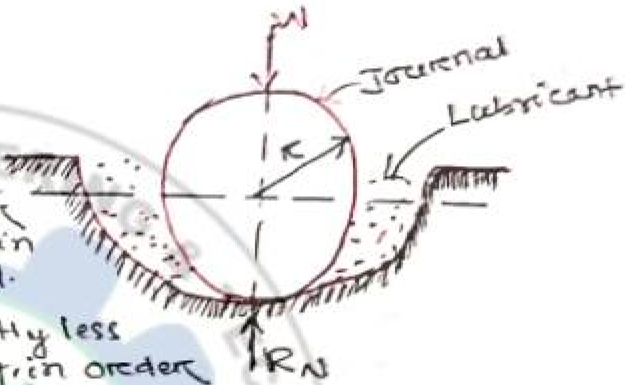
Note

* If $\eta < 50\%$ (Self-locking screw)
* If $\eta > 50\%$ (Overhauling screw)

Journal Bearing

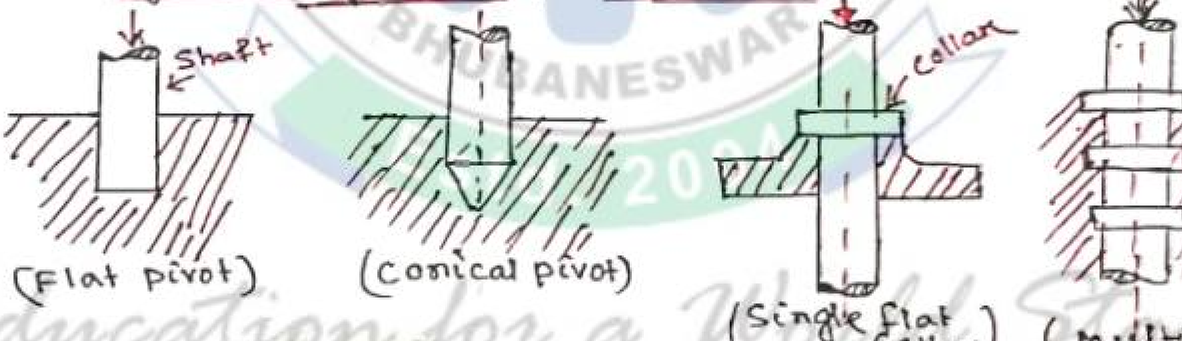
It forms a turning pair. The fixed outer element of a turning pair is called bearing. The portion of inner element (i.e shaft) which fits in the bearing is called journal.

The journal is slightly less in diameter than the bearing, in order to permit free movement of the journal in a bearing.



Friction of pivot & Collar Bearing

- The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivots may have a flat surface or conical surface.
- The collar may have flat or conical bearing surface, but the flat surface is most commonly used. The flat collar may be single collar & multi-collar.



(Flat pivot) (Conical pivot) (Single flat Collar) (Multiple flat Collar)

Assumption

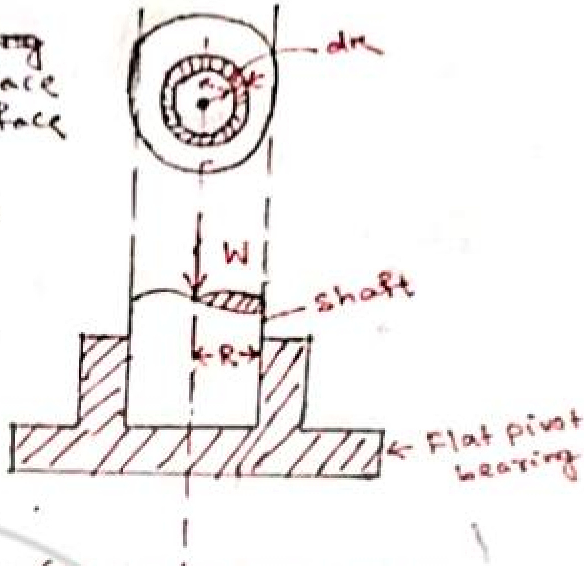
(i) The pressure is uniformly distributed throughout the bearing surface.
(ii) The wear is uniform throughout the bearing surface.

Flat pivot Bearing

When vertical shaft rotates in a flat pivot bearing the sliding friction will be along the surface of contact between the shaft & the bearing. It is also known as foot step bearing.

Let W = Load transmitted ~~along~~ over the bearing surface
R = Radius of bearing surface
p = Intensity of pressure
μ = co-efficient of friction

Considering two cases:→
(a) Uniform pressure case
(b) Uniform wear case



(Flat pivot or foot step bearing)

(a) Uniform pressure case

Intensity of pressure $p = \frac{W}{\pi R^2}$

Consider a ring of radius r & thickness of bearing surface dr.

∴ Area of the bearing surface $A = 2\pi r . dr$

Load transmitted to the ring $\delta W = p \times A$
$= p \times 2\pi r dr$

Frictional resistance or force $F_r = \mu \, \delta W$
$= 2\pi \mu p r . dr$

Frictional torque $T_r = F_r \times r$
$= 2\pi \mu p r . dr \times r$
$= 2\pi \mu p r^2 . dr$

Integrating above eqn within the limit 0 to R.

Total frictional torque $T = \int_0^R 2\pi \mu p r^2 . dr$
$= 2\pi \mu p \int_0^R r^2 . dr$
$= 2\pi \mu p \left[\frac{r^3}{3}\right]_0^R$
$= 2\pi \mu p \times \frac{R^3}{3}$
$= \frac{2}{3} \times \pi \mu p . R^3$
$= \frac{2}{3} \times \pi \mu \times \frac{W}{\pi R^2} \times R^3$

$$\boxed{T = \frac{2}{3} \times \mu W R}$$

(b) Considering uniform wear case

$p . r = C$ (constant)

$\Rightarrow p = \frac{C}{r}$

Area of the bearing surface $A = 2\pi r . dr$

Load transmitted to the ring $\delta W = p \times 2\pi r . dr$

$= \frac{C}{r} \times 2\pi r . dr$

$= 2\pi C . dr$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi C . dr = 2\pi C [r]_0^R = 2\pi C R$$

$$\Rightarrow C = \frac{W}{2\pi R}$$

Frictional torque acting on the ring

$T_r = 2\pi \mu p r^2 . dr$

$= 2\pi \mu \times \frac{C}{r} \times r^2 dr \quad \because p = \frac{C}{R}$

$= 2\pi \mu C r . dr$

Total frictional torque on bearing =

$$T = \int_0^R 2\pi \mu C r\, dr = 2\pi \mu C \int_0^R r . dr$$

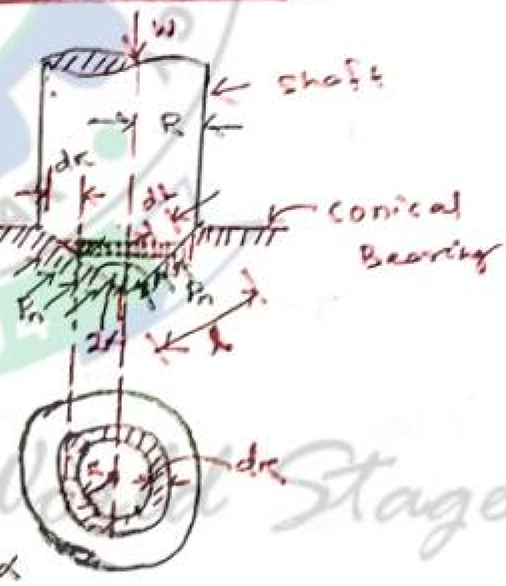
$$= 2\pi \mu C \left[\frac{r^2}{2}\right]_0^R = 2\pi \mu C . \frac{R^2}{2}$$

$$= \pi \mu C R^2$$

$$= \pi \mu \times \frac{W}{2\pi R} \times R^2$$

$$\boxed{T = \frac{1}{2} \times \mu W R}$$

Conical Pivot Bearing



Let p_n = Intensity of pressure $= \frac{W}{\pi R^2}$ $\left(\frac{\text{Load}}{\text{Area}}\right)$

α = Semi angle of cone

μ = coefficient of friction betn shaft & the bearing

R = Radius of shaft

Consider a small ring of radius r & the thickness dr. Let dl is the length of the ring.

$dl = dr \operatorname{cosec}\alpha$

Area of the ring $A = 2\pi r dl$

$= 2\pi r dr \operatorname{cosec}\alpha$

(i) Consider uniform pressure :-

Load acting on ring $(W_n) = p_n \times 2\pi r dr \operatorname{cosec}\alpha$

Frictional force or Resistance on ring

$F_r = \mu . W_n = \mu p_n 2\pi r dr \operatorname{cosec}\alpha$

$= 2\pi \mu p_n \operatorname{cosec}\alpha . r . dr$

Frictional torque $T_r = F_r \times r = 2\pi \mu p_n \operatorname{cosec}\alpha\, r . dr \times r$

$= 2\pi \mu p_n \operatorname{cosec}\alpha\, r^2 dr$

Total Torque $T = \int_0^R 2\pi \mu p_n \operatorname{cosec}\alpha\, r^2 \cdot dr$

$= 2\pi \mu p_n \operatorname{cosec}\alpha \int_0^R r^2 \cdot dr$

$= 2\pi \mu p_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_0^R$

$= 2\pi \mu p_n \operatorname{cosec}\alpha \frac{R^3}{3}$

Substituting the value of $p_n = \frac{W}{\pi R^2}$

$= 2\pi\mu \times \frac{W}{\pi R^2} \times \operatorname{cosec}\alpha \frac{R^3}{3}$

$$T = \frac{2}{3} \mu W R \operatorname{cosec}\alpha$$

Considering uniform wear

p_r = Normal intensity of pressure

$p_r \cdot r = C$ (constant)

$\Rightarrow p_r = \frac{C}{r}$

Load transmitted to the ring

$\delta W = p_r \times 2\pi r \cdot dr$

$= \frac{C}{r} \times 2\pi r \cdot dr$

$= 2\pi C \cdot dr$

Total load transmitted to the ring

$W = \int_0^R 2\pi C \cdot dr = 2\pi C \int_0^R dr$

$= 2\pi C [r]_0^R = 2\pi C R$

$\Rightarrow C = \frac{W}{2\pi R}$

Frictional torque acting on ring

$T_r = 2\pi \mu p_r \cdot \operatorname{cosec}\alpha\, r^2 \cdot dr$

$= 2\pi\mu \times \frac{C}{r} \times \operatorname{cosec}\alpha\, r^2 \cdot dr$

$= 2\pi\mu C \cdot \operatorname{cosec}\alpha \cdot r \cdot dr$

Total frictional torque acting on bearing

$T = \int_0^R 2\pi\mu C \cdot \operatorname{cosec}\alpha \cdot r \cdot dr$

$= 2\pi\mu C \cdot \operatorname{cosec}\alpha \int_0^R r \cdot dr$

$= 2\pi\mu C \cdot \operatorname{cosec}\alpha \left[\frac{r^2}{2}\right]_0^R$

$= 2\pi\mu C \cdot \operatorname{cosec}\alpha \cdot \frac{R^2}{2}$

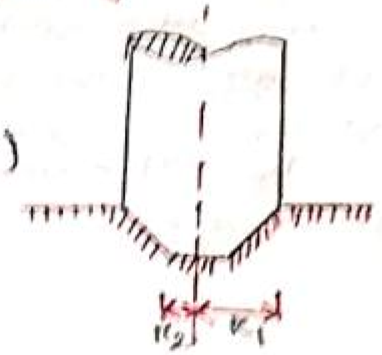
$= \pi\mu C \cdot \operatorname{cosec}\alpha \cdot R^2$

Substituting the value of C in above eqn

$T = \pi\mu \times \frac{W}{2\pi R} \times \operatorname{cosec}\alpha \cdot R^2$

$$T = \frac{1}{2} \times \mu W R \cdot \operatorname{cosec}\alpha$$

Trapezoidal conical pivot Bearing



r_1 = External radius
r_2 = Internal radius

Area of bearing surface
$$A = \pi (r_1^2 - r_2^2)$$

Intensity of pressure
$$P_n = \frac{W}{A} = \frac{W}{\pi(r_1^2 - r_2^2)}$$

(i) Considering uniform pressure

Total torque transmitted on bearing
$$T = 2\pi\mu P_n \operatorname{cosec}\alpha \, r^2 \cdot dr = \int_{r_2}^{r_1} 2\pi\mu P_n \operatorname{cosec}\alpha \, r^2 \cdot dr$$
$$= 2\pi\mu P_n \operatorname{cosec}\alpha \int_{r_2}^{r_1} r^2 \cdot dr = 2\pi\mu P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$
$$= 2\pi\mu P_n \operatorname{cosec}\alpha \left(\frac{r_1^3 - r_2^3}{3}\right)$$

Substituting the value of P_n,
$$= 2\pi\mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \operatorname{cosec}\alpha \times \left(\frac{r_1^3 - r_2^3}{3}\right)$$
$$\boxed{T = \frac{2}{3}\mu W \operatorname{cosec}\alpha \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)}$$

(ii) Considering uniform wear

Load transmitted to ring $\delta W = 2\pi c \cdot dr$

Total load transmitted $W = \int_{r_2}^{r_1} 2\pi c \cdot dr = 2\pi c \cdot [r]_{r_2}^{r_1}$
$$\Rightarrow W = 2\pi c (r_1 - r_2)$$
$$\Rightarrow c = \frac{W}{2\pi(r_1 - r_2)}$$

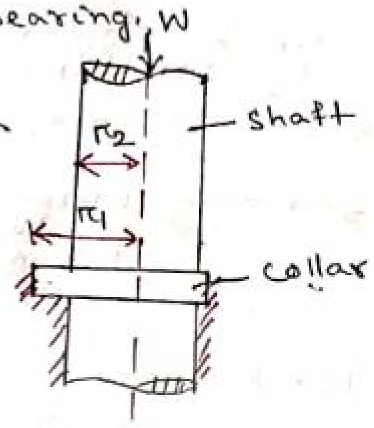
Torque acting on the ring considering uniform wear case is
$$T = 2\pi\mu \cdot c \operatorname{cosec}\alpha \, r \cdot dr$$

Total torque acting on the bearing
$$T = \int_{r_2}^{r_1} 2\pi\mu c \operatorname{cosec}\alpha \, r \cdot dr = 2\pi\mu c \operatorname{cosec}\alpha \int_{r_2}^{r_1} r \cdot dr$$
$$= 2\pi\mu c \operatorname{cosec}\alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$
$$= 2\pi\mu c \cdot \operatorname{cosec}\alpha \left(\frac{r_1^2 - r_2^2}{2}\right)$$
$$= \pi\mu c \operatorname{cosec}\alpha (r_1^2 - r_2^2)$$

Substituting the value of c in above
$$= \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \operatorname{cosec}\alpha (r_1^2 - r_2^2)$$
$$= \frac{1}{2}\mu W (r_1 + r_2) \operatorname{cosec}\alpha$$
$$\boxed{T = \mu W R \operatorname{cosec}\alpha} \quad \left\{ \because R = \frac{r_1 + r_2}{2} \right.$$
= Mean radius of Bearing.

Flat Collar Bearing :→

It is also known as thrust bearing.



Consider a flat collar bearing

Let r_1 = External radius of collar

r_2 = Internal radius of collar

Area of bearing surface

$$A = \pi(r_1^2 - r_2^2)$$

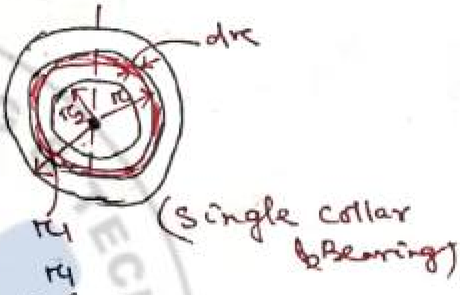
(i) Considering uniform pressure

Intensity of pressure $p = \frac{W}{A}$

$$= \frac{W}{\pi(r_1^2 - r_2^2)}$$

The frictional torque on the ring of radius r & thickness dr.

$$T_r = 2\pi\mu \cdot p \cdot r^2\, dr$$



Integrating this eqn within the limit r_2 to r_1

Total frictional torque

$$T_r = \int_{r_2}^{r_1} 2\pi\mu p r^2\, dr = 2\pi\mu p \int_{r_2}^{r_1} r^2\, dr$$

$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p \left(\frac{r_1^3 - r_2^3}{3}\right)$$

Substituting the value of p in above eqn

$$= 2\pi\mu \times \frac{W}{\pi(r_1^2 - r_2^2)} \times \frac{r_1^3 - r_2^3}{3}$$

$$\boxed{T = \frac{2}{3} \times \mu W \times \left[\frac{(r_1^3 - r_2^3)}{(r_1^2 - r_2^2)}\right]}$$

(ii) Considering uniform wear

Load transmitted on ring $\delta W = p_r \cdot 2\pi r\, dr$

$$= \frac{C}{r} \times 2\pi r\, dr$$

$$= 2\pi C\, dr$$

Total load transmitted to collar

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C \left[r\right]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$\Rightarrow C = \frac{W}{2\pi(r_1 - r_2)}$$

Frictional torque acting on ring

$$T_r = 2\pi\mu C r \cdot dr$$

$$\text{Total frictional torque } T = \int_{r_2}^{r_1} 2\pi\mu C r \cdot dr = 2\pi\mu C \int_{r_2}^{r_1} r \cdot dr$$

$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu C\left(\frac{r_1^2 - r_2^2}{2}\right)$$

$$= \pi\mu C(r_1^2 - r_2^2)$$

Substituting the value of C in above eqn

$$T = \pi \mu \times \frac{W}{2\pi (r_1 - r_2)} \times (r_1^2 - r_2^2)$$

$$\boxed{T = \frac{1}{2} \mu W (r_1 + r_2)}$$

Note

* Power absorbed in friction $\boxed{P = T \cdot \omega}$

ω = Angular speed $= \frac{2\pi N}{60}$ rad/s.

Multi-collar Bearing:→

(i) Number of collars (n)

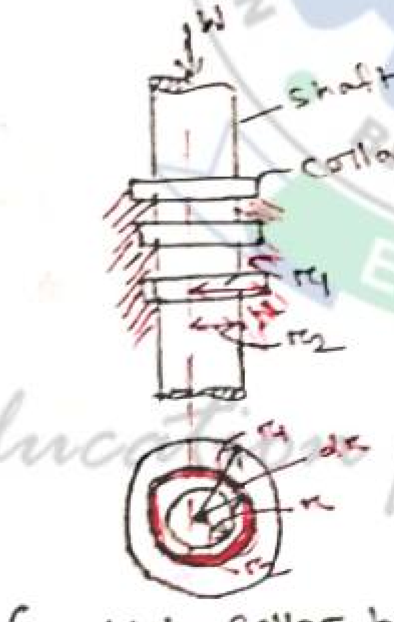
Intensity of pressure $\boxed{p = \frac{W}{n \cdot \pi (r_1^2 - r_2^2)}}$

(ii) Torque transmitted

$n = ?$

$$\boxed{T = \frac{2}{3} \mu W \left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2} \right)} \text{ (For uniform pressure)}$$

$$\boxed{T = \frac{1}{2} \mu W (r_1 + r_2)} \text{ (For uniform wear)}$$



(multiple collar bearing)

Friction clutch

Clutch:- It is a mechanical device used to connect or disconnect driven shaft from the driving shaft

↳ clutches are usefull in devices that have two rotating shafts

Friction clutch:- In this type of clutches, friction force is used to engage & disengage the clutch

↳ When the driver release the clutch pedal, the driven member & driving member of clutch, comes in contact with each other. A friction force works betⁿ these two parts.

↳ In this clutch, power transmission is achieved by means of friction betⁿ the contacting surfaces.

Types of friction clutch

(a) Disc or plate clutch → single disc clutch
→ multiple disc clutch

(b) cone clutch

(c) centrifugal clutch

Single disc or plate clutch

↳ Single plate clutch consist of a clutch plate whose both sides are coated with a friction material.

↳ It is also called dry clutch, because no lubricant is used as coolant.

↳ These surface have high co-efficient of friction.

Let, T = Torque transmitted by the clutch
p = Intensity of axial pressure
r_1 = External radius of friction surface
r_2 = Internal radius of friction surface
μ = coefficient of friction.

Considering uniform pressure

* Intensity of pressure / Average pressure

$$p \text{ or } p_{av} = \frac{W}{\pi(r_1^2 - r_2)^2}$$

* Total frictional torque (T)

$$T = n \mu W R$$

n = No. of pairs of friction or contact surface

$n = 2$

$$T = \frac{2}{3} \times \mu W \times \left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]$$

$$= \mu W R$$

R = mean radius of friction surface

$$R = \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$$

Considering uniform wear

* Axial thrust $W = 2\pi C(r_1 - r_2)$

* Total torque on friction surface (T)

$$T = n\mu W R$$

$n = 2$ bcz Both sides are effective

$$T = \frac{1}{2} \times \mu W (r_1 + r_2)$$

R = mean radius

$$R = \frac{r_1 + r_2}{2}$$

* The intensity of pressure is max^m at the inner radius r_2

$$p_{max} \times r_2 = C \Rightarrow p_{max} = \frac{C}{r_2}$$

* The intensity of pressure is min^m at the outer radius r_1.

$$p_{min} \times r_1 = C \Rightarrow p_{min} = \frac{C}{r_1}$$

Multiple Disc Clutch

- It is consist of more than three discs or plates, so that it is able to provide more torque.
- The inside discs are fastened to the driven shaft. The outside disc (Steel) (Bronze) are held by bolts & are fastened to the housing, which are keyed to the driving shaft.
- It is extensivly used in motor cars, machine tools etc.

n_1 = number of discs on driving shaft
n_2 = number of discs on driven shaft

* ∴ Number of pair of contact surface $n = n_1 + n_2 - 1$

* Total frictional torque $T = n\mu W R$

R = mean radius of friction surfaces

$$= \frac{2}{3}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right) \text{ (For uniform pressure)}$$

$$= \frac{r_1 + r_2}{2} \text{ (For uniform wear.)}$$

Objective Type

(1) The angle of inclination of the plane, at which the body begins to move down the plane, is called –

(a) Angle of friction (c) Angle of projection

(b) Angle of repose (d) None of the above

Ans:– a

(2) In a screw jack, the effort required to lift the load W is given by –

(a) $P = W\tan(\alpha-\phi)$ (c) $P = W\cos(\alpha-\phi)$

(b) $P = W\tan(\alpha+\phi)$ (d) $P = W\cos(\alpha+\phi)$

Ans:– b

(3) The efficiency of screw jack is given by

(a) $\frac{\tan(\alpha+\phi)}{\tan\alpha}$ (c) $\frac{\tan(\alpha-\phi)}{\tan\alpha}$

(b) $\frac{\tan\alpha}{\tan(\alpha+\phi)}$ (d) $\frac{\tan\alpha}{\tan(\alpha-\phi)}$

Ans:– b

(4) The radius of friction circle for a shaft of radius r rotating inside a bearing is

(a) $r\sin\phi$ (c) $r\tan\phi$

(b) $r\cos\phi$ (d) $r\cot\phi$

Ans:– a

(5) The efficiency of screw jack is maximum, when

(a) $\alpha = 45^\circ + \frac{\phi}{2}$ (c) $\alpha = 90^\circ + \phi$

(b) $\alpha = 45 - \frac{\phi}{2}$ (d) $\alpha = 90^\circ - \phi$

Ans:– b

(6) The max^m efficiency of a screw jack is

(a) $\frac{1-\sin\phi}{1+\sin\phi}$ (c) $\frac{1-\tan\phi}{1+\tan\phi}$

(b) $\frac{1+\sin\phi}{1-\sin\phi}$ (d) $\frac{1+\tan\phi}{1-\tan\phi}$

Ans:– a

(7) The frictional torque transmitted by a disc or plate clutch is same as that of

(a) Flat pivot bearing (c) conical pivot bearing

(b) Flat collar bearing (d) Trapezoidal pivot bearing

Ans:– b

(8) The frictional torque transmitted by a cone clutch is same as that of –

(a) Flat pivot bearing (c) Conical pivot bearing

(b) Flat collar bearing (d) Trapezoidal pivot bearing

Ans:– d

(9) The force of friction depends upon

(a) Nature of surface of contact

(b) material of objects in contact

(c) Both a & b

(d) None of the above

Ans:– C

(10) The body will move only when
(a) Force of friction = Applied force
(b) Force of friction < applied force
(c) Force of friction > applied force
(d) All of above
Ans:- b

(11) The force of friction (F) is equal to
(a) $\mu R/2$ (c) $2\mu R$
(b) μR (d) $\mu R/3$
Ans:- b

(12) When the two surfaces in contact have a thick layer of lubricant in between them, it is known as —
(a) Solid friction (c) Greasy friction
(b) Rolling friction (d) Film friction
Ans:- d

(13) ____ friction is the value of the limiting friction just before slipping occurs.
(a) Dynamic (c) Sliding
(b) Static (d) Rolling
Ans:- b

(14) ____ friction is the value of frictional force after slipping has occured.
(a) Dynamic (c) Sliding
(b) Static (d) Rolling
Ans:- a

(15) Coefficient of rolling friction is ____ than coefficient of sliding friction.
(a) Greater (c) Lesser
(b) Equal to (d) None of the above
Ans:- c

(16) Co-efficient of dynamic friction is ___ than static friction
(a) Greater (c) Lesser
(b) Equal to (d) None of the above
Ans:- c

(17) The coefficient of ~~friction~~ sliding friction for rubber on concrete is ___
(a) 0.030 (c) 0.18
(b) 0.70 (d) 0.004
Ans:- b

(18) The co-efficient of sliding friction for steel is —
(a) 0.030 (c) 0.18
(b) 0.70 (d) 0.004
Ans - c

(19) Which theory is used to determine safe design of bearings?
(a) Uniform wear theory
(b) Uniform pressure theory
(c) Both a & b
(d) None of the above
Ans:- b

(20) Which clutch is known as Wet Clutch?
(a) single plate clutch (c) Both 'a' & 'b'
(b) multi plate clutch (d) None of the above
Ans:- b

(21) How many jacks are there which can be used in screw jack?
(a) 2 (b) 3 (c) 4 (d) 5
Ans:- a

(22) Which of the following does not form the important part of the screw jack?
(a) Frame (b) Nut (c) Cup (d) Coupling
Ans:- d

(23) The transverse shear stress at the root of the threads in the nut can be given by? (Symbols have their usual meaning, Z = Number of threads in nut)
(a) $4W/\pi d z^2$ (b) $W/\pi d t z$ (c) $W/\pi d t z^2$ (d) None of these
Ans:- b

(24) Which type of friction in cup design is recommended for the set screw?
(a) Sliding (b) Rolling (c) Static (d) None of these
Ans:- b

(25) In which case is the wear more?
(a) Conventional power screw
(b) Recirculating power screw
(c) Equal in both the cases
(d) Can not be determined
Ans:- a

(26) Eddy current clutch is a type of friction clutch,
(a) Yes (c) It is a mechanical clutch
(b) No, its a electromagnetic clutch (d) Non of the listed
Ans:- b

(27) If number of contact surfaces are 5, then number of disks required in multi disk clutch are?
(a) 4 (b) 5 (c) 6 (d) can't be determined
Ans:- c (Disks = Contacting surfaces + 1)

(28) multi disk clutches are:-
(a) Plasma clutches (c) Depends upon the lubrication used
(b) Wet clutches (b) None of these
Ans:- b

(29) The coefficient of friction in multi disk plate clutch is ___
(a) High (c) medium
(b) Less (d) None of the above Ans:- b

(30) A jaw clutch is essentially a
(a) Positive clutch (b) cone clutch (c) Friction clutch (d) Disc clutch
Ans:- a

(31) The material used for lining of friction surfaces of a clutch should have ______ coefficient of friction.
(a) Low (c) Medium
(b) High (d) None of the above
Ans:- b

Belt Drives

↳ The belts are used to transmit power from one shaft to another by means of pulley.

The amount of power transmitted depends upon:-

- (i) The velocity of the belt.
- (ii) ~~Fee~~ The tension under which the belt is placed on the pulleys.
- (iii) The arc of contact betn the belt & smaller pulleys.
- (iv) The condition under which the belt is used.

* The selection of ~~motors~~ belt drives depends upon various factors are:→

(i) Speed of driving & driven shaft
(ii) Power to be transmitted.
(iii) Space available
(iv) Speed reductio ratio
(v) Centre distance betn the shafts.
~~(vi) Shaft layout~~
(vi) Shaft layout
(vii) Service condition

Types of Belt drives

(a) Light drives:→
↳ used to transmit small power.
↳ Belt speed upto 10 m/s
↳ ~~Ex~~ Uses: - Agricultural machines ~~tool~~
small machine tools

(b) medium drives:-
↳ used to transmit medium power.
↳ Belt speed over 10 m/s upto 22 m/s.
↳ Uses:→ machine tools

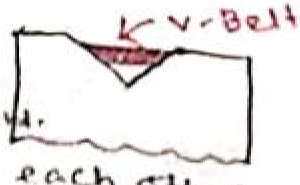
(c) Heavy drives:→
↳ used to transmit large power.
↳ Belt speed above 22 m/s.
↳ Uses:- Compressor, Generator

Types of Belt

(a) Flat belt:→

↳ mostly used in factories & workshops.
↳ medium amount of power is transmitted
↳ Two pulleys are not more than 8 metres apart.

(b) V-Belt:→



↳ mostly used in factories & workshop.
↳ Great amount of power is transmitted.
↳ Two pulleys are very near to each other.

(C) Circular belt or Rope

- Mostly used in factories & workshops.
- Great amount of power is transmitted.
- Two pulleys are more than 8 metres apart.

Material used for Belts

- The material used for belts are flexible & durable. It must have high co-efficient of friction.
- The various materials of belt are :→ Leather, Cotton, Rubber, Balata.
  - → Similar to rubber belt except balata gum is used in place of rubber.
  - → These belts are acid proof.
  - → The strength of balata belts is 25% higher than rubber belts.

Co-efficient of Friction betn Belt & Pulley (μ)

It depends upon the various factors :→

(i) The material of belt
(ii) The material of pulley
(iii) The slip of belt
(iv) The speed of belt.

According to C.G Barth, the co-efficient of friction betn the leather belt on cast iron pulley :-

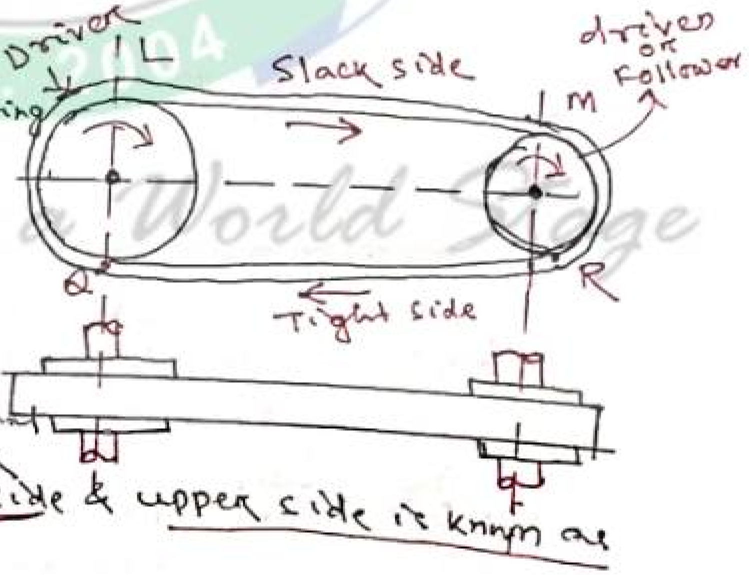
$$\mu = 0.54 - \frac{42.6}{152.6 + V}$$

V = Vel. of Belt in m/min.

Types of Flat Belt drives

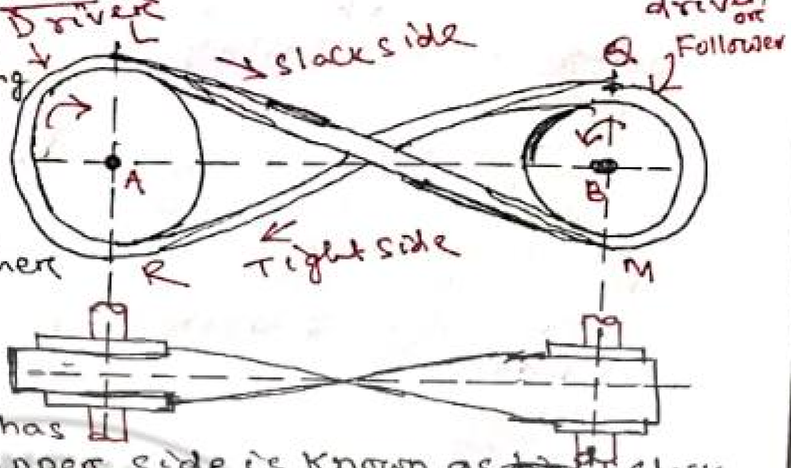
(a) Open Belt drive :→

- It is used in shafts arranged in ||al & rotating in same direction
- The driver 'A' pulls the belt from one side i.e lower side RQ & delivers it to the other side i.e higher side LM.
- The tension is more in the lower side, as compared to upper side. So, lower side is known as tight side & upper side is known as slack side.



(b) Crossed or Twist Belt drive

It is used with shafts arranged in ||al & rotating in opposite directions.



The driver pulls the belt from one side i.e RQ & delivers it to other side i.e LM.

The Lower side i.e RQ has more tension & the upper side i.e LM has the less tension. So, upper side is known as ~~tight~~ Slack Side & lower side is known as tight side.

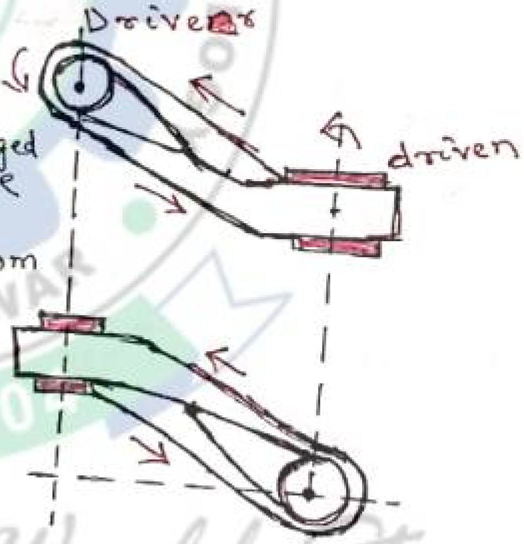
~~Hence~~ In this belt drive, a point where the belt crosses, so it rubs against each other & there will be excessive wear & tear. In order to avoid this problem shafts should be placed at a distance of 20b & the speed of belt should be less than 15 m/s.

b = width of the belt.

(c) Quarter turn Belt drive

It is also known as right angle belt drive.

It is is used with shafts arranged at right angles & rotating in one definite direction.

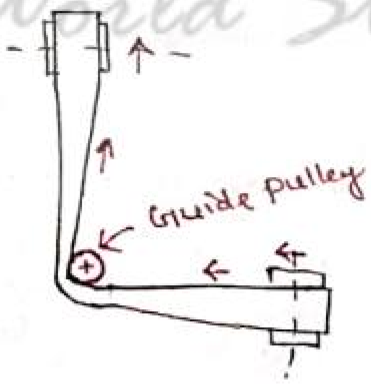


In order to preven the belt from leaving the pulley, the width of the face of the pulley (B) should be greater or equal to 1.4b.

i.e $B \geq 1.4b$

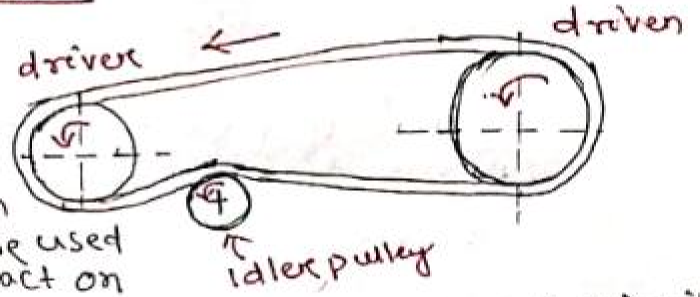
b = width of the belt

When reversible motion is desired, then a quarter turn belt drive with ~~ed~~ guide pulley may be used.



(d) Belt drive with idler pulley

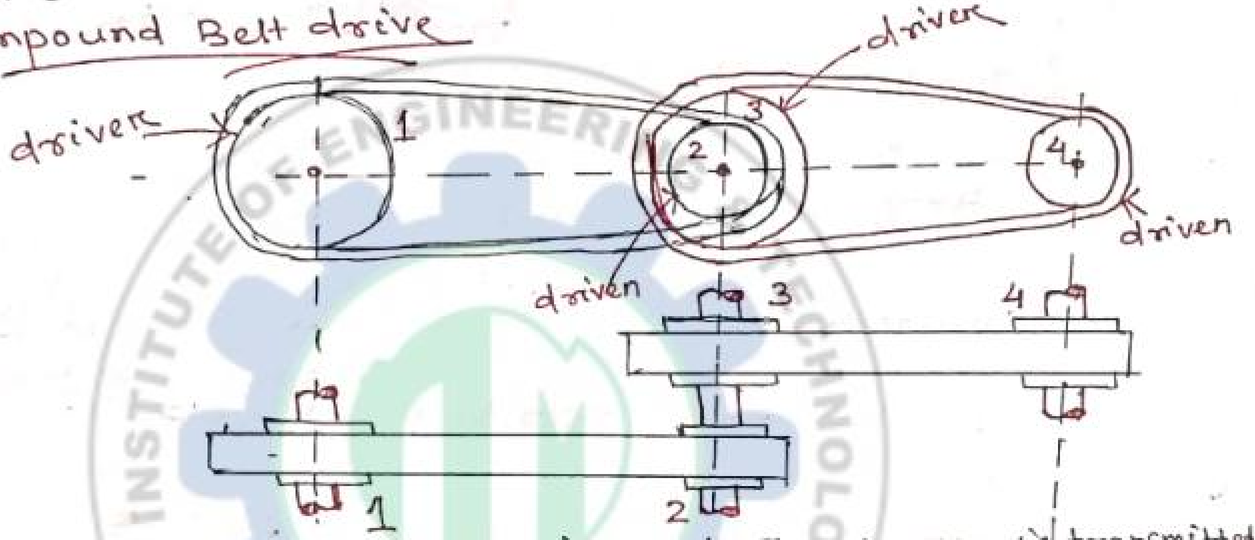
It is also known as ~~jucky~~ juckey pulley drive.



It is used with shafts arranged in ||al & when an open belt drive can not be used due to small angle of contact on the smaller pulley.

This type of drive is provided to obtained high velocity ratio.

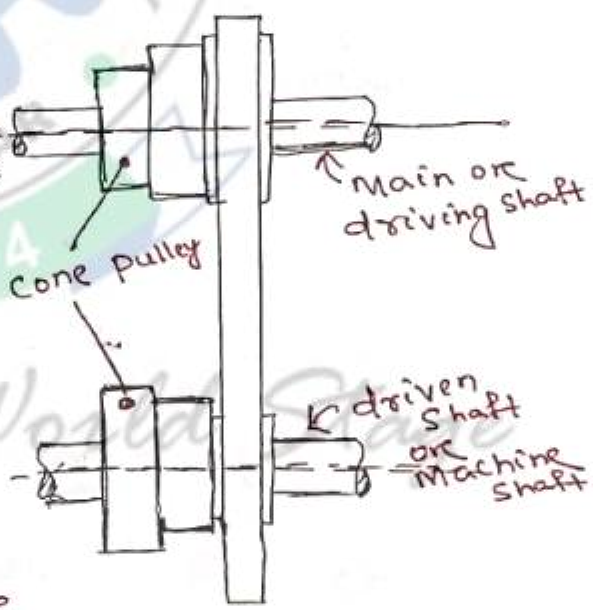
(e) Compound Belt drive



A compound belt drive is used when power is transmitted from one shaft to another through number of pulleys.

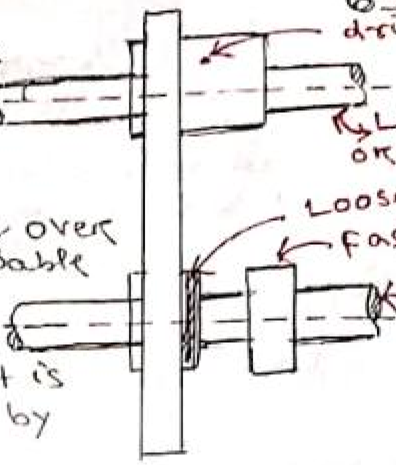
(f) Stepped or cone pulley

It is used for changing the speed of the driven shaft while the main or driving shaft runs at constant speed.



(g) Fast & Loose pulley drive

It is used when the driven or machine shaft is to be started or stopped whenever desired without interfering the driving or main shaft.



A pulley keyed to the machine shaft is called fast pulley & runs at same speed as that of machine shaft.

A loose pulley runs freely over the machine shaft & is incapable of transmitting any motion.

When the driven shaft is required to be stopped, the belt is pushed on to the loose pulley by means of sliding bar.

Velocity Ratio of Belt drive

The ratio betn the velocities of driver & the driven is known as velocity ratio.

Mathematically,

Let d_1 = dia. of driver N_1 = Speed of driver

d_2 = dia. of driven N_2 = Speed of driven

Length of belt that passes over the driver in one minute $= \pi d_1 N_1$

Length of belt that passes over the driven in one minute $= \pi d_2 N_2$

The length of belt that passes over the driver is equal to the length of belt that passes over the driven.

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \boxed{\frac{N_2}{N_1} = \frac{d_1}{d_2}}$$ Velocity ratio

If thickness of Belt (t) is considered, then velocity ratio

$$\boxed{\frac{N_2}{N_1} = \frac{d_1+t}{d_2+t}}$$

Note Velocity of Belt $\boxed{V = \frac{\pi d N}{60}}$ m/s

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60}$$

For driver $V_1 = \frac{\pi d_1 N}{60}$

For driven $V_2 = \frac{\pi d_2 N_2}{60}$

✓ Incase of compound Belt drive (4 pulleys)

$$\boxed{\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}}$$ $\left(\frac{\text{speed of last driven}}{\text{speed of 1st driver}}\right)$

✓ Incase of 6 pulleys :→

$$\boxed{\frac{N_6}{N_1} = \frac{d_1 \times d_3 \times d_5}{d_2 \times d_4 \times d_6}}$$

Slip of Belt :→

There is a firm frictional grips betⁿ the belts & pulleys surfaces, but sometimes this grips become insufficient. This cause some forward motion of the driver without carrying the belt with it & also cause some forward motion of the belt without carrying the driven pulley in it. This is called slip of belt.

It is expressed in percentage.

Let $S_1\%$ = slip betⁿ driver & Belt

$S_2\%$ = slip betⁿ Belt & driven

Velocity ratio $\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S_1 + S_2}{100}\right)$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{S}{100}\right) \quad \because \text{Total slip } S = S_1 + S_2$$

If thickness of belt (t) is considered, then velocity ratio

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{S}{100}\right)$$

Creep of Belt

When the belt passes from slack side to tight side, a certain portion of the belt extends & it contracts again when the belt passes from tight side to slack side. Due to this changes of length, there is a relative motion betⁿ the belt & pulley surfaces. This relative motion is called creep of Belt.

Velocity ratio $$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

Where σ_1 = stress in the belt on tight side

σ_2 = " " on slack side

E = Young's modulus for the material of belt.

Length of Open Belt drive :→

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

r_1 = Radius of larger pulley

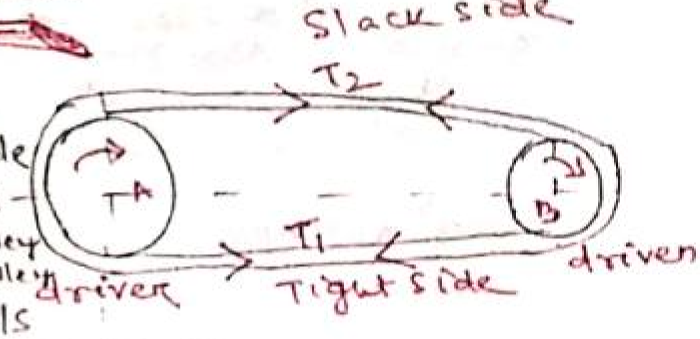
r_2 = Radius of smaller pulley

x = distance betⁿ the centre of two pulleys

Length of cross Belt drive :-

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

Power transmitted by Belt



Let
T_1 = Tension in tight side
T_2 = Tension in slack side
r_1 = Radius of driving pulley
r_2 = " " driven pulley
V = velocity of belt in m/s

Workdone $= (T_1 - T_2) \times V$ N-m/s

power transmitted $\boxed{P = (T_1 - T_2) \times V}$ Watt.

Ratio of Driving tensions for Flat Belt drive:→

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta}} \qquad \boxed{2.3\log\left(\frac{T_1}{T_2}\right) = \mu \cdot \theta}$$

μ = co-efficient of friction betn belt & pulley
θ = Angle of contact or Angle of Lap

Angle of contact or Lap (θ)

For open Belt drive:→ $\boxed{\sin\alpha = \frac{r_1 - r_2}{x}}$

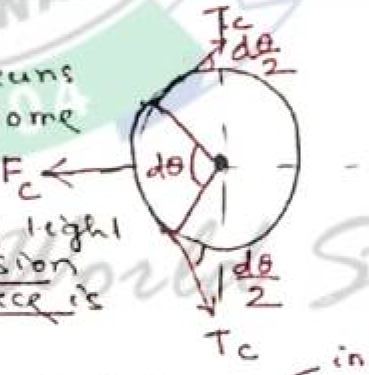
$$\boxed{\theta = (180^\circ - 2\alpha) \times \frac{\pi}{180} \text{ rad}}$$

For cross Belt drive:→ $\boxed{\sin\alpha = \frac{r_1 + r_2}{x}}$

$$\boxed{\theta = (180^\circ + 2\alpha) \times \frac{\pi}{180} \text{ rad}}$$

Centrifugal Tension (T_c)

Since the belt continously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both sides (i.e tight side & slack side). The tension caused by the centrifugal force is called centrifugal tension.



Let m = mass of the belt in kg = Area × length × density (in meter)
$b \times t \times L \times \rho$
L = length = 1 m
ρ = density
V = velocity of belt in m/s
T_c = centrifugal tension

$$\boxed{T_c = mv^2}$$

considering centrifual tension:-
Total tension in tight side $\boxed{T_{t_1} = T_1 + T_c}$
Total tension in slack side $\boxed{T_{t_2} = T_2 + T_c}$
power transmitted $P = (T_{t_1} - T_{t_2}) \times V$
$= (T_1 + T_c - T_2 - T_c) \times V$

$$\boxed{P = (T_1 - T_2) \times V}$$

Maximum Tension in the Belt :-

The max^m^ tension in the belt (T) is equal to the total tension in the tight side of belt (T_{t_1}).

Let σ = max^m^ safe stress

b = width of the belt

t = Thickness of the belt

max^m^ tension in the belt → $T = \sigma \times b \times t$

$T = T_{t_1}$ (Neglecting centrifugal tension)

$T = T_{t_1} + T_c$ (Considering centrifugal tension)

Condition for the transmission of max^m^ power :→

For max^m^ power $T_c = \frac{T}{3}$

$T_1 = \frac{2T}{3}$

velocity of belt for max^m^ power $V = \sqrt{\frac{T}{3m}}$

Inital Tension in the Belt :→

The motion of the belt from driver & driven is governed by a firm frictional grip, due to friction bet^n^ the belt & pulley. Inorder to increase this grip, the belt is tightened up. At this stage, the belt is subjected to some tension called inital tension.

Let T_0 = Inital tension in belt

T_1 = Tension in tight side of belt

T_2 = Tension in slack side of belt

α = co-efficient of increase of the belt length

$T_0 = \frac{T_1 + T_2}{2}$ (Neglecting centrifugal Tension)

$T_0 = \frac{T_1 + T_2 + 2T_c}{2}$ (Considering centrifugal Tension)

Gear Drives & its Terminology

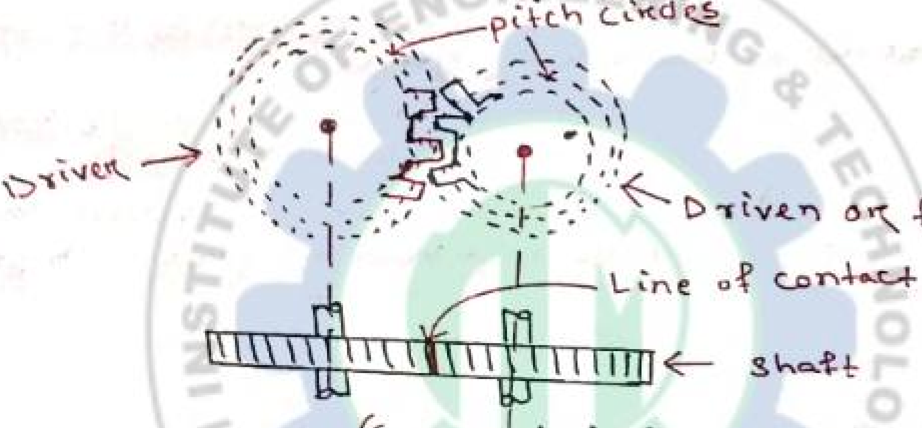
Gear or Tooth Wheel

↳ A friction wheel with the teeth cut on it, is known as toothed wheel or gear.

↳ A gear is a rotating machine part having cut teeth, which mesh with another toothed part to transmit torque.

Gear Drive

A gear drive requires two gears for operation. The two gears are cut & the driver gear receives force from the power output. Then the driver gear transfer power to the driven gear.



(Gear drive)

Advantages of Gear drive

(i) It transmit exact velocity ratio.
(ii) It may be used to transmit large power.
(iii) It has high efficiency.
(iv) It has reliable service
(v) It has compact layout

Disadvantages of Gear drive

(i) It requires special tools & equipment.
(ii) The error in cutting teeth may cause vibrations & noise during operation.

Classification of Gear or Toothed wheel :→

(i) According to the position of axes of the shaft:→

(a) Parallel gears :→

↳ The two parallel & co-planar shafts connected by the the gears, is known as spur gears. & the arrangement is known as spur gearing. (Here the teeth are ‖al to the axis of the wheel).

↳ When the teeth are inclined to the axis, then it is known as helical gearing.

↳ It may be single helical gearing & double helical gearing

(b) Non-Parallel or Intersecting gear

↳ The two non-parallel or intersecting, but coplanar shafts connected by gears, is called bevel gears & the arrangement is known as bevel gearing.

↳ When the teeth are inclined to the face of the bevel, then it is known as helical bevel gears.

(c) Non-intersecting & Non-parallel

↳ Two non-coplanar shaft connected by gears, is known as skew bevel gear or spiral gears. & the arrangement is known as spiral gearing.

(ii) According to the Velocity of Gears

(a) Low velocity :→ The gear having velocity less than 3m/s are known as low velocity gears.

(b) Medium velocity :→ The gears having velocity between 3m/s to 15 m/s are known as medium velocity.

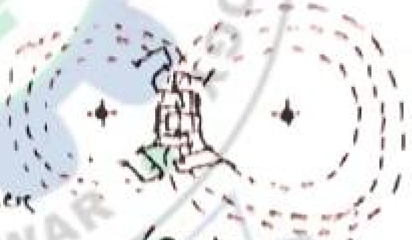
(c) High velocity :→ The gear having velocity more than 15m/s are known as High velocity gear or High speed gears.

(iii) According to the type of gearing

(a) External gearing :→

↳ When the gears of the two shafts mesh externally with each other, then it is known as external gearing.

↳ The larger of these two wheels is called spur wheel & the smaller wheel is called pinion.



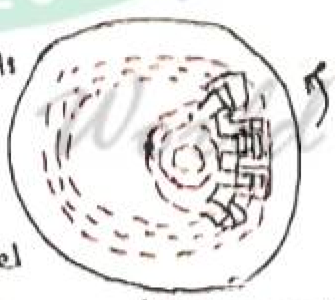
(External Gearing)

↳ In external gearing, the motion of two wheels is always unlike.

(b) Internal Gearing :→

↳ When the gears of the two shafts mesh internally with each other, then it is known as internal gearing.

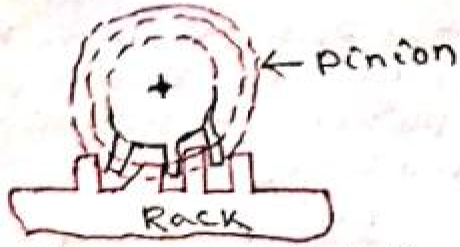
↳ The larger of these two wheels is called annular wheel & the smaller wheel is called pinion.



(Internal gearing)

↳ The motion of the two wheel is always like.

(c) Rack & pinion :→ Sometimes, the gears of the ~~shaft~~ ~~meshes~~ two shafts meshes externally & internally with each other in a straight line, known as Rack & pinion arrangement.



(Rack & pinion)

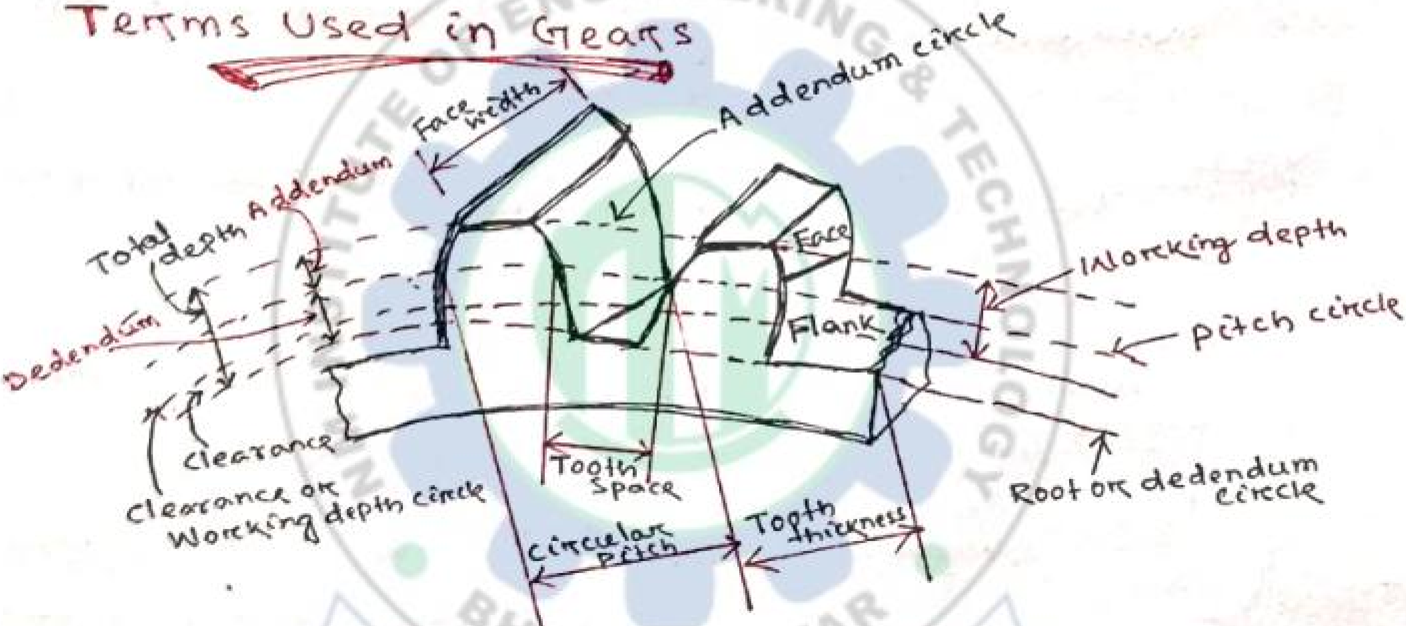
(iv) According to the position of teeth on the gear Surface

(a) Straight :→ Spur gear have Straight teeth.

(b) Inclined :→ Helical gears have teeth inclined to the wheel rim.

(c) Curved :→ Spiral gears have teeth curved over the rim surface.

Terms Used in Gears



Pitch circle :→ It is an imaginary circle, which give the same motion as the actual gear.

Pitch circle diameter :→ It is the diameter of pitch circle.
- ↳ It specified the size of the gear.
- ↳ It is also known as pitch diameter.

Pressure angle or Angle of obliquity :→ It is the angle betn the common normal to two gear teeth at the point of contact & the common tangent at the pitch point.

It is denoted by ϕ. The standard pressure angles are $14\frac{1}{2}^\circ$ & 20°.

Addendum :- It is the radial distance of a tooth from the pitch circle to the top of the tooth.

Dedendum :- It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

Lead :- It is the axial advance (Length along the axle) of a helical gear tooth during one complete turn i.e 360°.

Circular pitch (p_c) :→ It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth.

It is denoted by p_c.

Mathematically $\boxed{p_c = \frac{\pi D}{T}}$ D = dia. of pitch circle
T = Number of teeth on the wheel

Diametral pitch (p_d) :→ It is the ratio of the number of teeth to the pitch circle diameter. It is denoted by p_d.

Mathematically $\boxed{p_d = \frac{T}{D}}$ T = Number of teeth on the wheel
D = pitch circle diameter

Module (m) :→ It is the ratio of the pitch circle diameter to the number of teeth. It is denoted by m.

Mathematically $\boxed{m = \frac{D}{T}}$

Clearance :→ It is the radial distance from the top of the tooth to the bottom of the tooth.

Total depth :→ It is the radial distance between the addendum & dedendum circles of a meshing gear.

Face of Tooth :→ It is the surface of the ~~tooth~~ gear tooth above the pitch surface.

Flank of Tooth :→ It is the surface of the gear tooth below the pitch surface.

Face width :→ It is the width of the gear tooth measured parallel to its axis.

Path of contact :→ It is the path traced by a point ~~on the pitch circle~~ of contact of two teeth from the beginning to the end of engagement.

Arc of Contact :→ It is the path traced by a point on the pitch circle from the beginning to end of engagement of a pair of teeth.

It is consist of two parts. i.e

(a) **Arc of Approach :→** It is the portion of the path of contact from the beginning of the engagement to the pitch point.

(b) **Arc of Recess :→** It is the portion of the path of contact from the pitch point to the end of the engagement of a pair of teeth.

Contact Ratio :→ The ratio of the length of arc of contact to the circular pitch is known as contact ratio. i.e the number of pairs of teeth in contact.

Gear material

(i) Cast iron →
* Good wearing properties
* Good machinability
* Easy to producing complicated shape by casting.

(ii) Steel :→
* plain carbon steel or Alloy steel is used
* used for high strength gears

(iii) phosphor Bronze :– * widely used for warm gears inorder to reduce wear, which will be excessive with cast iron or steel gears.

Law of Gearing :→

It state that, the angular velocity ratio of all gears of a meshed gear system must remain constant, also the common normal at the point of contact must passes through the pitch point.

Gear Ratio :→ It also known as gear ratio of gear train or speed ratio of gear.

It is the ratio of the angular velocity of input gear to the angular velocity of output gear.

It can be calculated directly from the numbers of teeth on the gears in the gear train.

Objective Type

(1) The two parallel & coplanar shafts are connected by gears having teeth ||al to the axis of the shaft. This arrangement is called ___

(a) Spur gearing (c) Bevel gearing
(b) Helical gearing (d) Spiral gearing

Ans:- a

(2) The type of gear used to connect two non-parallel non-intersecting shafts are -

(a) Spur gears (c) Spiral gears
(b) Helical gears (d) None of these

Ans:- c

(3) An imaginary circle which by pure rolling action, gives the same motion as the actual gear is called -

(a) Addendum circle (c) Diametral pitch
(b) Dedendum circle ~~(d) pitch circle diameter~~ (d) Clearance circle

Ans:- c

(4) The size of the gear is usually specified by

(a) pressure angle (c) diametral pitch
(b) circular pitch (d) pitch circle diameter

Ans:- d

(5) The radial distance of a tooth from the pitch circle to the bottom of the tooth, is called

(a) Dedendum (c) Clearance
(b) Addendum (d) Working depth

Ans:- a

(6) The product of the dimetral pitch & circular pitch is equal to

(a) 1 (c) π
(b) $\frac{1}{\pi}$ (d) 2π

Ans:- c

(7) The module is the reciprocal of

(a) Diametral pitch (c) Pitch diameter
(b) circular pitch (d) None of these

Ans:- a

(8) Which is the incorrect relationship of gears?

(a) circular pitch × Diametral pitch = π
(b) module = P.C.D / No. of teeth
(c) Dedendum = 1.157 module
(d) Addendum = 2.157 module

Ans:- d

(9) Mitre gears are used for

(a) Great speed reduction (c) minimum axial thrust
(b) Equal speed (d) minimum backlash

Ans:- b

(10) The condition for correct gearing is

(a) pitch line velocities of teeth be same.
(b) Radius of curvature of two profiles be same
(c) Common normal to the pitch surface cuts the line of centres at a fixed point
(d) None of the above

Ans:- c

(11) Law of gearing ~~sta~~ satisfied if
(a) Two surfaces slide smoothly
(b) common normal at the point of contact passes through the pitch point on the line joining the centres of rotation.
(c) Number of teeth = P.C.D/module
(d) Addendum is greater than dedendum
Ans:- b

(12) Involute profile is preferred to cycloidal because
(a) The profile is easy to cut
(b) only one curve is required to cut.
(c) The rack has straight line profile & hence can be cut accurately.
(d) None of the above
Ans:- b

(13) The contact ratio for gear is
(a) Zero (b) Less than one (c) Greater than one
Ans:- c

(14) The max^m length of arc of contact for two mating gears, in order to avoid interference, is
(a) $(r+R)\sin\phi$
(b) $(r+R)\cos\phi$
(c) $(r+R)\tan\phi$
(d) None of these
Ans:- c

(15) Interference can be avoided in involute gears with 20° pressure angle by
(a) cutting involute correctly
(b) using as small number of teeth as possible
(c) using more than 20 teeth
(d) using more than 8 teeth
Ans:- c

(16) The ratio of face width to transverse pitch of a helical gear with α as the helix angle is normally
(a) more than $1.15/\tan\alpha$
(b) more than $1.05/\tan\alpha$
(c) more than $1/\tan\alpha$
(d) None of these
Ans:- a

(17) For speed ratio of 100, smallest gear box is obtained by using
(a) a pair of spur gears
(b) a pair of helical & a pair of spur gear compounded.
(c) a pair of bevel & a pair of spur gear compounded.
(d) a pair of helical & a pair of worm gear compounded.
Ans:- d

(18) Which gears are used for skew arrangement?
(a) spur gears on helical gears
(b) Helical, worm or hypoid gears
(c) Mitre gears
(d) None of the above
Ans:- b

(19) Bevel gears used for connecting intersecting shafts at 90° & have speed ratio 1:1 is known as
(a) Bevel gears
(b) Beveloid gears
(c) Metre gears
(d) None of the above
Ans:- c

(20) Tapered involute gears which can couple intersecting shafts, skew shafts, & parallel shafts are known as:-
(a) Bevel gears (c) mitre gears
(b) Beveloid gears (d) None of the above
Ans:- b

(21) Gears having teeth cut on the rotating face plane of the gear & mate with standard involute spur gears are known as:-
(a) mitre gears (c) spur gears on helical gears
(b) Face gear (d) None of the above
Ans:- b

(22) Which gears are used for obtaining large speed reduction betn non-intersecting shafts making an angle of 90° with each other.
(a) worm gear (c) mitre gear
(b) Beveloid gear (d) None of the above
Ans:- a

(23) Bevels connecting shafts other than 90° are
(a) worm gears (c) Angular bevel gears
(b) mitre gears (d) None of the above
Ans:- ~~b~~ c

(24) Bevels connecting non-intersecting shafts are
(a) Skew bevel gears (c) mitre gears
(b) Angular bevel gears (d) None of the above
Ans:- a

(25) The gear used to convert rotary motion into translating motion is
(a) worm & wheel (c) Rack & pinion
(b) Crown gear (d) Spiral Bevel gear
Ans:- c

(26) The point of contact of two pitch circles of mating ~~pair~~ gears is called ——
(a) pressure point (c) module
(b) pitch point (d) contact point
Ans:- b

(27) Which of the following does not give velocity ratio of gears?
(a) ω_2/ω_1 (c) D_1/D_2
(b) N_1/N_2 (d) T_1/T_2
Ans:- b

Gear Train

↳ Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.

↳ A gear train may consist of spur, bevel or spiral gears.

Types of Gear Train

The various types of gear trains are :-

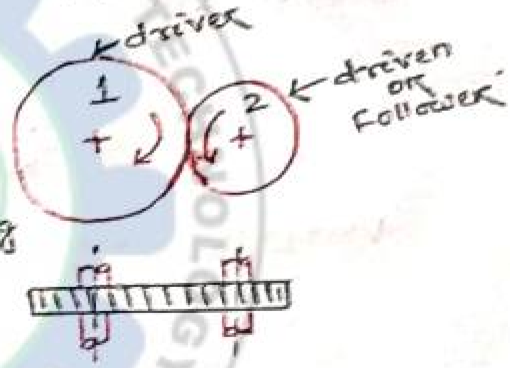
(a) Simple gear train
(b) Compound gear train
(c) Reverted gear train
(d) Epicyclic gear train

(a) Simple gear train

↳ When there is only one gear on each shaft, it is known as simple gear train.

↳ The gears are represented by their pitch circle.

Gear 1 drives gear 2, therefore gear 1 is called driver & the gear 2 is called driven or follower.



The motion of the driven gear is opposite to motion of the driving ~~pulley~~. gear.

Let N_1 = Speed of gear 1 in r.p.m
N_2 = Speed of gear 2 in rpm
T_1 = Number of teeth on gear 1.
T_2 = Number of teeth on gear 2.

↳ The ratio of the speed of the driver to the speed of the driven is known as speed ratio.

Mathematically $\frac{N_1}{N_2} = \frac{T_2}{T_1}$

↳ The ratio of the speed of the driven to the speed of the driver is known as train value.

mathematically $\frac{N_2}{N_1} = \frac{T_1}{T_2}$

Train value is the reciprocal of speed ratio.

↳ Sometimes, the distance between the two gears is large. In this case two methods are used:-

* By providing large sized gear
* By providing one or more intermediate gears

By providing large sized gear is very inconvenient & uneconomical method. But providing one or more intermediate gear is ~~very~~ very convenient & economical method.

Considering one intermediate gear:

Let N_1 = Speed of driver in rpm

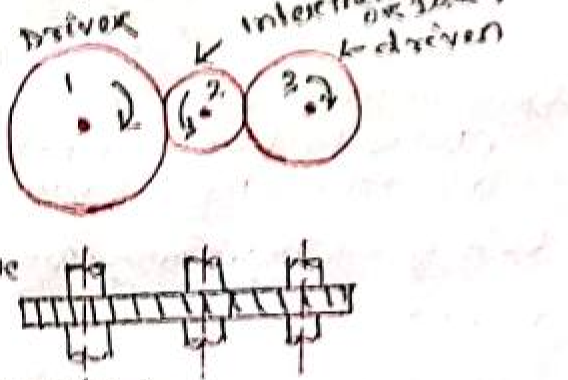
N_2 = Speed of intermedial gear in rpm

N_3 = Speed of driven in rpm

T_1 = No. of teeth on driver

T_2 = No. of teeth on intermediate gear

T_3 = No. of teeth on driven



Since the gear 1 mesh with gear 2

Speed ratio $\frac{N_1}{N_2} = \frac{T_2}{T_1}$ ——— (i)

Similarly the gear 2 mesh with gear 3

Speed ratio $\frac{N_2}{N_3} = \frac{T_3}{T_2}$ ——— (ii)

So, speed ratio of gear train is obtained by multiplying equ^n (i) & (ii)

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

$$\Rightarrow \boxed{\frac{N_1}{N_3} = \frac{T_3}{T_1}}$$

(Note)

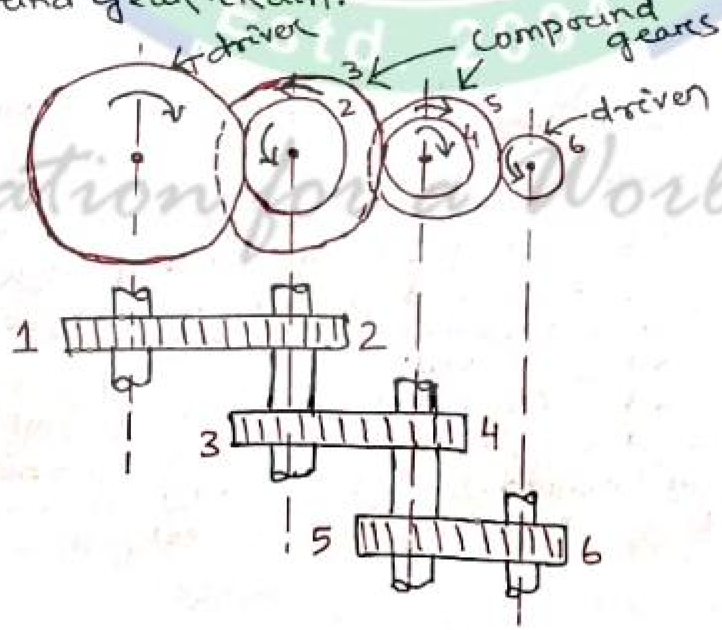
Purpose of Intermediate or Idle gear :->

(i) To connect gears, where a large centre distance is required.

(ii) To obtain the desired direction of motion of the driven gear (i.e clockwise or anticlockwise)

(b) Compound gear train

When there are more than one gear on a shaft, it is called compound gear train.



Let N_1 = Speed of driver i.e gear 1
T_1 = Number of teeth on driver or gear 1

N_2, N_3, N_4, N_5, N_6 = Speed of gear 2,3,4,5,6, respectivly.
T_2, T_3, T_4, T_5, T_6 = Number of teeth on gear 2,3,4,5,6, respectivly.

Gear 1 mesh with gear 2 :- $\frac{N_1}{N_2} = \frac{T_2}{T_1}$ ——— (i)

Gear 3 mesh with gear 4 :- $\frac{N_3}{N_4} = \frac{T_4}{T_3}$ ——— (ii)

Gear 5 mesh with gear 6 :- $\frac{N_5}{N_6} = \frac{T_6}{T_5}$ ——— (iii)

multiplying eqn (i), (ii) & (iii)

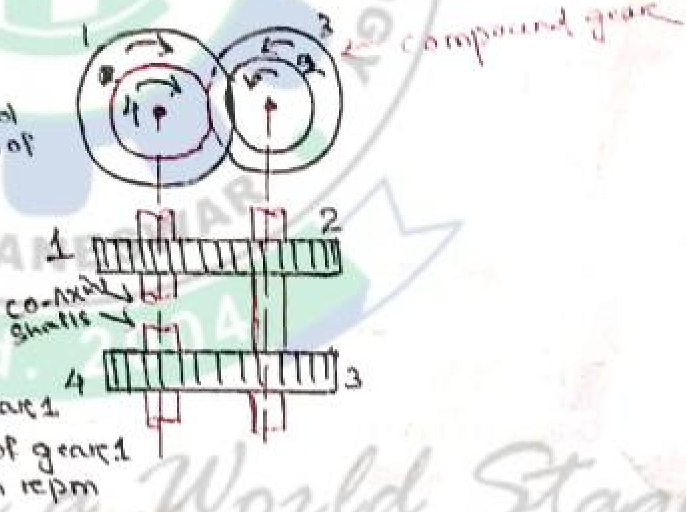
Speed ratio $\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$

Since the gear 2 & 3 & 4 & 5 are mounted on same shafts.
so, $N_2 = N_3$ & $N_4 = N_5$

$$\Rightarrow \boxed{\frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}}$$

(c) Reverted gear train

When the axes of the first gear & last gear are co-axial, then the gear train is known as reverted gear train.



* Gear 1 drives gear 2 in opposite direction.
* Gear 2 & 3 are compound gears & the direction of gear 2 & 3 are same.
* Gear 3 drives the gear 4, in the same direction as gear 1.

Let
T_1 = Number of teeth on gear 1
r_1 = Pitch circle radius of gear 1
N_1 = Speed of gear 1 in rpm
T_2, T_3, T_4 = No. of teeth on gear 2,3 & 4 respectivly.
r_2, r_3, r_4 = pitch circle radius of gear 2,3,4 respectivly.
N_2, N_3, N_4 = speed of gear 2, 3, 4 respectivly.

* The distance betn the centres of shafts of gear 1 & 2 as well as gear 3 & 4 are same.

so $\boxed{r_1 + r_2 = r_3 + r_4}$

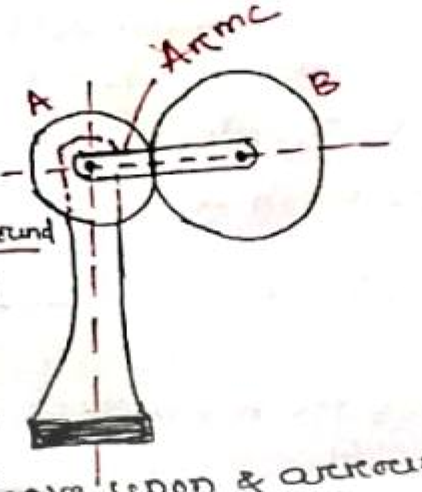
* The number of teeth on each gear is directly proportional to radius,

so, $\boxed{T_1 + T_2 = T_3 + T_4}$

* speed ratio $\boxed{\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}}$

(d) Epicylic gear train

If gear A is fixed & the arm is rotate about the axis of gear A, then the gear B is forced to rotate upon & around gear A. Such motion is called epicylic.



The gear train arranged in such a manner that one or more of their members move upon & around another member are known as epicyclic gear train.

Note

Uses of various gear train

(a) Simple gear train :→ Gearbox of Automobile
Lathe machines
Heavy duty press machine

(b) Compound gear train → Head stock of Lathe machines
Clock & watches

(c) Reverted gear train → Automobile transmission.
Lathe back gears
Industrial speed reducer
Clock

(d) Epicyclic gear train :→ Back gear of Lathe
Differential gear of automobile
Pulley blocks
Wrist watches

Velocity ratio of epicyclic gear train

Tabular method

Direction
clockwise → '−'
Anticlockwise → '+'

Consider a epicylic gear train
Let T_A = Number of teeth on gear A
T_B = Number of teeth on gear B

Step No	Conditions of motion	Revolutions of element		
		Arm C	Gear A	Gear B
1.	Arm fixed, gear A rotates through +1 revolution i.e 1 rev. anticlockwise	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed, gear A rotates through +x revolutions	0	+x	$-x \times \frac{T_A}{T_B}$
3.	Add +y revolutions to all elements	+y	+y	+y
4.	Total motion	+y	x+y	$y - x \times \frac{T_A}{T_B}$

Objective Type

(1) In a simple gear train, if the number of idle gears is odd, the motion of the driven gear will
(a) be same as that of driving gear
(b) be opposite as that of driving gear
(c) Depend upon the number of teeth on the driving gear
(d) None of the above
Ans:- a

(2) The train value of a gear train is
(a) Equal to velocity ratio of a gear train
(b) Reciprocal of velocity ratio of gear train
(c) Always greater than unity
(d) Always less than unity
Ans:- b

(3) In a clock mechanism, the gear train used to connect minute hand to the hour hand, is
(a) Epicyclic gear train
(b) Reverted gear train
(c) compound gear train
(d) simple gear train.
Ans:- b

(4) In a gear train, when the axes of the shaft, over which the gears are mounted, move relative to a fixed axis, is called
(a) simple gear train
(b) compound gear train
(c) Reverted gear train
(d) Epicyclic gear train
Ans:- d

(5) A differential gear in automobiles is used to
(a) Reduce speed
(b) Assist in changing speed
(c) Provide jerk-free movement to vehicle
(d) Help in turning
Ans:- d

(6) A differential gear in an automobile is a
(a) simple gear train
(b) compound gear train
(c) epicyclic gear train
(d) Reverted gear train
Ans:- c

(7) What is meant by an idle gear?
(a) Gears bet^n driver & driven gears
(b) Gears used when driver & driven gears moves in same direction.
(c) Both a & b
(d) None of the above
Ans:- c

(8) Which type of gear train & gears are used in Humpage gear box?
(a) Epicyclic gear train with bevel gears
(b) Epicyclic gear train with worm gears
(c) Reverted gear train with bevel gears
(d) Reverted gear train with worm gears.
Ans:- a

(9) The working depth of an involute gear is equal to

(a) Addendum (c) Addendum + Dedendum

(b) Dedendum (d) 2 X Addendum

Ans:- d

(10) Tooth thickness on pitch line of involute gear interms of module (m) is equal to

(a) 1.157m (c) 2m

(b) 1.167 m (d) 1.5708m

Ans:- d (Tooth thickness = 1.5708 X module)

(11) A fixed gear having 200 teeth is in mesh with another gear having 50 teeth. The two gears are connected by an arm, The number of turns made by the smaller gear for one revolution of arm about the centre of bigger gear is

(a) 2 (b) 4 (c) 3 (d) None of the above

Ans:- b ($N_1 = 200$, $N_2 = 50$, $\frac{N_1}{N_2}$ = No. of turns = $\frac{200}{50} = 4$)

(12) Which gear train is used for higher velocity ratios in a small space?

(a) Simple gear train (c) Reverted gear train

(b) Compound gear train (d) Epicyclic gear train

Ans:- d

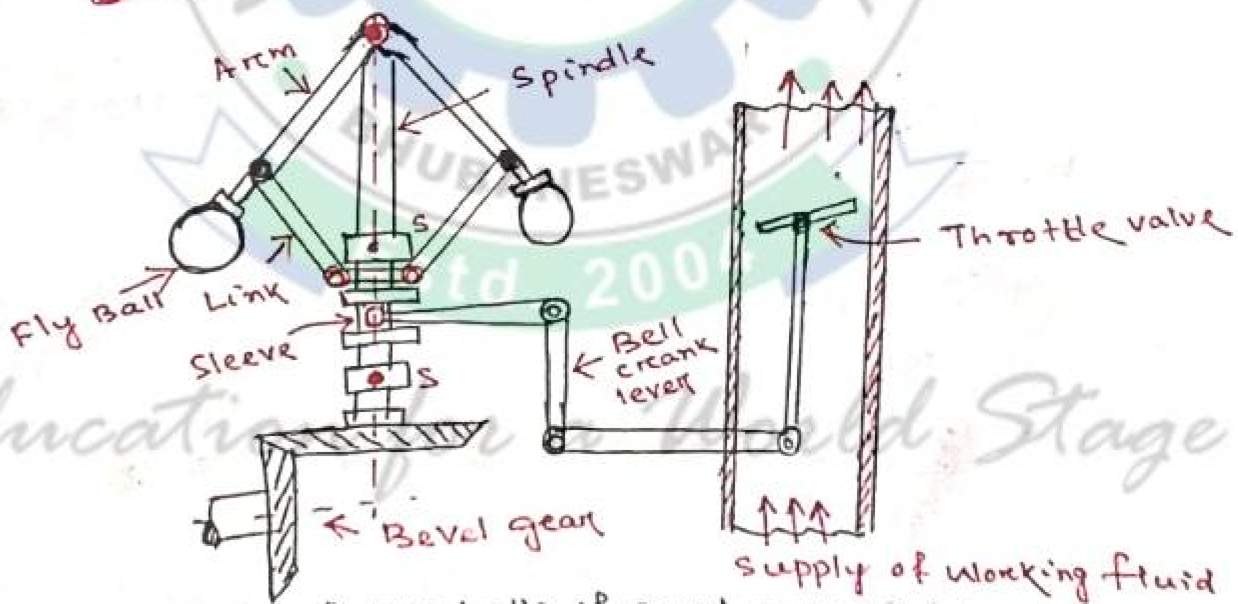
Governors

- The function of governor is to regulate the mean speed of the engine.
- When the load on an engine increases, its speed decreases & when the load of engine decreases, it speed increases.

Types: Governors are broadly classified into two types:-
(i) Centrifugal governor
(ii) Inertia governor

- Centrifugal governor
  - Pendulum type
    - Watt governor
  - Loaded type
    - Dead weight governor
      - Porter Governor
      - Proell governor
    - Spring ~~loaded~~ controlled governor
      - Hartnell governor
      - Hartung governor
      - Wilson-Hartnell governor
      - Pickering governor

Centrifugal Governor



- It is consist of two balls of equal mass, which are attached to the arm. These are known as governor balls or fly balls.
- The balls are revolved with a spindle, which is driven by the engine through bevel gears. The upper end of the arms are pivoted at the spindle
- The arms are connected by the links to the sleeve, which is keyed to the spindle.

↳ The balls & the sleeve rises when the spindle speed increases & the falls when the speed decreases. Inorder to limit the sleeve movement two stops s, s are provided. on the spindle.

↳ The sleeve is connected by a bell crank lever to a throttle valve. When sleeve rises, the supply of working fluid decrease & when the sleeve falls, the supply of working fluid increases.

↳ When the engine load increases:→

* Engine & governor speed decreases., so the centrifugal force on balls also decreases.
* As a result balls moves ~~outwards~~ inwards & sleeve moves ~~down~~ downwards.
* The downward movement of sleeve operates the throttle valve of the bell crank lever to increase the supply of working fluid & thus engine speed increases. (Power output increases)

↳ When the engine load decreases:→

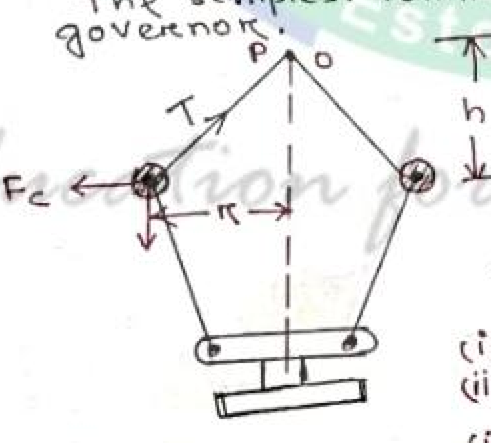
* Engine & govenor speed increases, so the centrifugal force on balls also decreases.
* As a result balls moves outwards & sleeve moves upwards.
* The upward movement of sleeve reduces the supply of the working fluid & hence engine speed decreases. (Power output decreases)

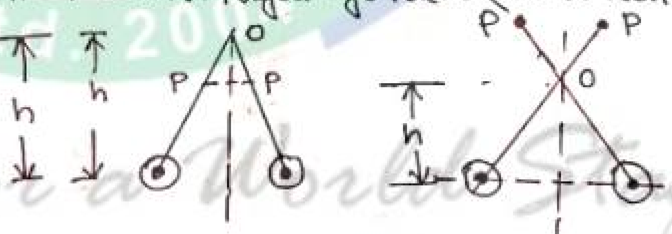
Height of a Governor (h)

It is the vertical distance from the centre of the ball to a point where the axes of the arm intersect on the spindle axis. It is denoted by h.

Watt Governor

The simplest form of centrifugal governor is a Watt governor.





The arm of the Watt governor connected to the spindle in 3 ways i.e

(i) pivot P, may be on spindle axle

(ii) pivot P, may be offset from the spindle axis.

(iii) pivot P, may be offset but the arm cross the axis at 'O'.

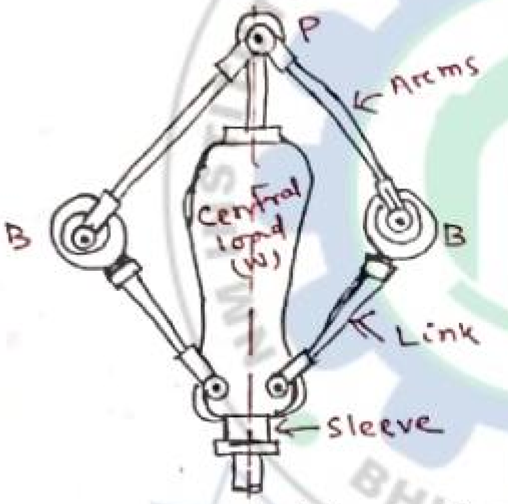
m = Mass of ball in kg
w = Weight of the ball in newton
T = Tension in the arm in newton
ω = Angular velocity in rad/s (Ball & arm) = $\frac{2\pi N}{60}$ rad/s
r = Radius of the path of rotation of ball
F_c = Centrifugal force on ball in Newton = $m\omega^2 r$
h = Height of governor in metres

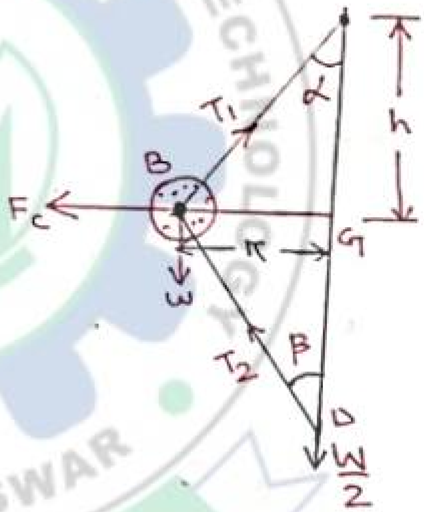
Height of Watt governor $h = \frac{895}{N^2}$ metres

Porter Governor

It is the modification of Watt's governor, with central load attached to the sleeve.

consider the forces acting on one-half of the governor





m = mass of ball in kg
w = weight of ball in ~~kg~~ newton = m·g
M = mass of central load in kg
W = Weight of central load in newton = M·g
r = Radius of rotation in metre
h = Height of governor in metres
N = Speed of ball in r.p.m
ω = Angular speed of balls in rad/s = $\frac{2\pi N}{60}$ rad/s
F_C = Centrifugal force acting on ball = $m \cdot \omega^2 \cdot r$
T_1 = Force or Tension in the arm in newton
T_2 = Force or Tension in the link in newton
α = Angle of inclination of arm (upper link)
β = Angle of inclination of link (Lower link)

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

$$q = \frac{\tan\alpha}{\tan\beta}$$

if $q = 1$ ($\tan\alpha = \tan\beta$)

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

If Frictional force (F) is considered which is acting on the sleeve :→

$$N^2 = \frac{m \cdot g + \left(\frac{M \cdot g \pm F}{2}\right)(1+q)}{m \cdot g} \times \frac{895}{h}$$

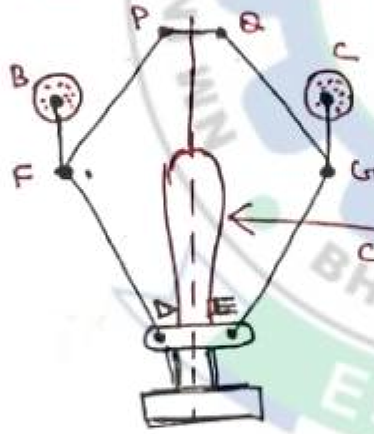
When $q = 1$,

$$N^2 = \frac{m \cdot g + (M \cdot g \pm F)}{m \cdot g} \times \frac{895}{h}$$

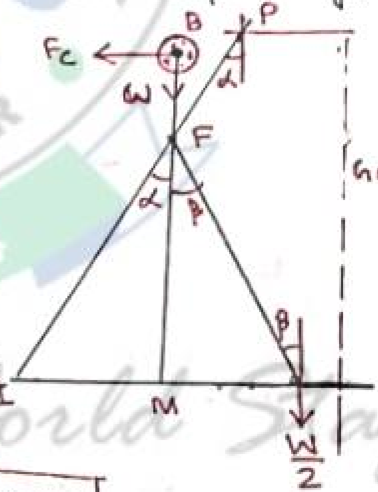
'−' → When sleeve moves downward frictional force acts - upward (min^m speed)

'+' → sleeve moves upward frictional force acts - downward (max^m speed)

Proell governor



The proell governor has the balls fixed at B & C to the extension of the links DF & EG which are pivoted at P & Q respectively.



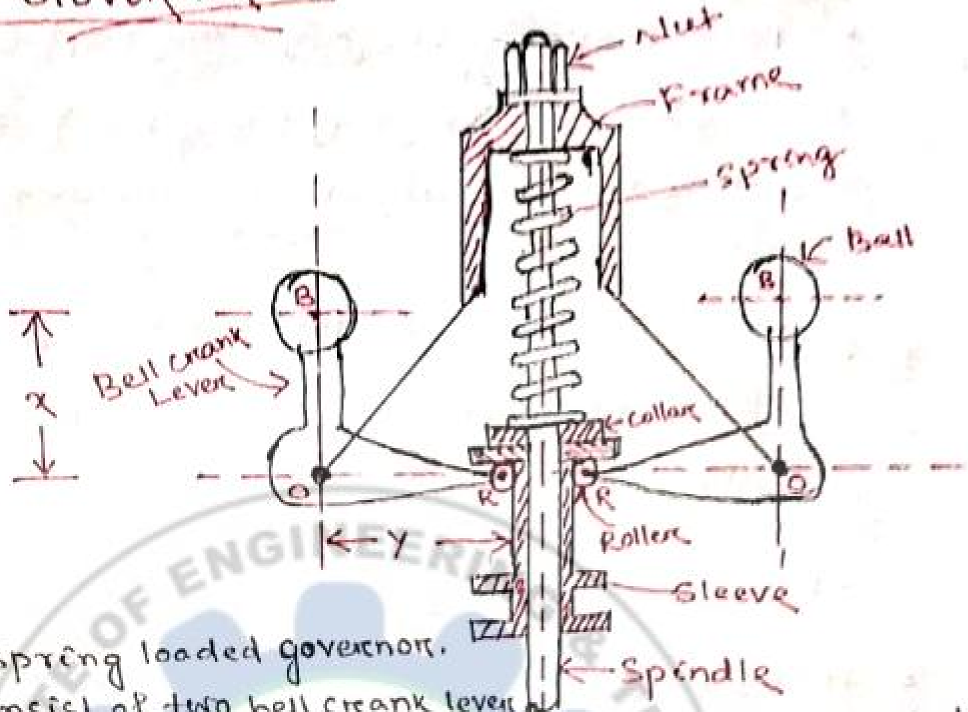
Consider one-half of the governor. The lar BM is drawn on ID. I is the instantaneous centre.

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right] \times \frac{895}{h}$$

If $q = 1$, $\alpha = \beta$

$$N^2 = \frac{FM}{BM}\left[\frac{m+M}{m}\right] \times \frac{895}{h}$$

Hartnell Governor



- It is a spring loaded governor.
- It is consist of two bell crank lever pivoted at the point O, O to the frame. The frame is attached to the governor spindle & rotate with it.
- Each lever carries a ball at the vertical arm OB & roller at the horizontal arm OR.
- The spring force may be adjusted by screwing a nut up or down on the sleeve.

m = mass of each ball in kg
M = mass of sleeve in kg
r_1 = min^m radius of rotation in metre
r_2 = max^m radius of rotation in metre
ω_1 = Angular speed at min^m radius in rad/sec.
ω_2 = Angular speed at max^m radius in rad/s
S_1 = Spring force exerted on sleeve at ω_1 in newton
S_2 = Spring force exerted on sleeve at ω_2 in newton
F_{c_1} = centrifugal force at ω_1 in Newton $= m\omega_1^2 r_1$
F_{c_2} = centrifugal force at ω_2 in Newton $= m\omega_2^2 r_2$
S = Stiffness of spring or force required to compress the spring by one mm.
x = Length of ball arm of the lever (vertical)
y = Length of sleeve arm of the lever (Horizontal)
r = Radius of rotation when the governor is in mid-position.
h = Compression of the spring when the radius of rotation changes from r_1 to r_2.

$$\boxed{h = (r_2 - r_1)\frac{y}{x}}$$

$$\Rightarrow \boxed{r_2 = r_1 + h\left(\frac{x}{y}\right)}$$

$$a_1 = \frac{r - r_1}{x}$$

$$a_2 = \frac{r_2 - r}{x}$$

~~Eqn~~

* $M \cdot g + S_1 = \frac{2}{y_1}(F_{c_1} \times x_1 - m \cdot g \times a_1)$ (For min^m position)
* $M \cdot g + S_2 = \frac{2}{y_2}(F_{c_2} \times x_2 + m \cdot g \times a_2)$ (For max^m position)

Neglecting the obliquity effect of the arm i.e

$$x_1 = x_2 = x$$
$$y_1 = y_2 = y$$

& Weight of ball $(m \cdot g)$

* then

$$M \cdot g + S_1 = 2F_{c_1} \times \frac{x}{y}$$ (Minimum position)

$$M \cdot g + S_2 = 2F_{c_2} \times \frac{x}{y}$$ (Maximum position)

Stiffness $S = \frac{S_2 - S_1}{h}$

* Inital compression of the spring $= \frac{\text{Stiffness of spring at min}^m \text{ position}}{\text{Stiffness of spring}}$

$$= \frac{S_1}{S}$$

—— x ——

Sensitiveness of Governor

It is defined as the ratio of the difference betn the max^m & min^m speeds to the mean speed.

$$\text{Sensitiveness of Governor} = \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$ (Interms of Angular speed)

N_1 = min^m speed
N_2 = Max^m speed
N = Mean speed $= \frac{N_1 + N_2}{2}$

Stability of a Governor

A Governor is said to be stable if there is only one radius of rotation of governor balls at which the governor is in equilibrium. (If speed increases, the radius of rotation of governor balls increases)

A Governor is said to be unstable if the radius of rotation decreases as the speed increases.

Isochronous Governor

A governor is said to be isochronous,

When the speed is constant i.e range of speed is zero ($N_2 = N_1$) for all radius of rotation of the balls, neglecting friction.

Let consider porter governor

$$N_1^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1}$$

$$N_2^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2}$$

Here if range of speed $N_2 - N_1 = 0$ or $N_2 = N_1$

Then the equn becomes

$\boxed{h_1 = h_2}$, which is impossible incase of porter governor.

Hence, a porter governor can not be isochronous.

Objective Type

(1) The height of a Watt's governor is

(a) directly proportional to speed

(b) Inversely proportional to speed

(c) Zero

(d) None of these

Ans:- b

(2) A Watt's governor can work satisfactorily at speed from —

(a) 60 to 80 rpm

(b) 80 to 100 rpm

(c) 100 to 200 rpm

(d) 200 to 300 rpm

Ans:- a

(3) When the sleeve of a porter governor moves upwards, the governor speed ——

(a) Increases

(b) Decreases

(c) remain unaffected

(d) First increase & then decrease

Ans:- a

(4) When the sleeve of porter governor moves downward, the governor speed ——

(a) Increases

(b) Decreases

(c) remain constant

(d) First increases & then decreases

Ans:- b

(5) A Hartnell governor is a

(a) Dead weight governor

(b) Pendulum type governor

(c) Spring loaded governor

(d) Inertia governor

Ans- c

(6) Which is the pendulum type governor?

(a) Watt's governor

(b) porter governor

(c) Hartnell governor

(d) None of the mentioned

Ans:- a

(7) A governor is said to be isochronous when equilibrium speed of all radii of rotation of the balls with in the working range.

(a) Is constant
(b) Varies uniformly
(c) Is not contant above
(d) None of the above

Ans:- a

(8) Which is the dead weight type governor?

(a) Porter governor
(b) Hartnell governor
(c) Wilson-Hartnell governor
(d) Harting governor

Ans:- a

(9) Height of governor is distance measured from

(a) the centre of the two balls
(b) the centre of balls mass to the point of intersection of upper arms.
(c) the centre of balls mass to the point of intersection of Lower arms.
(d) the point of intersection of upper arms to the point of intersection of lower links.

Ans:- b

(10) Which of the following governor can never be isochronous?

(a) Watt governor
(b) Proell governor
(c) Porter governor
(d) Hartnell governor

Ans:- c

(11) Hartnell governor could be classified under the head of ——

(a) Inertia type governor
(b) Pendulum type governor
(c) Centrifugal type governor
(d) Dead weight type governor
(e) None of the above

Ans:- c

(12) The governor used in gramophone is of the following type:→

(a) Pickering
(b) Porter
(c) Hartnell
(d) Watt
(e) Harting

Ans:- a

(13) Practically the sensitiveness of watt & porter governor are:-

(a) Same
(b) Half
(c) Double
(d) 4 times
(e) None of the above

Ans:- a

(14) The quality of a governor can be judged by its —

(a) Stability
(b) sensitivity
(c) Effort & power
(d) All of the above
(e) None of the above

Ans:- e

(15) For same lift of sleeve, range of speed of Proell governor as compared to porter governor is

(a) Less
(b) more
(c) Equal
(d) Half

Ans:- b

(16) In a Hartnell governor, if the stifness of spring is increased, governor will become ——

(a) more sensitive
(b) Less sensitive
(c) Hunting
(d) Isochronous

Ans:- b

(17) For a governor running at constant speed, the force acting on the sleeve is ——

(a) constant
(b) maximum
(c) minimum
(d) zero

Ans:- D

(18) A governor is said to be hunting, if the speed of the engine
(a) Remains contant at the mean speed
(b) is above the mean speed
(c) is below the mean speed
(d) Fluctuates continuously above & below the mean speed
Ans:- d

(19) The power of governor is equal to
(a) $\frac{c^2}{1+2c}(m+M)h$
(b) $\frac{3c^2}{1+2c}(m+M)h$
(c) $\frac{2c^2}{1+2c}(m+M)h$
(d) $\frac{4c^2}{1+2c}(m+M)h$
Ans:- d

(20) When the relation betn the controlling force (F) & radius of rotation (r) for a spring controlled governor is $F_c = a.r + b$, then governor will be
(a) Stable (b) unstable (c) Isochronous
Ans:- b

(21) For a governor, if F_c is the controlling force, r is the radius of rotation of the balls, the stability of the governor will be ensured when
(a) $\frac{dF_c}{dr} > \frac{F_c}{r}$ (b) $\frac{dF_c}{dr} < \frac{F_c}{r}$ (c) $\frac{dF_c}{dr} = 0$ (d) None of these
Ans:- a

(22) Governor is used in automobile to
(a) Decrease the variation of speed
(b) maximize the fuel economy
(c) Limit the vehicle speed
(d) maintain constant engine speed
Ans:- c

Balancing

(i) It is defined as the process of designing or modifying a machine in which unbalanced forces is minimum.

(ii) Balancing is the process of attempting to improve the mass distribution of a body so that it rotates in its bearings without unbalanced centrifugal forces.

Types → Static Balancing
→ Dynamic Balancing

Static Balancing :→

(i) A rotating mass is said to be statically balanced if the rotating mass can rest, without turning, at any angular position in its bearings.

(ii) This condition is attained when the sum of the centrifugal forces on the rotating mass due to unbalanced masses is zero.

(iii) A body is said to be in static balance when its centre of gravity is in the axis of rotation.

(iv) The requirement of static ~~balancing is~~ balance is that the sum of all forces on the moving system must be zero.

$$\sum F - ma = 0$$

(v) It is also known as single plane balance, which means that the masses which are generating the inertia forces are in or nearly in the same plane.

Dynamic Balancing

(i) A rotating mass is said to be dynamically balanced when it does not vibrate in its running state.

(ii) To make a rotating mass dynamically balanced, it must first be statically balanced.

(iii) Dynamic balance is a balance due to the action of inertia forces.

(iv) A body is said to be in dynamic balance when the resultant moments which involved in the acceleration of different moving parts is equal to zero.

Advantages of Balancing

(i) Increase quality of operation.
(ii) Minimize vibration
(iii) Minimize Audible & signal noises
(iv) Increase bearing Life.
(v) Minimize power loss.

Balancing of Rotating mass

The process of providing the second mass inorder to counteract the effect of the centrifugal force of the first mass, is called balancing of rotating masses.

The various types of balancing of rotating masses are:

(a) Balancing of a single rotating mass by a single mass rotating in the same plane.

(b) Balancing of a single rotating mass by two masses rotating in different planes.

(c) Balancing of different masses rotating in the same plane.

(d) Balancing of different masses rotating in different plane.

(a) Balancing of a single rotating mass by a single mass rotating in the same plane :→

Consider a disturbing mass of m attached to a shaft rotating at ω rad/s.

Let r_1 is the radius of rotation of mass m_1

∴ centrifugal force exerted by the mass m_1 on the shaft

$$F_{c_1} = m_1 \cdot \omega^2 \cdot r_1 \quad \text{—(i)}$$

Inorder to counteract the effect of centrifugal force, a balancing mass m_2 may be attached in the same plane of rotation

∴ centrifugal force due to mass m_2.

$$F_{c_2} = m_2 \cdot \omega^2 \cdot r_2 \quad \text{—(ii)}$$

Equating equ^n (i) & (ii)

$$m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2$$

$$\Rightarrow m_1 \cdot r_1 = m_2 \cdot r_2$$

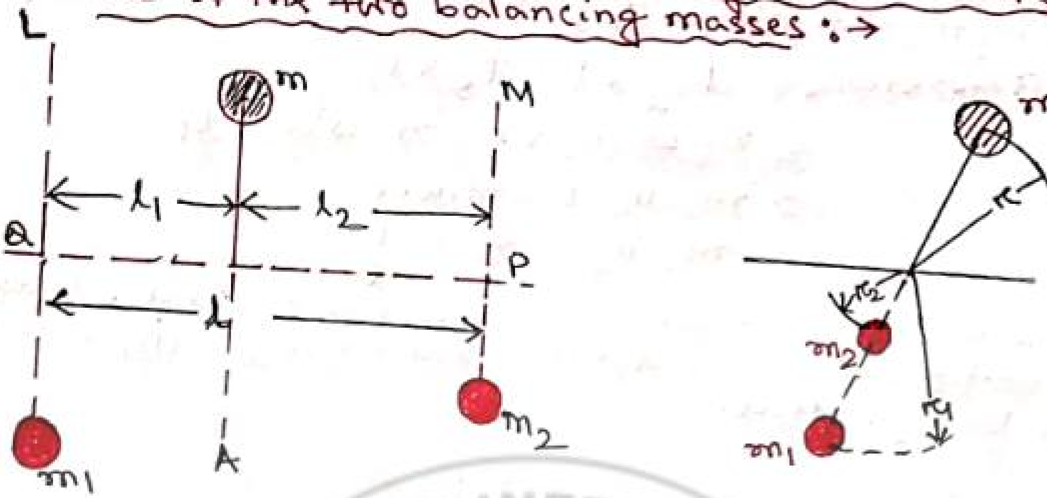
(b) Balancing of a single rotating mass by two masses rotating in different planes :→

Two Cases :-

(i) The plane of the disturbing mass may lie in betn the planes of the two balancing masses.

(ii) The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

(i) When the plane of the disturbing mass lies in between the planes of the two balancing masses :→



Consider a disturbing mass m laying in a plane A, to be balanced by two rotating masses m_1 & m_2 lying in plane L & M.

Let r, r_1 & r_2 be the radius of rotation of the masses in plane A, L, M respectivly.

Let l_1 = distance betⁿ the planes A & L
l_2 = " " " " A & M
l = " " " " L & M

Centrifugal force exerted by mass m.

$$F_C = m \cdot \omega^2 \cdot r$$

Centrifugal force exerted by mass m_1

$$F_{C_1} = m_1 \cdot \omega^2 \cdot r_1$$

Centrifugal force exerted by mass m_2

$$F_{C_2} = m_2 \cdot \omega^2 \cdot r_2$$

∴ Net centrifugal force $F_C = F_{C_1} + F_{C_2}$

$$\Rightarrow m \cdot \omega^2 \cdot r = m_1 \cdot \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\Rightarrow \boxed{m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2} \quad \text{——— (i)}$$

Inorder to find the dynamic force at the bearing Q of the shaft, taking moments about P.

Therefore

$$F_{C_1} \times l = F_C \times l_2$$

$$\Rightarrow m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\Rightarrow m_1 \cdot r_1 \times l = m \cdot r \cdot l_2$$

$$\Rightarrow m_1 \cdot r_1 = m \cdot r \cdot \frac{l_2}{l} \quad \text{——— (ii)}$$

Similarly, the dynamic force at the bearing P of a shaft, taking moment about Q,

Therefore, $F_{C_2} \times l = F_C \times l_1$

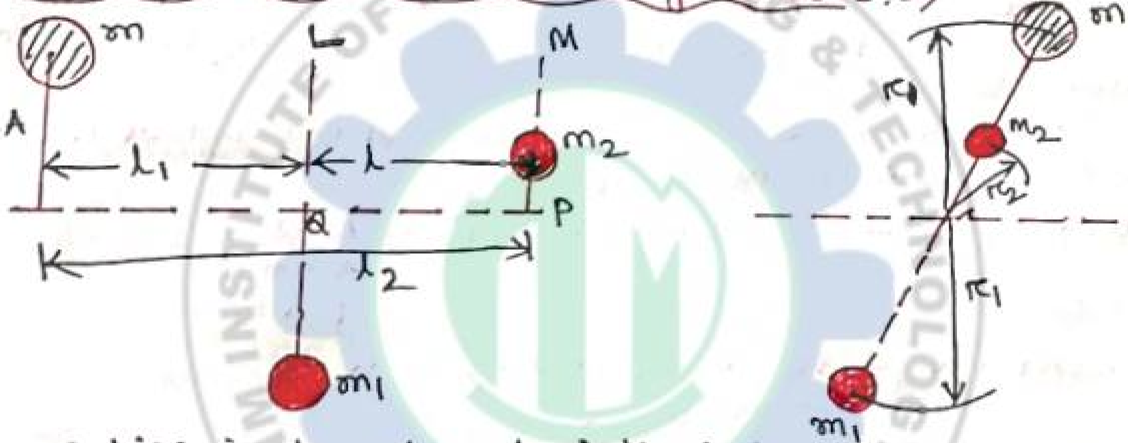
$$\Rightarrow m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\Rightarrow m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1$$

$$\Rightarrow m_2 \cdot r_2 = m \cdot r \cdot \frac{l_1}{l} \quad \text{——— (iii)}$$

The equ^n(i) represents the condition for static balance, but in order to achieve dynamic balance, equ^{n}s (ii) & (iii) must also be satisfied.

(ii) When the plane of the disturbing mass lies on one end of the planes of the balancing masses: →



m lies in the plane A. & the balancing masses lies in the plane L & M.

~~From the figure Fc~~ The condition must be satisfied inorder to balance the system is

$$\boxed{F_C + F_{C_2} = F_{C_1}}$$

$$\Rightarrow m \cdot \omega^2 \cdot r + m_2 \cdot \omega^2 \cdot r_2 = m_1 \cdot \omega^2 \cdot r_1$$

$$\Rightarrow m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \quad \text{——— (i)}$$

The dynamic force at bearing Q, (Balancing force in plane L), taking moment about P.

$$F_{C_1} \times l = F_C \times l_2$$

$$\Rightarrow m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\Rightarrow m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2$$

$$\Rightarrow m_1 \cdot r_1 = m \cdot r \frac{l_2}{l} \quad \text{——— (ii)}$$

The dynamic force at bearing P of the shaft (Balancing force in plane (M). Taking moment about Q.

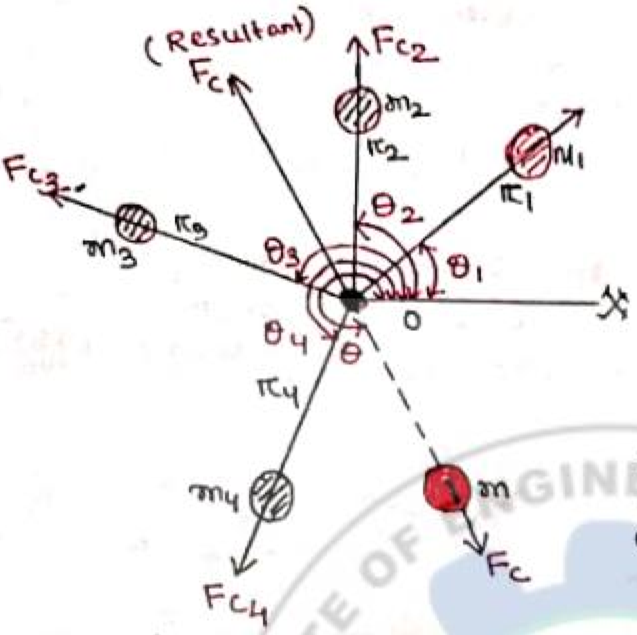
$$F_{C_2} \times l = F_C \times l_1$$

$$\Rightarrow m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\Rightarrow m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1$$

$$\Rightarrow m_2 \cdot r_2 = m \cdot r \cdot \frac{l_1}{l} \quad \text{——— (iii)}$$

(c) Balancing of several masses rotating in the same plane



Consider several masses of magnitude m_1, m_2, m_3 & m_4 at a distance of r_1, r_2, r_3 & r_4 from the axis of rotation of the shaft.

Let $\theta_1, \theta_2, \theta_3$ & θ_4 angle of these masses with the horizontal line OX.

Let these masses rotates about an axis through 'O', with a constant angular speed ω rad/s.

Analytical Method

(i) Find out centrifugal force exerted by the each mass on the rotating shaft.

(ii) Resolve the centrifugal forces horizontally (ΣH) & vertically (ΣV) & their sum. ($\Sigma H + \Sigma V$)

Horizontal centrifugal force

$$\Sigma H = m_1 \cdot r_1 \cos\theta_1 + m_2 \cdot r_2 \cos\theta_2 + \ldots\ldots$$

Vertical centrifugal force

$$\Sigma V = m_1 \cdot r_1 \sin\theta_1 + m_2 \cdot r_2 \sin\theta_2 + \ldots\ldots$$

(iii) magnitude of the resultant centrifugal force

$$F_c = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

(iv) If θ is the angle, which the resultant force make with the horizontal, then

$$\tan\theta = \frac{\Sigma V}{\Sigma H}$$

(v) The balancing force is then equal to the resultant force, but opposite direction.

(vi) Now find out the magnitude of the balancing mass, such that

$$F_c = m \cdot r$$

m = Balancing mass
r = Radius of rotation.

Problem

Four masses m_1, m_2, m_3 & m_4 are 200kg, 300kg, 240kg & 260 kg respectivly. The corresponding radii of rotation are 0.2m, 0.15m, 0.25m, & 0.3m respectivly. The angles betn successive masses are 45°, 75°, & 135°. Find the position & magnitude of the balance mass required, if its radius of rotation is 0.2m.

Soln

$m_1 = 200kg$, $m_2 = 300kg$, $m_3 = 240kg$, $m_4 = 260kg$

$r_1 = 0.2m$, $r_2 = 0.15m$, $r_3 = 0.25m$, $r_4 = 0.3m$

$\theta_1 = 0°$, $\theta_2 = 45°$, $\theta_3 = 45 + 75 = 120°$, $\theta_4 = 45° + 75° + 135° = 255°$

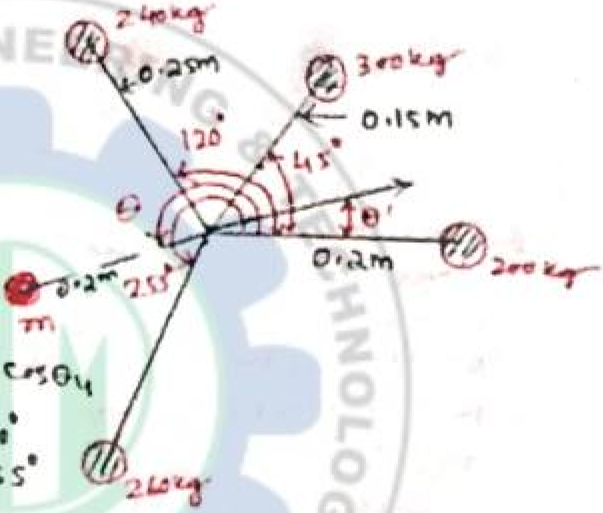
$r = 0.2m$

$m_1 \cdot r_1 = 200 \times 0.2 = 40 kg\text{-}m$

$m_2 r_2 = 300 \times 0.15 = 45 kg\text{-}m$

$m_3 r_3 = 240 \times 0.25 = 60 kg\text{-}m$

$m_4 r_4 = 260 \times 0.3 = 78 kg\text{-}m$



Resolving Horizontally

$\Sigma H = m_1 r_1 \cos\theta_1 + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_4 r_4 \cos\theta_4$

$= 40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°$

$= 40 + 31.8 - 30 - 20.2$

$= 21.6 \ kg\text{-}m$

Resolving Vertically $\Sigma V = m_1 r_1 \sin\theta_1 + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_4 r_4 \sin\theta_4$

$= 40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°$

$= 0 + 31.8 + 52 - 75.3 = 8.5 \ kg\text{-}m$

Resultant $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2 kg\text{-}m$

Now $m \cdot r = R = 23.2$

$\Rightarrow m = \frac{23.2}{r} = \frac{23.2}{0.2} = 116 kg$ Ans

$\tan\theta' = \frac{\Sigma V}{\Sigma H} = \frac{8.5}{21.6} = 0.3935$

$\Rightarrow \theta' = 21.48°$

Therefore the angle of the balancing mass from horizontal mass of 200kg

$\theta = 180° + 21.48° = 201.48°$ Ans

(d) Balancing of Several masses rotating in different Planes :→

↳ When several masses revolve in different planes, they may be transferred to a reference plane (R.P)

↳ R.P may be defined as the plane passing through a point on axis of rotation & ⊥ar to it.

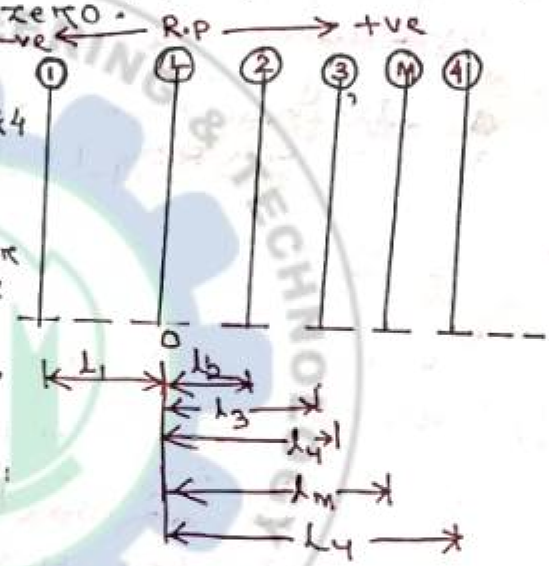
↳ In order to have a complete balance of the several revolving masses in different planes, two conditions must be satisfied.

(i) The force in the reference plane must balance i.e resultant force must be zero.

(ii) The couples about the reference plane must balance i.e resultant couple must be zero.

$m_1, m_2, m_3, \& m_4$ are the four masses revolving in planes 1,2,3 & 4 respectively.

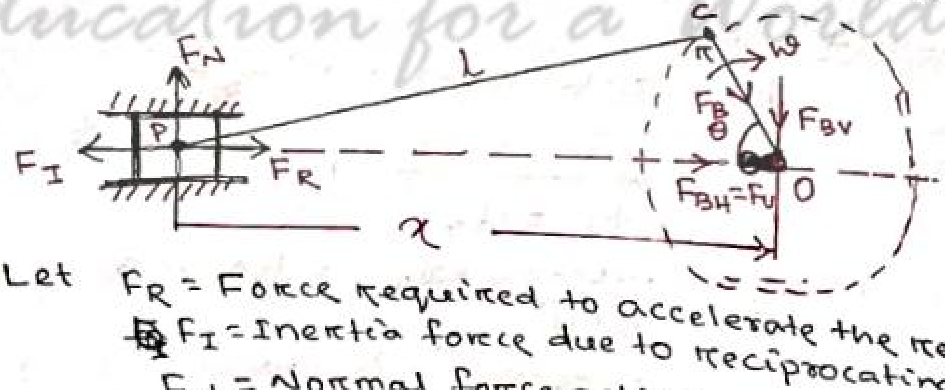
If we take L as the reference plane. The distances of all the other planes to the left to the reference plane may be taken as negative. & those to the right as positive.



Principles of Balancing of Reciprocating Parts :→

The resultant of all the forces acting on the body of the engine due to inertia forces only is known as unbalanced force or shaking force. If the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but unbalanced couple will be present.

Consider a horizontal reciprocating engine mechanism.



Let F_R = Force required to accelerate the reciprocating parts

F_I = Inertia force due to reciprocating parts.

F_N = Normal force acting on cross-head guides

F_B = Force acting on the crank shaft bearing or main bearing.

↳ The force $F_{BH} = F_U$ is an unbalanced force or shaking force & required to be properly balanced.

↳ F_R & F_I Horizontal component ↳ F_B, F_I — These forces are equal & opposite in direction.

↳ The force F_N & the vertical component of F_B i.e F_{BV} are equal & opposite & thus form a Shaking Couple of magnitude $F_N \times x$ or $F_{BV} \times x$.

↳ The purpose of balancing the reciprocating masses is to eliminate the shaking force & a shaking couple. By adding appropriate balancing masses, we can reduce shaking force & shaking couple, but it is not practical to eliminate them completly. Thus the reciprocating masses are only partially balanced.

Primary & Secondary Unbalanced Forces of reciprocating Masses:-

Let m = mass of reciprocating parts

l = Length of Connecting rod

r = Radius of the crank

θ = Angle of inclination of the crank

ω = Angular speed of crank

n = Ratio of length of connecting rod to the crank radius $= \frac{l}{r}$

Accelaration of the reciprocating parts

$$a_R = \omega^2 . r \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

Inertia force or force required to accelerate the reciprocating parts

$$F_I = F_R = \text{mass} \times \text{Acceleration}$$

$$= m.\omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

∴ Unbalanced force

$$F_U = m.\omega^2.r\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$= m\omega^2 r\cos\theta + m\omega^2 r \times \frac{\cos 2\theta}{n}$$

$$= F_P + F_S$$

The term $(m\omega^2 r\cos\theta)$ is known as primary unbalanced force. (F_P)

The term $(m\omega^2 r \times \frac{\cos 2\theta}{n})$ is known as Secondary unbalanced force.

* The F_p is maximum, when $\theta = 0^\circ$ or 180°. So, F_p is max^m twice in one revolution of crank.

max^m primary unbalanced force $F_{P(max)} = m \cdot \omega^2 \cdot r$

* The F_s is maximum, when $\theta = 0^\circ, 90^\circ, 180^\circ$ & 360°. So F_s is max^m four times in one revolution of crank.

max^m secondary unbalanced force

$$F_{s\,(max)} = m \cdot \omega^2 \times \frac{r}{n}$$

$$\boxed{F_{P(max)} = \frac{1}{n} \times F_{S\,(max)}}$$

Causes of unbalancing

(a) Thermal Distortion :-> It occurs when the metal is expose to temperature which expands when in contact with heat. When the temperature increases heat increases which causes the machinary parts to expands.

(b) Distortion from Stress :-> Stress rellief should be provided in the metal component without which the machine component will distort itself to adjust.

(C) Buildups & Deposits :-> metal components are exposed to oil which may be easily distorted without a maintenance & inspection oil can seep into the ports causes unbalance.

Effects of unbalance

(i) vibration
(ii) Noise
(iii) Decreased life of bearings
(iv) Unsafe work conditions
(v) Reduced machine life
(vi) Increased maintenance

Objective Type

(1) The balancing of rotating & reciprocating parts of an engine is necessary when it runs at ___

(a) Slow speed (b) Medium speed (c) High speed

Ans:- C

(2) A disturbing mass m_1 attached to a rotating shaft may be balanced by a single mass m_2 attached in the same plane of rotation as that of m_1 such that

(a) $m_1 \cdot r_2 = m_2 \cdot r_1$ (b) $m_1 r_1 = m_2 r_2$ (c) $m_1 \cdot m_2 = r_1 \cdot r_2$

Ans:- b

(3) For static balancing of a shaft

(a) The net dynamic force acting on the shaft is equal to zero.

(b) the net couple due to the dynamic forces acting on the shaft is equal to zero.

(c) Both a & b

(d) None of the above

Ans:- a

(4) For dynamic balancing of a shaft

(a) The net dynamic force acting on the shaft is equal to zero.

(b) The net couple due to dynamic forces acting on the shaft is equal to zero

(c) Both a & b

(d) None of the above

Ans:- C

(5) In order to have a complete balance of the several revolving masses in different planes

(a) The resultant force must be zero

(b) The resultant couple must be zero

(c) Both the resultant force & couple must be zero.

(d) None of the above.

Ans:- C

(6) The primary unbalanced force is max^m when the angle of inclination of the crank with the line of stroke is

(a) 0° (b) 90° (c) 180° (d) 360°

(7) Secondary forces in reciprocating mass on engine frame are

(a) of same frequency as of primary forces.

(b) twice the frequency as of primary forces

(c) Fourtimes the frequency as of primary forces

(d) None of the above

Ans:- b

(8) At which angle primary unbalanced force in reciprocating engine mechanism is maximum?

(a) 0° (b) 90° (c) 360° (d) All of the above

Ans:- a

(9) Secondary force in reciprocating engine mechanism is caused due to ___

(a) S.H.M of reciprocating parts

(b) oscillation of reciprocating parts

(c) obliquity of arrangement of reciprocating parts

(d) All of the above.

Ans:- C

(10) Which of the following factors are not responsible for unbalancing in rotating system?
(a) Errors
(b) Tolerances
(c) Shape of the rotor
(d) None of the above
Ans:- d

(11) Often an unbalance of forces is produced in rotary or reciprocating machinery due to the ——
(a) centripetal forces
(b) Centrifugal forces
(c) Friction forces
(d) Inertia forces
Ans:- d

(12) Which of the following is true for centrifugal force causing unbalance?
(a) Direction changes with rotation
(b) Magnitude changes with rotation
(c) Direction & magnitude both changes with rotation.
(d) Direction & magnitude both remain unchanged with rotation.
Ans:- a

(13) In a revolving rotor, the centrifugal force remains balanced as long as the centre of the mass of rotor lies ——
(a) Below the axis of shaft
(b) on the axis of shaft
(c) Above the axis of shaft
(d) Away from the axis of shaft
Ans:- b

(14) If the unbalanced ~~force~~ system is not set right then.
(a) static forces developed.
(b) Dynamic forces developed
(c) Tangential forces developed
(d) Radial forces developed
Ans:- a

(15) What is not the effect of unbalanced forces?
(a) Load on bearings
(b) Dangerous vibrations
(c) Stresses in various members
(d) violation of conservation of mass principle
Ans:- d

(16) What is the effect of rotating mass of a shaft on a system?
(a) Bend the shaft
(b) Twist the shaft
(c) Extend the shaft
(d) Compress the shaft
Ans: - a

Vibration

- It is the oscillation or repetitive motion of an object around an equilibrium position. This means that all parts of the body are moving together in the same direction at any point in time.
- Vibration means quickly moving back & forth (up & down) about a point of equilibrium.

Mechanical vibration

It is defined as the measurement of a periodic ~~[illegible]~~ process of oscillations with respect to an equilibrium point.

Vibratory motion

- When an object is displaced from its fixed position & made to move to & fro periodically, it is known as vibratory motion.
- A vibratory motion happens when a particle is vibrated.

Exp:- Movement of simple pendulum, movement of string in various instruments like Guitar, movement of cell phone in vibration mode.

Difference betn oscillatory motion & vibratory motion

Oscillatory motion

It means to & fro movement of body about its mean position. It can be defined as distance covered by movement about its equilibrium position.

Vibratory motion

Vibratory motion can be in all direction, it is the physical change brought about due to the movement of the body.

Note → When the elastic bodies such as spring, beam & a shaft are displaced from the equilibrium position by the application of external forces & then released, they are subjected to a vibratory motion.

Terms used in vibratory motion :-

(a) **Time period or period of vibration** :→ It is the time interval after which the motion is repeated itself. It is expressed in second.

(b) **Cycle** :- It is the motion completed during one time period.

(c) **Frequency** :- It is the number of cycles described in one second. The S.I unit of frequency is Hertz (Hz).

(d) **Amplitude** :- It is the distance covered by a point on a vibrating body measured from its equilibrium position. It is the measured of how far the object moves outwards from the starting position. Represented by A & SI unit is meter.

Types of vibratory motion

Free or Natural vibration :→

↳ When no external force acts on the body, after giving it an initial displacement, then the body is said to be under free or natural vibration.

↳ The frequency of free vibration is called free or natural frequency.

Forced vibration

↳ When the body vibrates under the influence of external force, then the body is said to be forced vibration.

↳ The vibration have the same frequency as the applied force.

Damped vibration :→

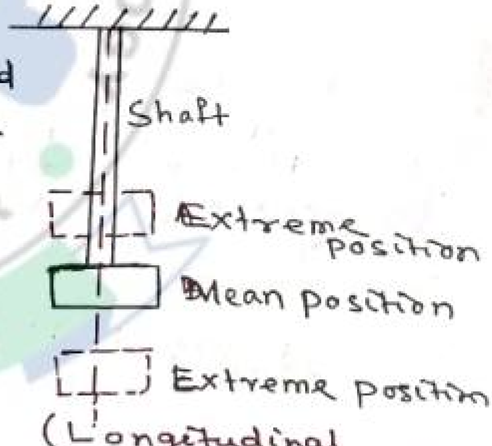
When there is a reduction in amplitude over every cycle of vibration, the motion is said to be damped vibration.

Types of Free vibrations

(a) Longitudinal vibration :-

When the particles of the shaft moves parallel to the axis of the shaft, then the vibration is known as longitudinal vibration.

In this case, the shaft is elongated & shortened alternately & thus the tensile & compressive stresses are induced alternately in the shaft.

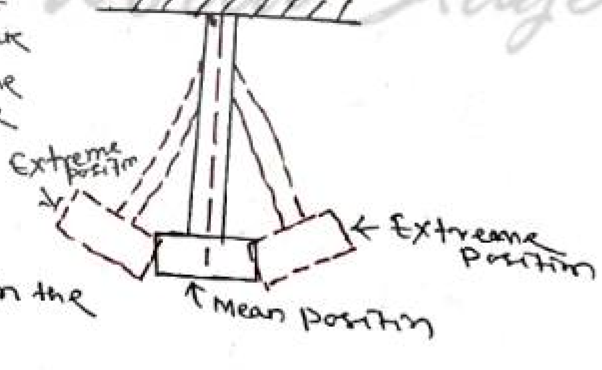


(Longitudinal Vibration)

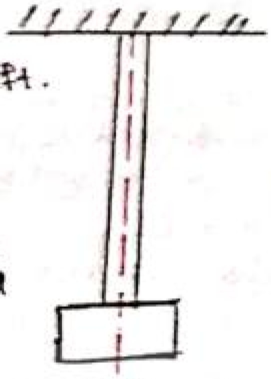
(b) Transverse vibration

When the particle of the shaft moves approximately perpendicular to the axis of the shaft, then the vibration is known as transverse vibration.

In this case, the shaft is straight & bent alternately & bending stresses are induced in the shaft.



(C) Torsional Vibration



When the particle of the shaft move in a circle about the axis of the shaft, then the vibrations are known as torsional vibration.

In this case, the shaft is twisted & untwisted alternately & the torsional shear stresses are induced in the shaft.

Natural Frequency of Free Longitudinal Vibrations

Consider a spring

Let S = Stiffness of spring in N/m
m = mass of the body in kg
W = weight of the body in Newtons
= m·g
δ = Deflection of spring in Newton

* Time period $t_p = \frac{2\pi}{\omega}$

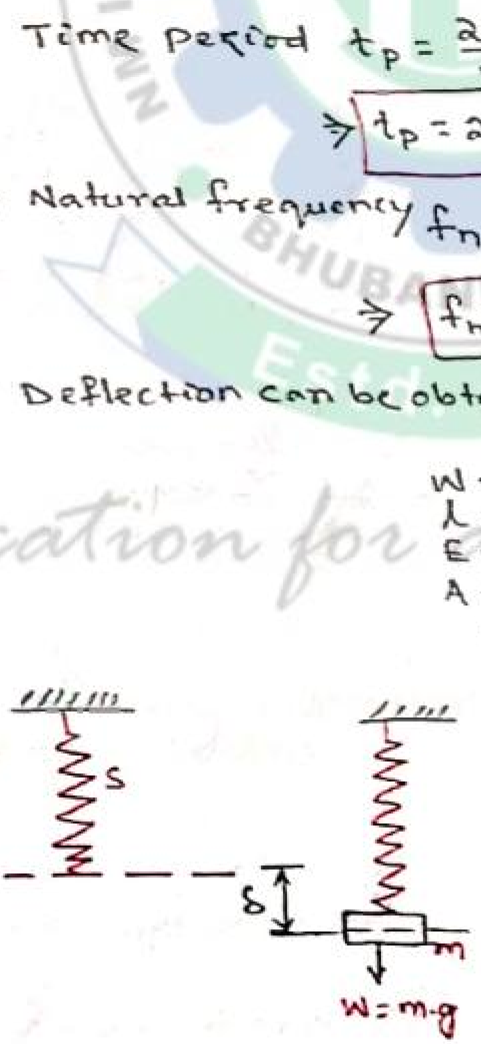
$$\Rightarrow \boxed{t_p = 2\pi\sqrt{\frac{m}{s}}} \quad \because \omega = \sqrt{\frac{s}{m}}$$

* Natural frequency $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

$$\Rightarrow \boxed{f_n = \frac{0.4985}{\sqrt{\delta}}\ Hz} \quad \because g = 9.81\ m/s^2$$

* Deflection can be obtained by $\boxed{\delta = \frac{W \cdot l}{E \cdot A}}$

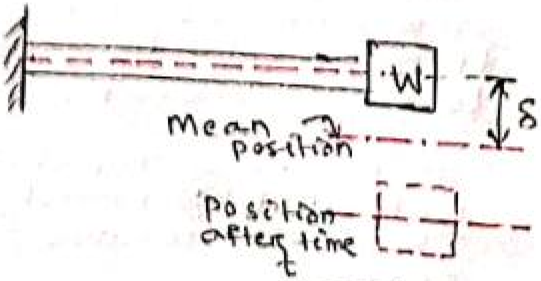
W = Load attached
l = Length of spring
E = Young's modulus of the material of spring
A = Cross-sectional area of spring.
$= \frac{\pi}{4}d^2$



$$\boxed{f_n = \frac{1}{t_p}}$$

Natural frequency of Free Transverse Vibration

Consider a shaft, whose one end is fixed & the other end carries a weight W.



S = Stiffness of shaft
δ = deflection of shaft
m = mass of the body = $\frac{W}{g}$

Time period $t_p = 2\pi\sqrt{\frac{m}{S}}$

$$f_n = \frac{1}{t_p}$$

Natural frequency $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

The deflection can be obtained from, static deflection of a cantilever beam

$$\delta = \frac{Wl^3}{3EI}$$

W = Load at free end
l = Length of the shaft
E = Young's modulus for the material of shaft
I = moment of inertia of the shaft in m^4.
$= \frac{\pi}{64} \times d^4 \; m^4$
d = dia. of shaft

Critical or Whirling speed of a Shaft

↳ The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.

↳ The critical or whirling speed is the same as the natural frequency of the transverse vibration, but its unit will be r.p.s.

critical speed (ω_c, , N_c) $N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$

$$N_c = \frac{0.4985}{\sqrt{\delta}} \text{ r.p.s}$$

Damping Factor or Damping Ratio

The ratio of the actual damping coefficient (C) to the critical damping coefficient (C_c) is known as damping factor.

Mathematically

$$\text{Damping factor} = \frac{c}{c_c} = \frac{c}{2m\omega_n} \quad (\because C_c = 2m\omega_n)$$

ω_n = Natural circular frequency of vibration of system rad/s
ω = circular frequency of the system in rad/s.

Logarithmic Decrement

It is defined as the natural logarithm of the amplitude reduction factor. ~~amplitude~~

→ It is the ratio of any two successive amplitudes on the same side of the mean position.

Logarithmic Decrement $$\delta = \frac{2\pi \times c}{\sqrt{(c_c)^2 - c^2}}$$

c = Actual damping coefficient

c_c = critical damping coefficient.

Effect of vibration in Mechanical System

Reduce bearing life.

Damage equipment

Create noise

Safety problem

consume excessive power

Damage product quality

Causes of vibration

(i) Imbalance :→ Imbalance could be caused by manufacturing defects or maintenance issues.

As machine speed increases, the effects of imbalance become greaters. Imbalance can severely reduce bearing life.

(ii) Misalignment :→ Misalignment may be caused during assembly or develop over time, due to thermal expansion, component shifting or improper reassembly after maintenance. The resulting vibration may be radial or axial or both.

(iii) Wear :-

→ The bearing rollers will cause a vibration each time they travel over the damaged area.

→ A gear tooth that is heavily chipped or a belt drive that is breaking down also can produce vibration.

(iv) Looseness :→

→ If the component that is vibrating has loose bearings or is loosely attached to its mounts, then it create vibration.

Remedies of vibration

(i) There should be no rigid connection bet^n the units (machine, engine; vibrating body etc) & the base otherwise the undesired vibration will occur. It damage the supporting structure.

(ii) It should be ensured that the isolators remain together in case the damping material fails (rubber, cork, belt, etc)

(iii) The materials normally used for isolation are rubber, belt, cork, metallic springs, etc. These are put between the foundation & the vibrating body.

(iv) Rubber is an isolator usefull for shearing load. It sound transmissibity is very low. But it is preferred for light loads & high frequency oscillation.

(v) The damping factor of belt is high, so it is used for low frequency ratios.

(vi) Cork is suitable for compressive loads. At high loads it becomes flexible.

Objective Type

(1) When there is a reduction in amplitude over every cycle of vibration, then the body is said to have

(a) Free vibration (b) Forced vibration (c) damped vibration.

Ans:- c

(2) Longitudinal vibrations are said to be occur when the particles of a body moves

(a) Perpendicular to its axis (c) in a circle about its axis

(b) parallel to its axis (d) None of the above

Ans:- b

(3) When a body is subjected to transverse vibration, the stress induced a body will be

(a) Shear stress (c) compressive stress

(b) Tensile Stress (d) None of the above

Ans:- b

(4) The natural frequency (in Hz) of free longitudinal vibrations is equal to

(a) $\frac{1}{2\pi}\sqrt{\frac{s}{m}}$ (c) $\frac{0.4985}{\sqrt{\delta}}$

(b) $\frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$ (d) Any one of these

Ans:- d

(5) The factor which affects the critical speed of the shaft is

(a) Diameter of the disc (c) Eccentricity

(b) Span of the shaft (d) All of these

Ans:- d

(6) In under damped vibrating system, if x_1 & x_2 are the successive values of the amplitude on the same side of the mean position, then the logarithmic decrement is equal to —

(a) x_1/x_2 (c) $\log_e(x_1/x_2)$

(b) $\log(x_1/x_2)$ (d) $\log(x_1 \cdot x_2)$

Ans:- b

(7) The ratio of the max^m displacement of the forced vibration to the deflection due to the static force, is known as

(a) damping factor (c) Logarithmic decrement

(b) Damping coefficient (d) magnification factor

Ans:- d

(8) In vibration isolation system, if ω/ω_n is less than $\sqrt{2}$, then for all values of the damping factor, the transmissibility will be

(a) Less than unity (c) Greater than unity

(b) Equal to unity (d) Zero

Ans:- c

(9) In vibration isolation system, if $\omega/\omega_n > 1$, then the phase difference between the transmitted force & the disturbing force is

(a) 0° (c) 180°

(b) 90° (d) 270°

Ans:- c

(10) When there is a reduction in amplitude over every cycle of vibration then the body is said to have
(a) Free vibration
(b) Forced vibration
(c) Damped vibration
(d) None of these
Ans:- c

~~(11) Longitudinal vibrations are said to occur when the particles of a body moves:-~~

(11) Which type of vibrations are also known as transient vibrations?
(a) Undamped vibrations
(b) Damped vibrations
(c) Torsional vibrations
(d) Transverse vibrations
Ans:- b

(12) During transverse vibrations, shaft is subjected to which type of stresses?
(a) Tensile stresses
(b) Torsional shear stresses
(c) Bending stresses
(d) All of the above
Ans:- c

(13) What is deterministic vibrations?
(a) Vibrations caused due to known exciting force.
(b) Vibrations caused due to unknown exciting force.
(c) Vibrations which are aperiodic in nature
(d) None of the above
Ans:- a

(14) Which of the following vibrations are classified according to magnitude of actuating force?
(a) Torsional vibrations
(b) Deterministic vibrations
(c) Transverse vibrations
(d) All of the above
Ans:- b

(15) What are discrete parameter systems?
(a) Systems which have infinite number of degree of freedom
(b) System which have finite number of degree of freedom.
(c) System which have no degree of freedom.
(d) None of the above.
Ans:- b

(16) Which among the following is the fundamental equation of S.H.M?
(a) $x + (K/m) \times x = 0$
(b) $x + \omega^2 x = 0$
(c) $x + (K/m)^2 x = 0$
(d) $x^2 + \omega x^2 = 0$
Ans:- b

(17) In damped free vibration, which parameters indicate vibrations?
(a) Natural frequency
(b) Rate of decay of amplitude
(c) Both a & b
(d) None of the above
Ans:- c

(18) What is meant by critical damping coefficient?
(a) Frequency of damped free vibrations is less than zero.
(b) The motion is aperiodic in nature.
(c) Both a & b
(d) None of the above
Ans:- b