STRUCTURAL DESIGN II

(Diploma 5th sem)

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1.1 Introduction

The structure of a building (or other object) is the part which is responsible for maintaining the shape of the building under the influence of the forces, loads and other environmental factors to which it is subjected. It is important that the structure as a whole (or any part of it) does not fall down, break or deform to an unacceptable degree when subjected to such forces or loads. The study of structures involves the analysis of the forces and stresses occurring within a structure and the design of suitable components to cater for such forces and stresses.

As an analogy, consider the human body. Human body comprises a skeleton of 206 bones which constitutes the structure of human body. If any of those bones were to break, or if any of the joints between those bones were to disconnect or seize up, the injured body would 'fail' structurally (and cause a great deal of pain).

Examples of structural components (or 'members', as Engineers and Architects call them) include:
• steel beams, columns, roof trusses and space frames;
• reinforced concrete beams, columns, slabs, retaining walls and foundations;
• timber joists, columns, glulam beams and roof trusses;
• masonry walls and columns.

1.2 What is an engineer?

The word 'engineer' comes from the French word ingénieur, which refers to someone who uses his/her ingenuity to solve problems. An engineer is a problem-solver.

A structural engineer solves the problem of ensuring that a building – or other structure – is adequate (in terms of strength, stability, cost, etc.) for its intended use.

1.3 The structural engineer in the context of related professions

some of the professionals involved in the design of buildings include the following:

• the architect;
• the structural engineer;
• the quantity surveyor.

Of course, this is not an exhaustive list. There are many other professionals involved in building design (for example, building surveyors and project managers) and many more trades and professions involved in the actual construction of buildings.

The architect is responsible for the design of a building with particular regard to its appearance and environmental qualities such as light levels and noise insulation.

The structural engineer is responsible for ensuring that the building can safely withstand all the forces to which it is likely to be subjected, and that it will not deflect or crack unduly in use.

The quantity surveyor is responsible for measuring and pricing the work to be undertaken – and for keeping track of costs as the work proceeds.

So, in short:

(1) The architect makes sure the building looks good.
(2) The (structural) engineer ensures it will stand up.

(3) The quantity surveyor ensures its construction is economical.

1.4 Structural understanding

The basic function of a structure is to transmit loads from the position of application of the load to the point of support and thus to the foundations in the ground. Let us for the time being consider a load as being any force acting externally on a structure.)

Any structure must satisfy the following criteria:
(1) Aesthetics (it must look nice).
(2) Economy (it mustn't cost more than the client can afford – and less if possible).
(3) Ease of maintenance.
(4) Durability. This means that the materials used must be resistant to corrosion, spalling (pieces falling off), chemical attack, rot or insect attack.
(5) Fire resistance. While few materials can completely resist the effects of fire, it is important for a building to resist fire long enough for its occupants to be safely evacuated.

In order to ensure that a structure behaves in this way, one need to develop an understanding and awareness of how the structure works.

1.5 Safety and serviceability

There are two main requirements of any structure: it must be safe and it must be serviceable. 'Safe' means that the structure should not collapse – either in whole or in part. 'Serviceable' means that the structure should not deform unduly under the effects of deflection, cracking or vibration.

Safety

A structure must carry the expected loads without collapsing as a whole and without any part of it collapsing. Safety in this respect depends on two factors:

(1) The loading the structure is designed to carry has been correctly assessed.
(2) The strength of the materials used in the structure has not deteriorated.

From this it is evident that one needs to know how to determine the load on any part of a structure. Furthermore, One also needs to know that materials deteriorate in time if not properly maintained: steel corrodes, concrete may spall or suffer carbonation, timber will rot. The structural engineer must consider this when designing any particular building.
Serviceability

A structure must be designed in such a way that it doesn't deflect or crack unduly in use. It is difficult or impossible to completely eliminate these things – the important thing is that the deflection and cracking are kept within certain limits. It must also be ensured that vibration does not have an adverse effect on the structure – this is particularly important in parts of buildings containing plant or machinery.

If, when one walks across the floor of a building, one feels the floor deflect or 'give' underneath one's feet, it may lead one to be concerned about the integrity of the structure. Excessive deflection does not necessarily mean that the floor is about to collapse, but because it may lead to such concerns, deflection must be 'controlled'; in other words, it must be kept within certain limits. To take another example, if a lintel above a doorway deflects too much, it may cause warping of the door frame below it and, consequently, the door itself may not open or close properly. Cracking is ugly and may or may not be indicative of a structural problem. But it may, in itself, lead to problems. For example, if cracking occurs on the outside face of a reinforced concrete wall then rain may penetrate and cause corrosion of the steel reinforcement within the concrete.

1.6 Composition of a building structure

A building structure contains various elements, the adequacy of each of which is the responsibility of the structural engineer.

A roof protects people and equipment in a building from weather. Walls can have one or more of several functions. The most obvious one is load bearing – in other words, supporting any walls, floors or roofs above it. But not all walls are load bearing. Other functions of a wall include the following:

- partitioning, or dividing, rooms within a building – and thus defining their shape and extent;
- weatherproofing;
- thermal insulation – keeping heat in (or out);
- noise insulation – keeping noise out (or in);
- fire resistance;
- security and privacy;
- resisting lateral (horizontal) loads such as those due to retained earth, wind or water.

A floor provides support for the occupants, furniture and equipment in a building. Floors on an upper level of a building are always suspended, which means that they span between supporting walls or beams. Ground floor slabs may sit directly on the ground beneath.

Staircases provide for vertical movement between different levels in a building.

Foundations represent the interface between the building's structure and the ground beneath it. A foundation transmits all the loads from a building into the ground in such a way that settlement (particularly uneven settlement) of the building is limited and failure of the underlying soil is avoided.

In a building it is frequently necessary to support floors or walls without any interruption or division of the space below. In this case, a horizontal element called a beam will be used. A beam transmits the loads it supports to columns or walls at the beam's ends.

A column is a vertical loadbearing element which usually supports beams and/or other columns above. Laymen often call them pillars or poles or posts. Individual elements of a structure, such as beams or columns, are often referred to as members.

1.7 Need to learn about structures

If one is studying architecture, one may be wondering why one needs to study structures at all. However, as an architect, it is important that one understands the principles of structural behaviour. On larger projects architects work in inter-disciplinary teams which usually include structural engineers. It is therefore important to understand about structural engineering. Remember – if one has difficulty in getting one's model to stand up, it is unlikely that the real thing will stand up either!

1.8 Basic aspects of structures

Structural engineers use the following words (amongst others, of course) in technical discussions:

• force
• reaction
• stress
• moment.

None of these words is new to any bode; they are all common English words that are used in everyday speech. However, in structural engineering each of these words has a particular meaning.

Force

A force is an influence on an object (for example, part of a building) that may cause movement. For example, the weight of people and furniture within a building causes a vertically downwards force on the floor, and wind blowing against a building causes a horizontal (or near horizontal) force on the external wall of the building.

Reaction

If one stands on a floor, the weight of your body will produce a downward force into the floor. The floor reacts to this by pushing upwards with a force of the same magnitude as the downward force due to your body weight.

This upward force is called a reaction, as its very presence is a response to the downward force of your body. Similarly, a wall or a column supporting a beam will produce an upward reaction as a response to the downward forces the beam transmits to the wall (or column) and a foundation will produce an upward reaction to the downward force in the column or wall that the foundation is supporting.

The same is true of horizontal forces and reactions. If one pushes horizontally against a wall, one's body is applying a horizontal force to the wall – which the wall will oppose with a horizontal reaction.

Stress

Stress is internal pressure. A heavy vehicle parked on a road is applying pressure to the road surface – the heavier the vehicle and the smaller the contact area between the vehicle's tyres and the road, the greater the pressure.

As a consequence of this pressure on the road surface, the parts of the road below the surface will experience a pressure which, because it is within an object (in this case, the road) is termed a stress. Because the effect of the vehicle's weight is likely to be spread, or dispersed, as it is transmitted downwards within the road structure, the stress (internal pressure at a point) will decrease the further down you go within the road's construction. So, stress is internal pressure at a given point within, for example, a beam, slab or column. It is likely that the intensity of the stress will vary from point to point within the object. Stress is a very important concept in structural engineering.

Moment

A moment is a turning effect. When one uses a spanner to tighten a nut, mechanically wind up a clock or turn the steering wheel on one's car, one is applying a moment.

1.9 How do structures or parts of structure behave

1.9.1 Compression

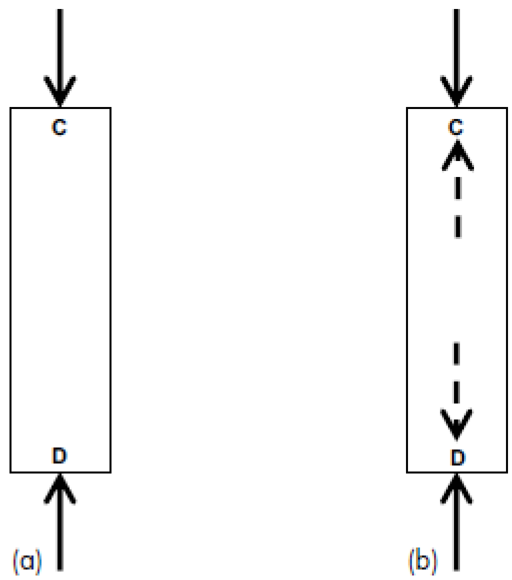


Figure 1.1 A column in compression

Figure 1.1 (a) shows an elevation – that is, a side-on view – of a concrete column in a building. The column is supporting beams, floor slabs and other columns above and the load, or force, from all of these is acting downwards at the top of the column. This load is represented by the downward arrow at the top of the column. Intuitively, we know that the column is being squashed by this applied load – it is experiencing compression.

A downward force must be opposed by an equal upward force (or reaction) if the building is stationary – as it should be. This reaction is represented by the upward arrow at the bottom of the column in Fig. 1.1 (a). Now, not only must the rules of equilibrium (total force up = total force down) apply for the column as a whole; these rules must apply at any and every point within a stationary structure.

Let's consider what happens at the top of the column – specifically, point C in Fig. 1.1 (b). The downward force shown in Fig. 1.1 (a) at point C must be opposed by an upward force – also at point C. Thus there will be an upward force within the column at this point, as represented by the upward broken arrow in Fig. 1.1 (b). Now let's consider what happens at the very bottom of the column – point D in Fig. 1.1 (b). The upward force shown in Fig. 1.1 (a) at point D must be opposed by a downward force at the same point. This is represented by the downward broken arrow in Fig. 1.1 (b).

Look at the direction of the broken arrows in Fig. 1.1 (b). These arrows represent the internal forces in the column. One will notice that they are pointing away from each other. This is always the case when a structural element is in compression: the arrows used to denote compression point away from each other.

Tension

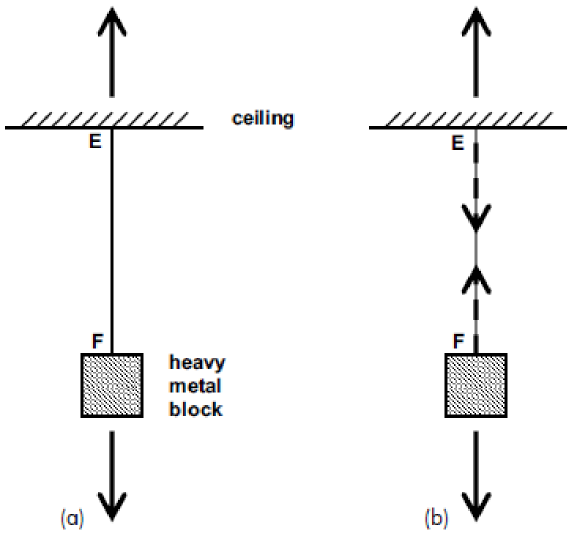


Figure 1.2 A piece of string in tension

Figure 1.2 shows a heavy metal block suspended from the ceiling of a room by a piece of string. The metal block, under the effects of gravity, is pulling the string downwards, as represented by the downward arrow. The string is thus being stretched and is therefore in tension.

For equilibrium, this downward force must be opposed by an equal upward force at the point where the string is fixed to the ceiling. This opposing force is represented by an upward arrow in Fig. 1.2 (a). Note that if the ceiling wasn't strong enough to carry the weight of the metal block, or the string was improperly tied to it, the weight would come crashing to the ground and there would be no upward force (or reaction) at this point. As with the column considered above, the rules of equilibrium (total force up = total force down) must apply at any and every point within this system if it is stationary.

Let's consider what happens at the top of the string. The upward force shown in Fig. 1.2 (a) at point E must be opposed by a downward force – also at this point. Thus there will be a downward force within the string at this point, as represented by the downward broken arrow in Fig. 1.2 (b).

Now let's consider what happens at the very bottom of the string – at the point where the metal block is attached (point F). The downward force shown in Fig. 1.2 (a) at point F must be opposed by an upward force at this point. This upward force within the string at this point is represented by the upward broken arrow in Fig. 1.2 (b).

Look at the direction of the broken arrows in Fig. 1.2 (b). These arrows represent the internal forces in the string. One will notice that they are pointing towards each other. This is always the case when a structural element is in tension: the arrows used to denote tension point towards each other. (An easy way to remember this principle is the letter T, which stands for both Towards and Tension.)

The standard arrow notations for members in (a) tension and (b) compression are shown in Fig. 1.3.

Note: Tension and compression are both examples of axial forces – they act along the axis (or centre line) of the structural member concerned.

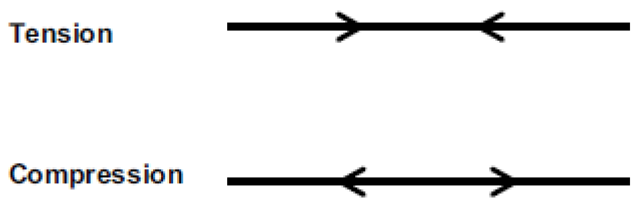


Figure 1.3 Arrow notations for tension and compression

Bending

Consider a simply supported beam (that is, a beam that simply rests on supports at its two ends) subjected to a central point load. The beam will tend to bend, as shown in Fig. 1.4.

The extent to which the beam bends will depend on four things:
(1) The material from which the beam is made. One would expect a beam made of rubber to bend more than a concrete beam of the same dimensions under a given load.

(2) The cross-sectional characteristics of the beam. A large diameter wooden tree trunk is more difficult to bend than a thin twig spanning the same distance.

(3) The span of the beam. Anyone who has ever tried to put up bookshelves at home will know that the shelves will sag to an unacceptable degree if not supported at regular intervals. (The same applies to the hanger rail inside a wardrobe. The rail will sag noticeably under the weight of all those clothes if it is not supported centrally as well as at its ends.)

(4) The load to which the beam is subjected. The greater the load, the greater the bending. The bookshelves will sag to a greater extent under the weight of heavy encyclopedias than they would under the weight of a few light paperback books.

If One carries on increasing the loading, the beam will eventually break. Clearly, the stronger the material, the more difficult it is to break. A timber ruler is quite easy to break by bending;

a steel ruler of similar dimensions might bend quite readily but it's unlikely that one would manage to break it with your bare hands!

This is evidently one way in which a beam can fail – through excessive bending. Beams must be designed so that they do not fail in this way.

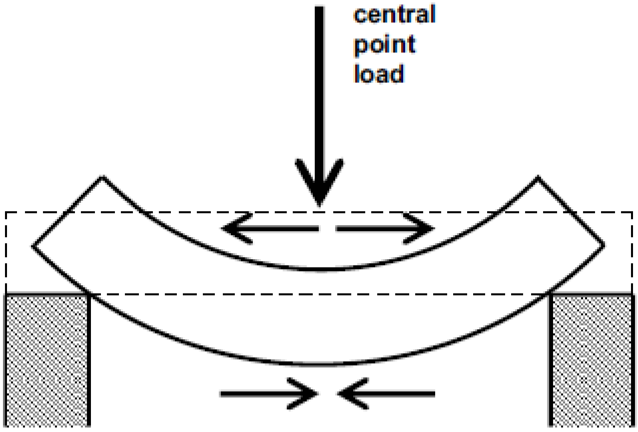


Figure 1.4 Bending in a beam.

Shear

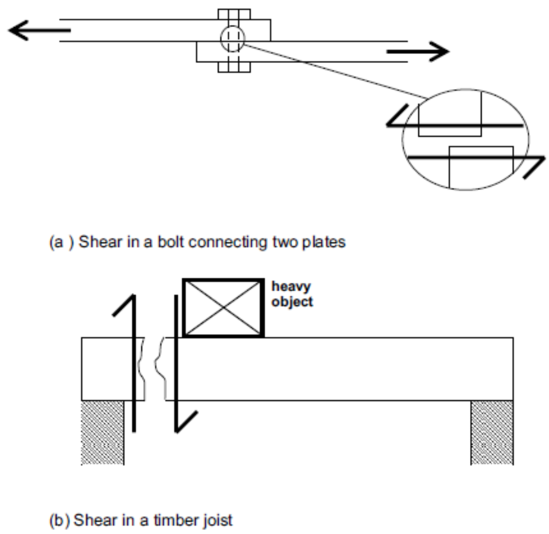


Figure 1.5 Concept of shear

Consider two steel plates that overlap each other slightly, with a bolt connecting the two plates through the overlapping part, as shown in Fig. 1.5 (a). Imagine now that a force is applied to the top plate, trying to pull it to the left. An equal force is applied to the bottom plate, trying to pull it to the right. Let's now suppose that the leftward force is slowly increased, as is the rightward force. (Remember that the two forces must be equal if the whole system is to remain stationary.) If the bolt is not as strong as the plates, eventually we will reach a point when the bolt will break. After the bolt has broken, the top part of it will move off to the left with the top plate and the bottom part will move off to the right with the bottom plate.

Let’s examine in detail what happens to the failure surfaces (that is, the bottom face of the top part of the bolt and the top face of the bottom part of the bolt) immediately after failure. As you can see from the ‘exploded’ part of Fig. 1.5 (a), the two failure surfaces are sliding past each other. This is characteristic of a shear failure.

We’ll now turn our attention to a timber joist supporting the first fl oor of a building, as shown in Fig. 1.5 (b). Let’s imagine that timber joists are supported on masonry walls and that the joists themselves support floorboards, as would be the case in a typical domestic dwelling – such as, perhaps, the house you live in. Suppose that the joists are inappropriately undersized – in other words, they are not strong enough for the loads they are likely to have to support.

Now let’s examine what would happen if a heavy object – for example, some large piece of machinery – was placed on the floor near its supports, as shown in Fig. 1.5 (b). If the heavy object is near the supporting walls, the joists may not bend unduly. However, if the object is heavy enough and the joists are weak enough, the joist may simply break. This type of failure is analogous to the bolt failure discussed above. With reference to Fig. 1.5 (b), the right-hand part of the beam will move downwards (as it crashes to the ground), while the left-hand part of the beam will stay put – in other words, it moves upwards relative to the downward-moving right-hand part of a beam. So, once again, we get a failure where the two failure surfaces are sliding past each other: a shear failure. So a shear failure can be thought of as a cutting or slicing action. So, this is a second way in which a beam can fail – through shear. Beams must be designed so that they do not fail in this way. (Incidentally, the half-headed arrow notation shown in Fig. 1.5 is the standard symbol used to denote shear.)

1.10 Structural Elements and their behaviour

Beams

Beams may be simply-supported, continuous or cantilevered, as illustrated in Fig. 1.6. They are subjected to bending and shear under load, and the deformations under loading are shown by broken lines.

A simply-supported beam rests on supports, usually located at each end of the beam. A continuous beam spans two or more spans in one unbroken unit; it may simply rest on its supports, but more usually it is gripped (or fixed) by columns above and below it. A cantilever beam is supported at one end only; to avoid collapse, the beam must be continuous over, or rigidly fixed at, this support.

Beams may be of timber, steel or reinforced or prestressed concrete.

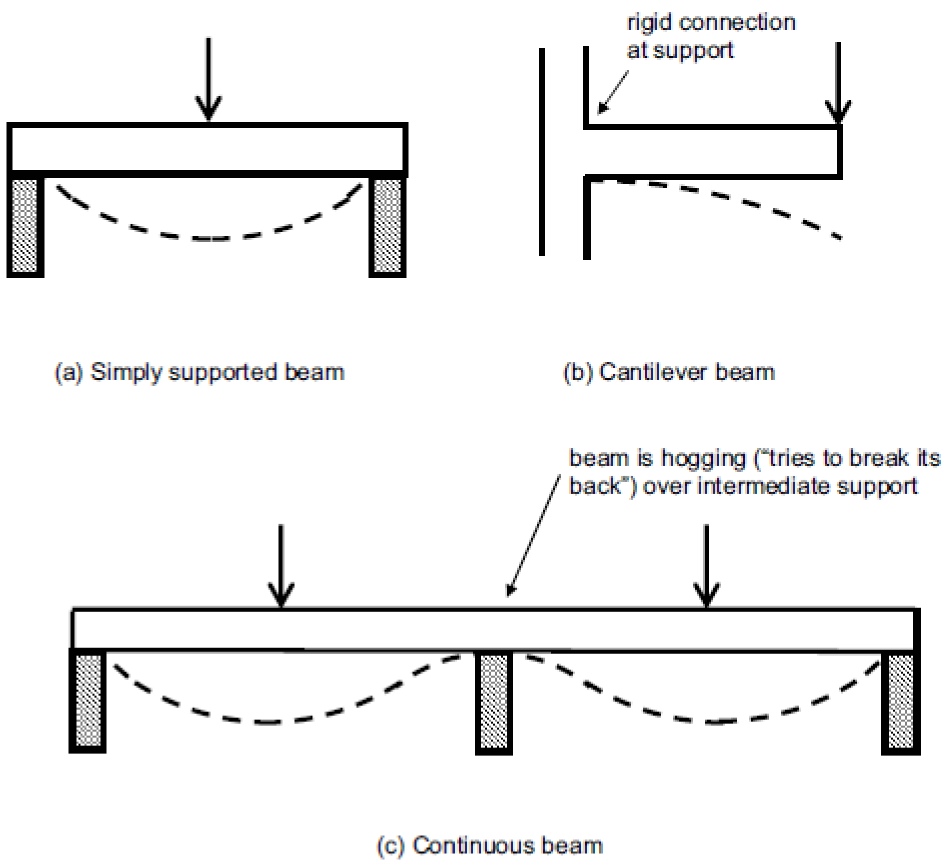


Figure 1.6 Beam types

Slabs

As with beams, slabs span horizontally between supports and may be simply supported, continuous or cantilevered. But unlike beams, which are usually narrow compared with their depth, slabs are usually wide and relatively shallow and are designed to form flooring – see Fig. 1.7.

Slabs may be one-way spanning, which means they are supported by walls on opposite sides of the slab, or two-way spanning, which means that they are supported by walls on all four sides. This description assumes that a slab is rectangular in plan, as is normally the case. Slabs are usually of reinforced concrete and in buildings they are typically 150–300 millimetres in depth. Larger than normal spans can be achieved by using ribbed or waffle slabs, as shown in Fig. 1.7 (c) and (d). Like beams, slabs experience bending.

Columns

Columns (or 'pillars' or 'posts') are vertical and support axial loads, thus they experience compression. If a column is slender or supports a nonsymmetrical arrangement of beams, it will also experience bending, as shown by the broken line in Fig. 1.8 (a). Concrete or masonry columns may be of square, rectangular, circular or cruciform cross-section, as illustrated in Fig. 1.8 (b). Steel columns may be H or hollow section.

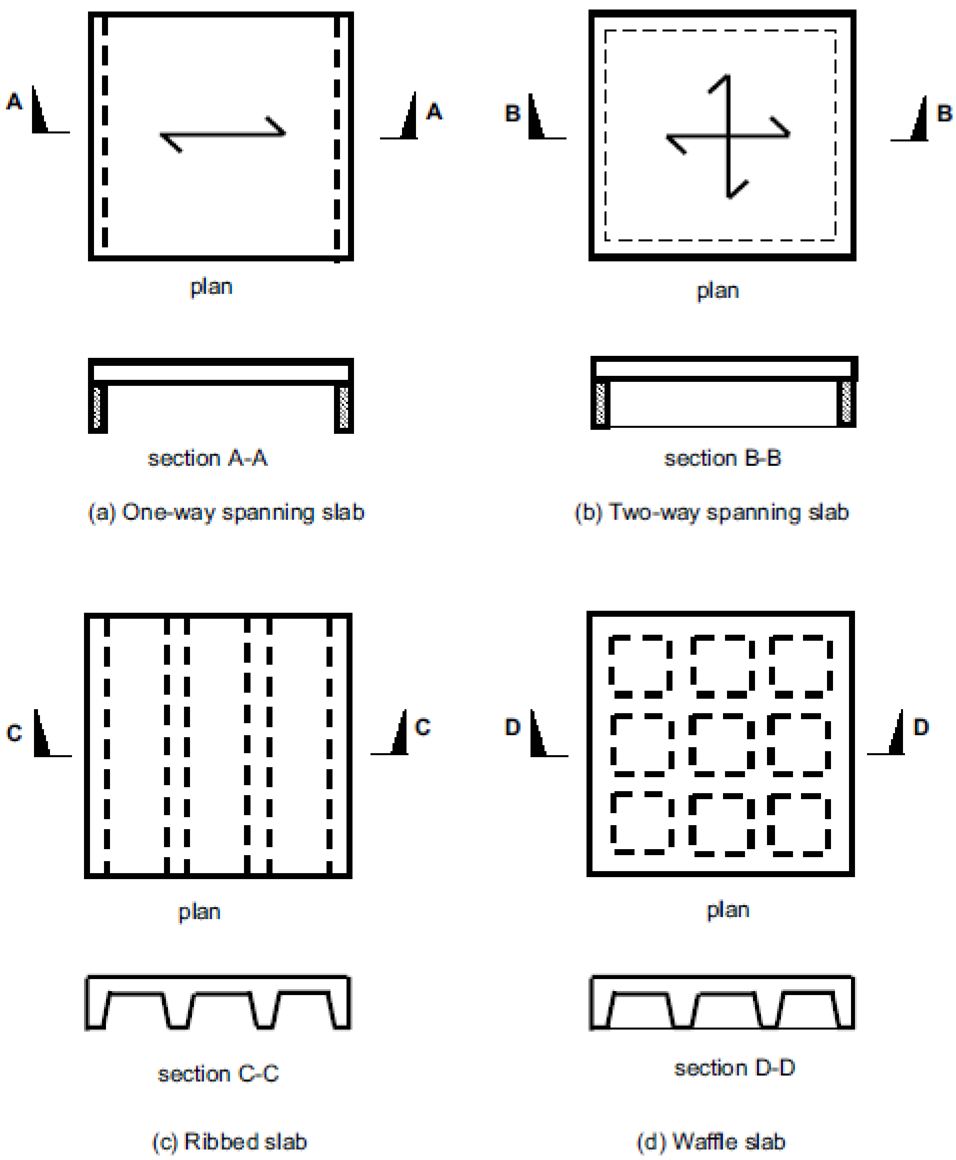


Figure 1.7 Slab types

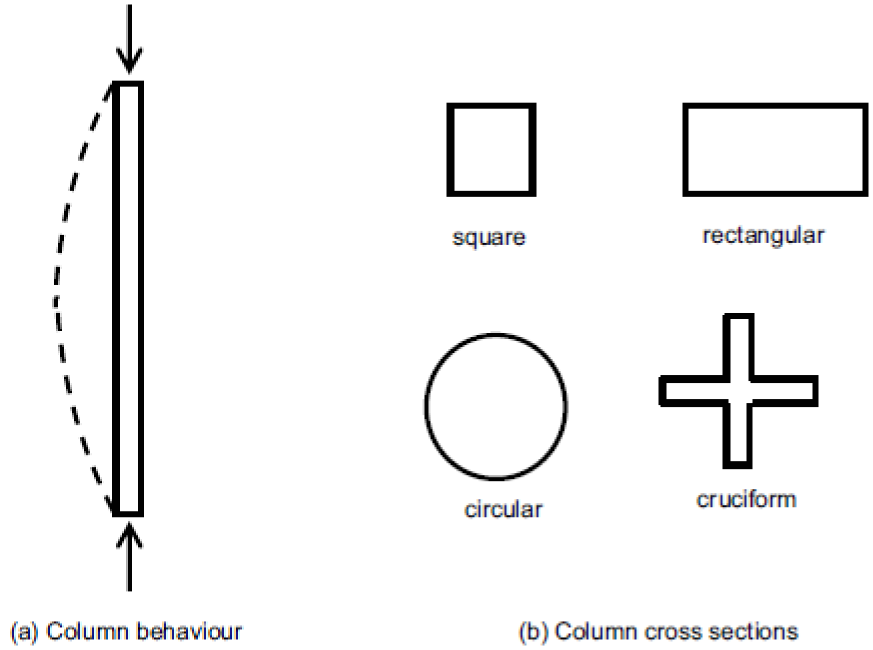


Figure 1.8 Column types

Walls

Like columns, walls are vertical and are primarily subjected to compression, but they may also experience bending. Walls are usually of masonry or reinforced concrete. As well as conventional flat-faced walls you might encounter fin or diaphragm walls, as shown in Fig. 1.9. Retaining walls hold back earth or water and thus are designed to withstand bending caused by horizontal forces, as indicated by the broken line in Fig. 1.9 (c).

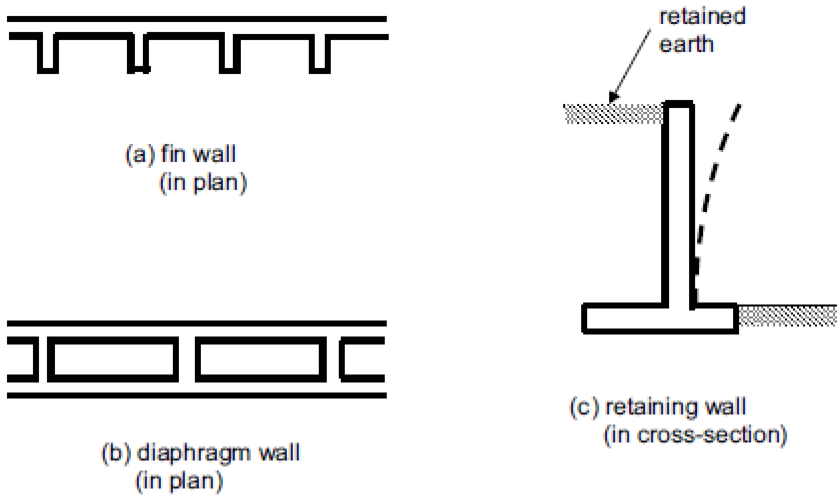


Figure 1.9 Wall types

Foundations

As mentioned previously, everything designed by an architect or civil or structural engineer must stand on the ground – or at least have some contact with the ground. So foundations are required, whose function is to transfer loads from the building safely into the ground. There are various types of foundation. A strip foundation provides a continuous support to load bearing external walls. A pad foundation provides a load-spreading support to a column. A raft foundation takes up the whole plan area under a building and is used in situations where the alternative would be a large number of strip and/or pad foundations in a relatively small space. Where the ground has low strength and/or the building is very heavy, piled foundations are used.

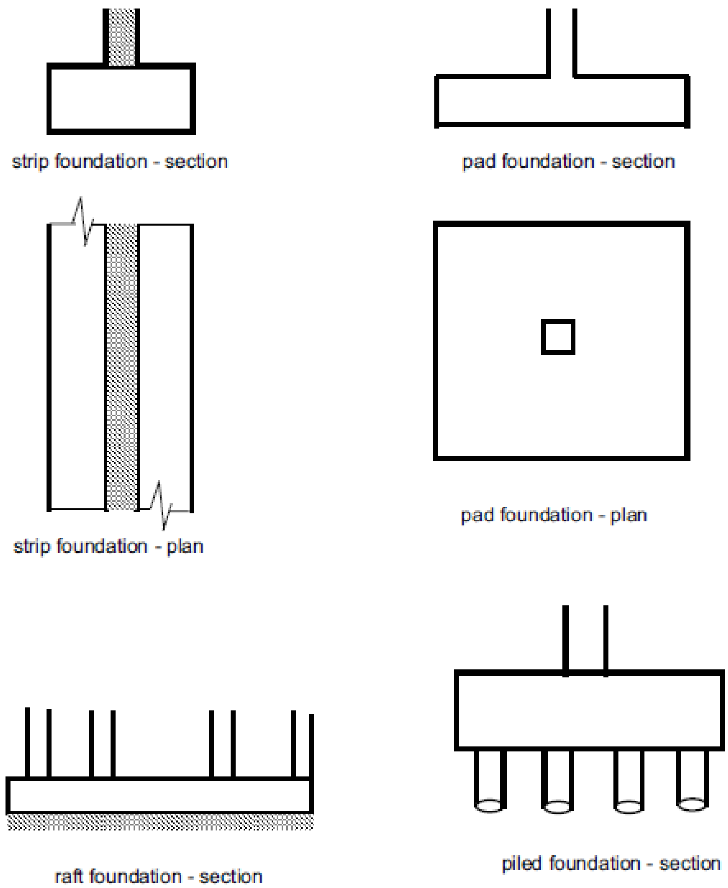


Figure 1.10 Foundation types

These are columns in the ground which transmit the building's loads safely to a stronger stratum. All these foundation types are illustrated in Fig. 1.10. Foundations of all types are usually of concrete, but occasionally steel or timber may be used for piles.

Arches

The main virtue of an arch, from a structural engineering point of view, is that it is in compression throughout. This means that materials that are weak in tension – for example, masonry – may be used to span considerable distances. Arches transmit large horizontal thrusts into their supports, unless horizontal ties are used at the base of the arch. It is to cope with these horizontal thrusts that fl ying buttresses are provided in medieval cathedrals – see Fig. 1.11.

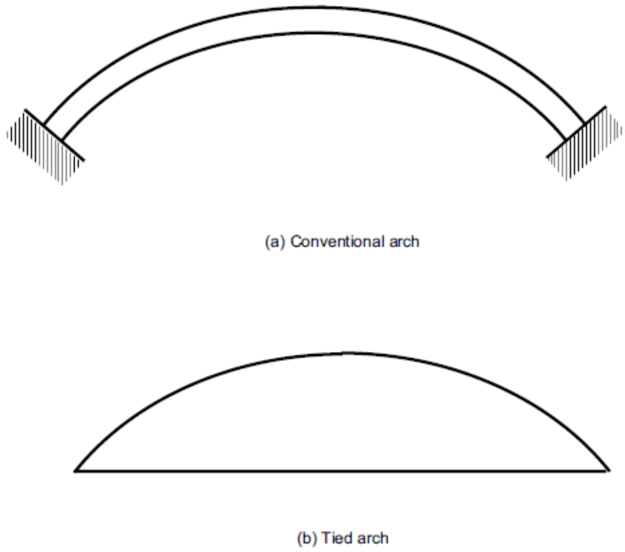


Figure 1.11 Arch types

Trusses

A truss is a two- or three-dimensional framework and is designed on the basis that each ‘member’ or component of the framework is in either pure tension or pure compression and does not experience bending. Trusses are often used in pitched roof construction: timber tends to be used for domestic construction and steel caters for the larger roof spans required in industrial or commercial buildings. Lattice girders, which are used instead of solid deep beams for long spans, work on the same principle – see Fig. 1.12.

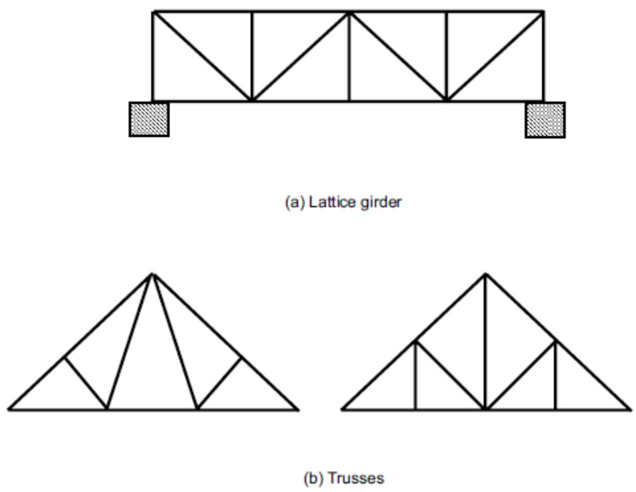


Figure 1.12 Truss types

Portal Frames

A portal frame is a rigid framework comprising two columns supporting rafters. The rafters may be horizontal or, more usually, inclined to support a pitched roof. Portal frames are usually of steel but may be of precast concrete. They are usually used in large single-storey structures such as warehouses or out-of-town retail sheds – see Fig. 1.13.

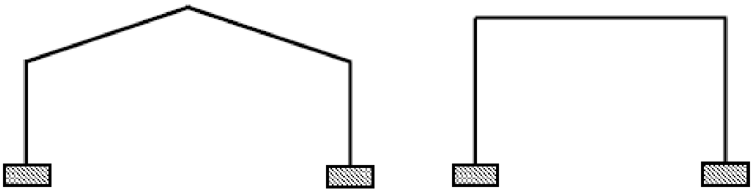
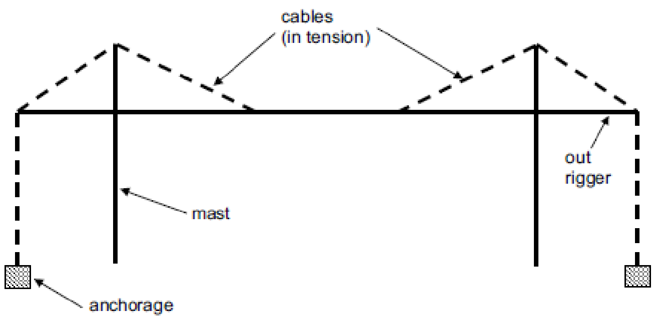


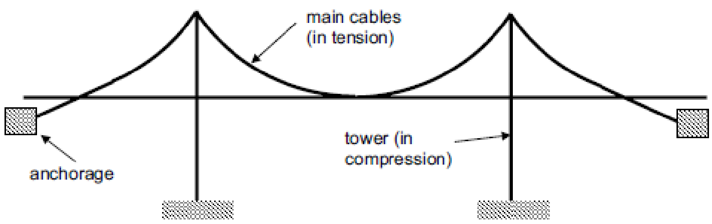
Figure 1.13 Portal frame types

Cable stayed and suspension structures

Cable stayed structures are usually bridges but are sometimes used in building structures where exceptionally long spans are required. Instead of being supported from below by columns or walls, the span is supported from above at certain points by cables which pass over supporting vertical masts and horizontal outriggers to a point in the ground where they are firmly anchored. The cables are in tension and must be designed to sustain considerable tensile forces – see Fig. 1.14.



(a) Cable-stayed structure (in cross-section)



(b) Suspension bridge

Figure 1.14 cable stayed and suspension structures

Cross section types

There is an infi nite range of cross-sectional shapes available. Standard sections are illustrated in Fig. 3.16.

• Beams and slabs in timber and concrete are usually rectangular in cross-section.

• Concrete columns are usually of circular, square, rectangular or cruciform cross-section (see above).

Steel beams are usually of ‘I’ or hollow section.
• Steel columns are usually of ‘H’ or hollow section.
• Prestressed concrete beams are sometimes of ‘T’, ‘U’ or inverted ‘U’ section.
• Members of steel trusses are sometimes of channel or angle sections.
• Steel Z purlins (not illustrated) are often used to support steel roofi ng or cladding.

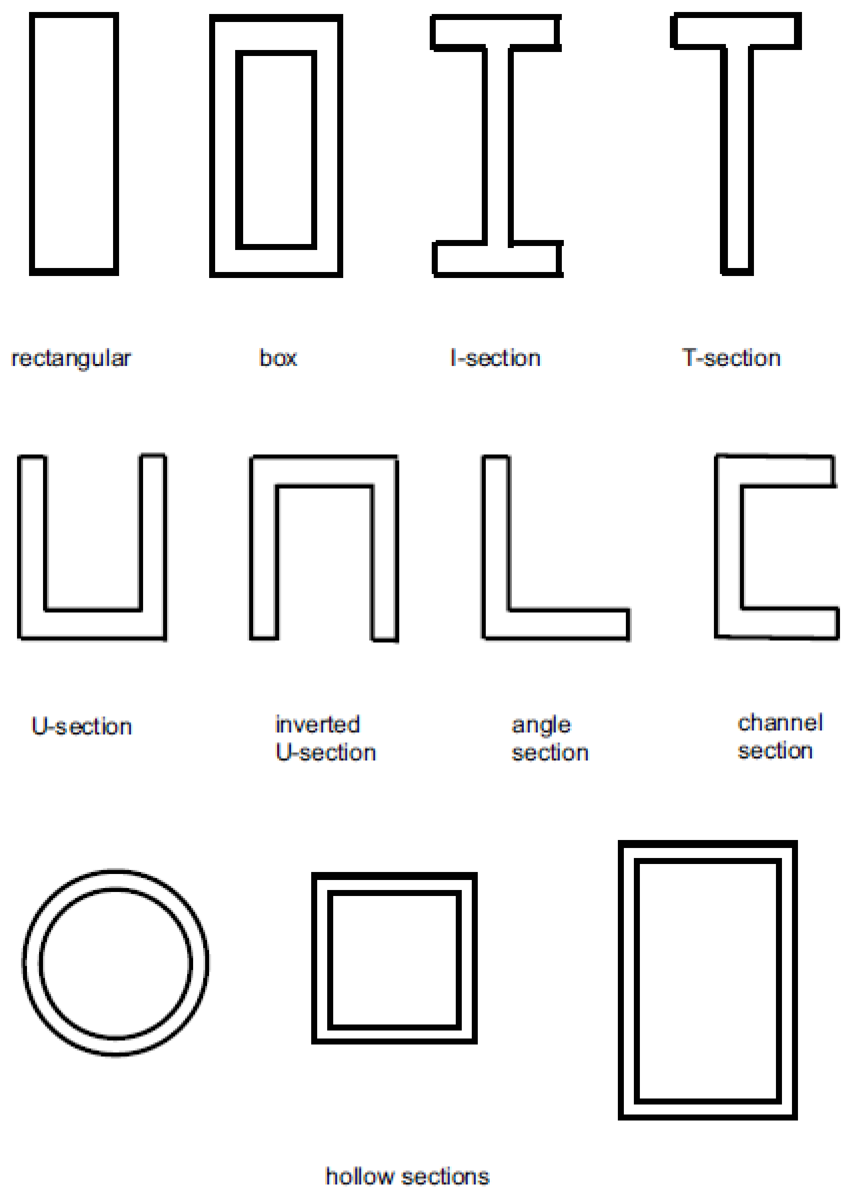


Figure 1.15 Cross section types

Lecture 2 Introduction to **statically determinate/ indeterminate** structure with reference to 2D and 3D structures. Free body diagram of structure.

2.1 What is a Pin ?

Let us imagine that you are inside a building and you will probably be in sight of a door. If it's a conventional door (not a sliding one, for example), it will have hinges on it. What are the hinges for? Well, they make it possible for you to open the door by rotating it about the vertical axis on which the hinges are located.

Figures 2.1 (a) and 2.1 (b) show the plan view of a door, in its shut and partially open positions respectively, along with part of the adjoining wall. You could approach this door and open it or shut it, partially or totally, at will. The hinges make it possible for you to do this by facilitating rotation. Had the door been rigidly fixed to the wall you would not have been able to open it at all. One other point to note: although you can open or shut the door at will, nothing you do to the door will affect the portion of the wall on the other side of the hinges. It remains unmoved. To put it another way, the hinges do not transmit rotational movements into the wall. This is a particularly important concept and is the basis of the analysis of pin jointed frames.

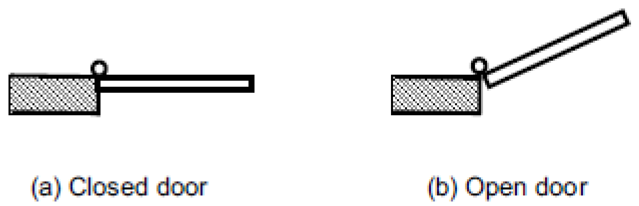


Figure 2.1 A door viewed from above

The word pin, as used in structural engineering, is analogous to the hinge in a door. A pin is indicated symbolically as a small unfilled circle.

Consider two steel rods connected by a pin joint, as shown in Fig. 2.2. The two rods are initially in line as shown in Fig. 2.2 (a) and the left-hand rod is subsequently rotated about 30 degrees anticlockwise, as shown in Fig. 2.2 (b). The right-hand rod is not affected by this rotational movement of the left-hand rod.

A pin, then, has two important characteristics:
(1) A pin permits rotational movement about itself.
(2) A pin cannot transmit turning effects, or moments.

Figure 2.2 Steel rods connected by a pin

2.2 Different types of supports

Up till now we've been talking about supports (to beams, etc.) and indicating them as upward arrows without giving any thought to the type or nature of the support. As we shall see, there are three different types of support.

2.2.1 Roller Supports

Imagine a person on roller skates standing in the middle of a highly polished floor. If you were to approach this person and give him (or her) a sharp push from behind (not to be recommended without discussing it with them first!), they would move off in the direction you pushed them. Because they are on roller skates on a smooth fl oor, there would be minimal friction to resist the person's slide across the floor.

A roller support to part of a structure is analogous to that person on roller skates: a roller support is free to move horizontally. Roller supports are indicated using the symbol shown in Fig. 2.3 (a). You should recognise that this is purely symbolic and a real roller support will probably not resemble this symbol. In practice a roller support might comprise sliding rubber bearings, for example, or steel rollers sandwiched between steel plates, as shown in Fig. 2.3 (b).

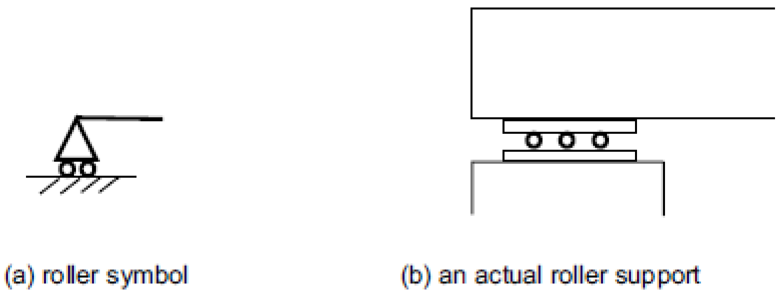


Figure 2.3 Roller symbolically and in reality

2.2.2 Pinned Support

Consider the door hinge analogy discussed above. A pinned support permits rotation but cannot move horizontally or vertically – in exactly the same way as a door hinge provides rotation but cannot itself move from its position in any direction.

2.2.3 Fixed Support

Form your two hands into fists, place them about a foot apart horizontally and allow a friend to position a ruler on your two fists so that it is spanning between them. Your fists are safely supporting the ruler at each end. Now remove one of the supports by moving your fist out from underneath the ruler. What happens? The ruler drops to the floor. Why? You have removed one of the supports and the remaining single support is not capable of supporting the ruler on its own – see Figs 2.4 (a) and (b).

However, if you grip the ruler between your thumb and remaining fingers at one end only, it can be held horizontally without collapsing. This is because the firm grip provided by your hand prevents the end of the ruler from rotating and thus falling to the floor – see Fig. 2.4 (c).

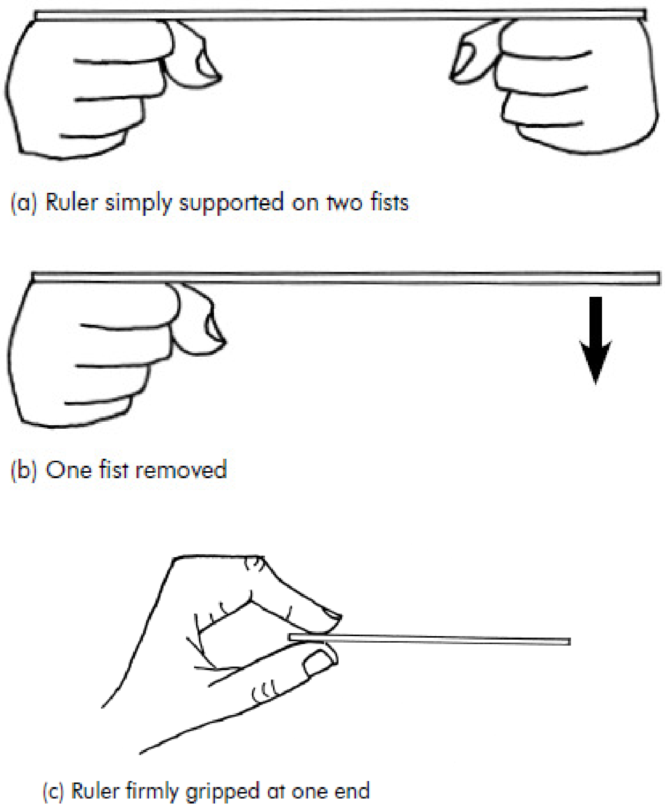


Figure 2.4 What is a fixed support ?

In structures, the support equivalent to your gripping hand in the above example is called a fixed support. As with your hand gripping the ruler, a fixed support does not permit rotation. There are many situations in practice where it is necessary (or at least desirable) for a beam or slab to be supported at one end only – for example, a balcony. In these situations, the single end support must be a fixed support because, as we've seen, a fixed support does not permit rotation and hence does not lead to collapse of the structural member concerned – see Fig. 2.5. Like a pinned support, a fixed support cannot move in any direction from its position. Unlike a pinned support, a fixed support cannot rotate. So a fixed support is fixed in every respect.

Now you've got a mental picture of each of the three different types of support (roller, pinned and fixed), let's revisit each of them and take our study of them a stage further. We are going to do this in the context of reactions and moments.

2.3 Restraints

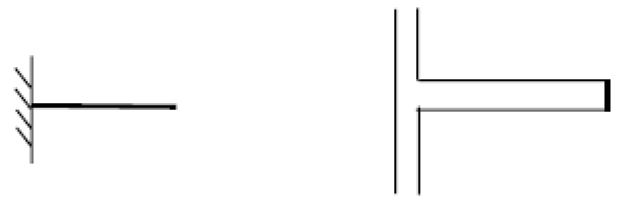
Let's consider each of the following as being a restraint:
(1) Vertical reaction
(2) Horizontal reaction
(3) Resisting moment.

2.3.1 Restraints experienced by different types of support

2.3.1.1 Roller support

Let's return to our roller skater standing on a highly polished floor. As the floor is supporting him, it must be providing an upward reaction to counteract the weight of the skater's body. However, we've already seen that if we push our skater, he will move. The rollers on the skates, and the frictionless nature of the floor, mean that the skater can offer no resistance to our push. In other words, the skater can provide no horizontal reaction to our pushing (in contrast to a solid wall, for example, which would not move if leaned on and therefore would provide a horizontal reaction).

There is also nothing to stop the skater from falling over (i.e. rotating). We can see from the above that a roller support provides one restraint only: vertical reaction. (There is no horizontal reaction and no moment.)



(a) fixed support symbol (b) an actual fixed support

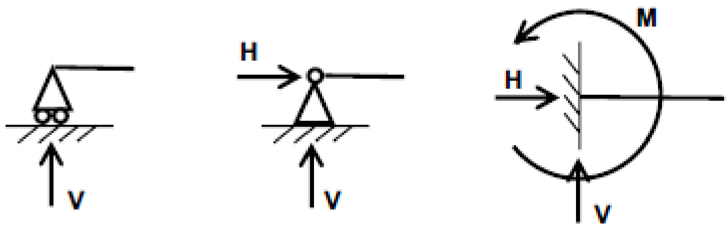
Figure 2.5 Fixed support symbolically and in reality

2.3.1.2 Pinned support

As discussed above, a pinned support permits rotation (so there is no resistance to moment), but as it cannot move horizontally or vertically there must be both horizontal and vertical reactions present. So, a pinned support provides two restraints: vertical reaction and horizontal reaction. (There is no moment.)

2.3.1.3 Fixed Support

We saw above that a fixed support is fixed in every respect: it cannot move either horizontally or vertically and it cannot rotate. This means there will be both horizontal and vertical reactions and, if it cannot rotate, there must be a moment associated with the fixed support. Incidentally, this moment is called a fixed end moment. So, a fixed support provides three restraints: vertical reaction, horizontal reaction and moment.



(a) roller support (b) pinned support (c) fixed support

Figure 2.6 Restraints provided by various support types.

To summarise:
• A roller support provides one restraint: vertical reaction.
• A pinned support provides two restraints: vertical reaction and horizontal reaction.
• A fixed support provides three restraints: vertical reaction, horizontal reaction and moment.

This is illustrated in Fig. 2.6.

2.4 Solution of Equilibrium Equations

We know from our knowledge of mathematics about the following:
• If we have the same number of unknowns as we have equations, a mathematical problem can be solved.
• But if we have more unknowns than equations, a mathematical problem cannot be solved.

Relating this to structural analysis, if we look back to the procedure we used for calculating reactions, we'll see that we were solving three equations. These equations were represented by:

(1) Vertical equilibrium (total force up = total force down)
(2) Horizontal equilibrium (total force right = total force left)
(3) Moment equilibrium (total clockwise moment = total anticlockwise moment).

2.5 Statically Determinate vs Statically indeterminate

As we have three equations, we can use them to solve a problem with up to three unknowns in it. In this context, an unknown is represented by a restraint, as defined earlier. (Remember, a roller support has one restraint, a pinned support has two restraints and a fixed support has three restraints.) Once these reactions are evaluated, we could determine the internal stress resultants (reactions, axial forces, moments etc.) in the structure. Correct solution for reaction and internal stresses must satisfy the equations of static equilibrium for the entire structure. They must also satisfy equilibrium equations for any part of the structure as a free body. A sketch depicting the free body with the associated forces and internal stresses is called a **free body diagram** (FBD). Hence a structural system with up to three restraints is solvable – such a system is said to be statically determinate (SD) – while a structural system with more than three restraints is not solvable (unless we use advanced structural techniques) – such a system is said to be statically indeterminate (SI).

So if we inspect a simple structure, examine its support and thence count up the number of restraints, we can determine whether the structure is statically determinate (up to three restraints in total) or statically indeterminate (more than three restraints).

Let's look at the three examples shown in Fig. 2.7.

Example 1
This beam has a pinned support (two restraints) at its left-hand end and a roller support (one restraint) at its right-hand end. So the total number of restraints is (2 + 1) = 3, therefore the problem is solvable and is statically determinate (SD).

Example 2
This pin-jointed frame has a pinned support (two restraints) at each end. So the total number of restraints is (2 + 2) = 4. As 4 is greater than 3, the problem is not solvable and is statically indeterminate (SI).

Example 3
This beam has a fixed support (three restraints) at its left-hand end and a roller support (one restraint) at its right-hand end. So the total number of restraints is (3 + 1) = 4, therefore, again, the problem is not solvable and is statically indeterminate (SI).

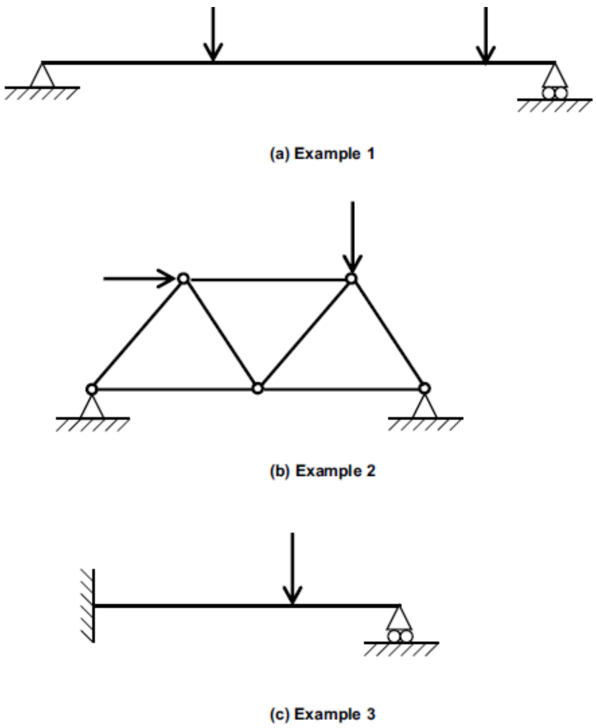


Figure 2.7 Statical determinancy

Sample Problems
Problem 1:
Determine whether each of the structures given in Fig. 2.8 is statically determinate (SD) or statically indeterminate (SI).

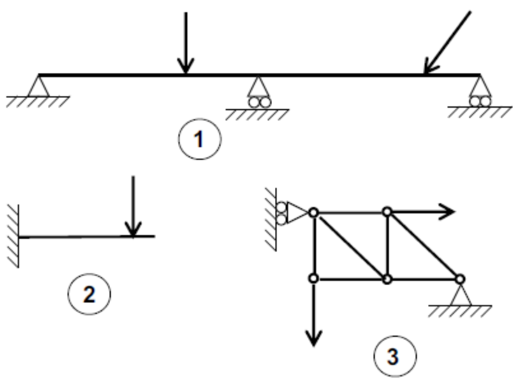


Figure 2.8 Statical determinancy Sample problems

Lecture 3 Introduction to **statically determinate/ indeterminate** structure with reference to 2D and 3D structures. Free body diagram of structure.

3.1 Stability

It is essential for a structure to be strong enough to be able to carry the loads and moments to which it will be subjected. But strength is not sufficient: the structure must also be stable.

3.2 Stability of structural frameworks

Many buildings and other structures have a structural frame. Steel buildings comprise a framework, or skeleton, of steel. We are going to consider the build-up of a framework from scratch. Our framework will consist of metal rods ('members') joined together at their ends by pins. (The concept of a pin, which is a type of connection that facilitates rotation) Consider two members connected by a pin joint, as shown in Fig. 3.1 (a). Is this a stable structure? (In other words, is it possible for the two members to move relative to each other?) As the pin allows the two members to move relative to one another, this is clearly not a stable structure.

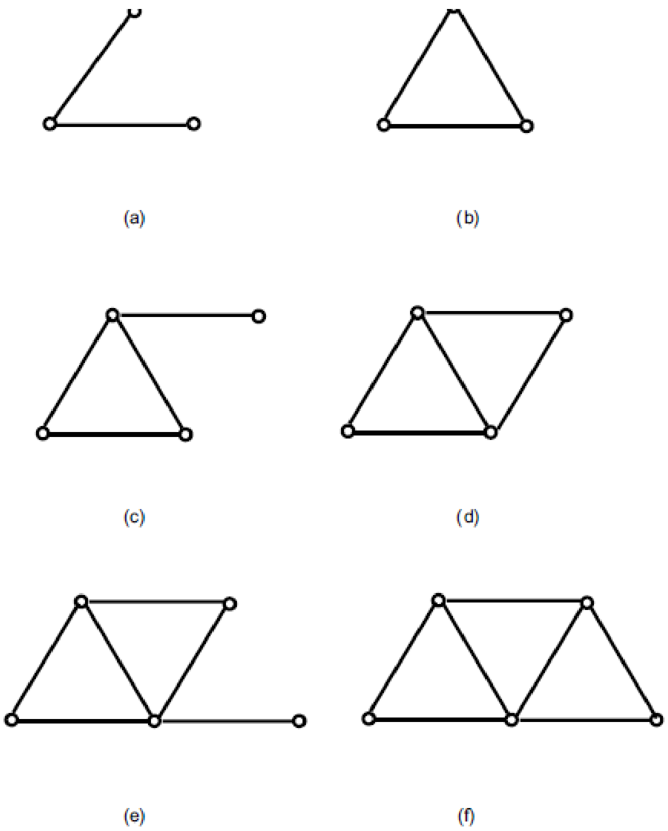


Figure 3.1 Building up a framework

Now, let's add a third member to obtain three members connected by pin joints to form a triangle, as shown in Fig. 3.1 (b). Is this a stable structure? Yes, it is because even though the joints are pinned, movement of the three members relative to each other is not possible. So this is a stable, rigid structure. In fact, the triangle is the most basic stable structure, as we will mention again in the following discussion. If we add a fourth member we produce the frame shown in Fig. 3.1 (c). Is this a stable structure? No it is not. Even though the triangle within it is stable, the 'spur' member is free to rotate relative to the triangle, so overall this is not a stable structure. Consider the frame shown in Fig. 3.1 (d), which is achieved by adding a fifth member to the previous frame. This is a stable structure. If you are unsure of this, try to determine which individual member(s) within the frame can move relative to the rest of the frame. You should see that none of them can and therefore this is a stable structure. This is why you often see this detail in structural frames as 'diagonal bracing', which helps to ensure the overall stability of a structure.

Let's add yet another member to obtain the frame shown in Fig. 3.1 (e). Is this a stable structure? No, it is not. In a similar manner to the frame depicted in Fig. 3.1 (c), it has a spur member which is free to rotate relative to the rest of the structure. Adding a further member we can obtain the frame shown in Fig. 3.1 (f) and we will see that this is a rigid, or stable, structure.

We could carry on ad infinitum in this vein, but I think you can see that a certain pattern is emerging. The most basic stable structure is a triangle (Fig. 3.1 (b)). We can add two members to a triangle to obtain a 'new' triangle. All of the frames that comprise a series of triangles (Figs 3.1 (d) and (f)) are stable; the remaining ones, which have spur members, are not.

Let's now see whether we can devise a means of predicting mathematically whether a given frame is stable or not. In Table 3.1 each of the six frames considered in Fig. 3.1 is assessed. The letter m represents the number of members in the frame and j represents the number of joints (note that unconnected free ends of members are also considered as joints). The column headed 'Stable structure?' merely records whether the frame is stable ('Yes') or not ('No').

Table 3.1 Is a structure stable?

	m	j	Stable structure?	$2j - 3$	**Is $m = 2j - 3$?**
Figure 3.1 (a)	2	3	No	3	No
Figure 3.1 (b)	3	3	Yes	3	Yes
Figure 3.1 (c)	4	4	No	5	No
Figure 3.1 (d)	5	4	Yes	5	Yes
Figure 3.1 (e)	6	5	No	7	No
Figure 3.1 (f)	7	5	Yes	7	Yes

It can be shown that if $m = 2j - 3$ then the structure is stable. If that equation does not hold, then the structure is not stable. This is borne out by Table 3.1: compare the entries in the column headed 'Stable structure?' with those in the column headed 'Is $m = 2j - 3$?'.

3.2 Internal stability of a framed structure-a summary

(1) A framework which contains exactly the correct number of members required to keep it stable is termed a ***perfect frame***. In these cases, $m = 2j - 3$, where m is the number of members in the frame and j is the number of joints (including free ends). Frames (b), (d) and (f) in Fig.3.1 are examples.

(2) A framework having less than the required number of members is unstable and is termed a ***mechanism***. In these cases, $m < 2j - 3$. Frames (a), (c) and (e) in Fig. 3.1 are examples. In each case, one member of the frame is free to move relative to the others.

(3) A framework having more than this required number is 'over-stable' and contains redundant members that could (in theory at least) be removed. Examples follow. In these cases, $m > 2j - 3$. These frames are statically indeterminate (SI) which means that the frames cannot be mathematically analysed without resorting to advanced structural techniques.

Examples

For each of the frames shown in Fig. 3.2, use the equation $m = 2j - 3$ to determine whether the frame is (a) a perfect frame (SD), (b) a mechanism (Mech) or (c) statically indeterminate (SI). Where the frame is a mechanism, indicate the manner in which the frame could deform. Where the frame is statically indeterminate, consider which members could be re-

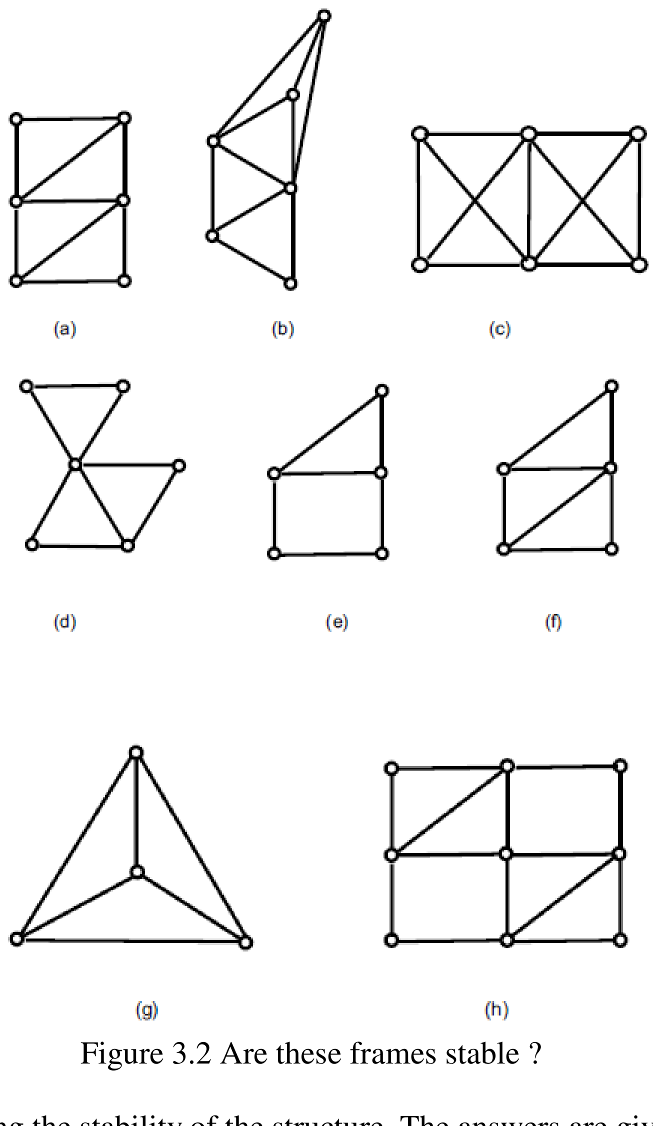


Figure 3.2 Are these frames stable ?

moved without affecting the stability of the structure. The answers are given in Table 3.2. The frames shown in Figs 3.2 (b), (c) and (g) are statically indeterminate. This means they are over-stable and that one or more members may be removed without compromising stability. In the case of Fig. 3.2 (b), any one member can be removed from the top part of the frame and the structure would still be stable. In Fig. 3.2 (c), two members could be removed without compromising stability – but the two members to be removed should be chosen with care. A sensible choice would be to remove one diagonal member from each of the two squares. In Fig. 3.2 (g), any one member could be removed. The frames shown in Figs 3.2 (d), (e) and (h) are mechanisms. This means that a part of the frame is able to move relative to another part of the frame. In Fig. 3.2 (d), the upper triangle is free to rotate about the frame's central pin independently of the lower part of the frame. In Fig. 3.2 (e), the square part of the frame is free to deform, or collapse, as we shall see in a later example. The mode of deformation of the frame in Fig. 3.2(h) is less easy to visualise. It is shown in Fig. 3.3.

Table 3.2 Stability of frames shown in Fig. 3.2

	m	j	$2j-3$	Is $m = 2j - 3$? (or > or <)	**Stability type**
Figure 3.2 (a)	9	6	9	=	SD
Figure 3.2 (b)	10	6	9	>	SI
Figure 3.2 (c)	11	6	9	>	SI
Figure 3.2 (d)	8	6	9	<	Mech
Figure 3.2 (e)	6	5	7	<	Mech
Figure 3.2 (f)	7	5	7	=	SD
Figure 3.2 (g)	6	4	5	>	SI
Figure 3.2 (h)	14	9	15	<	Mech

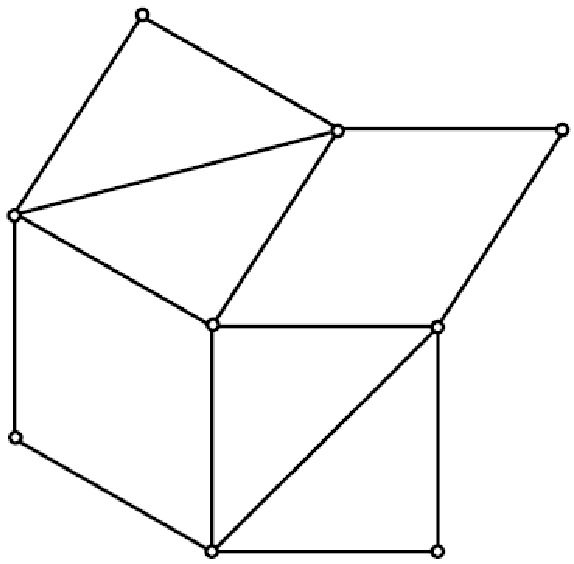
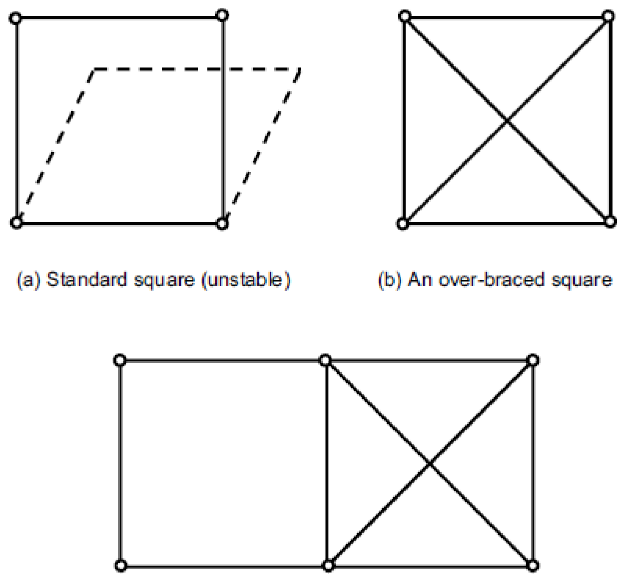


Figure 3.3 Deformation of frame shown in Figure 3.2 (h)

3.3 General cases

Look at frames (a) and (b) in Fig. 3.4. If we apply the $m = 2j - 3$ formula to the standard square depicted in Fig. 3.4 (a), we will find that it is unstable, or a mechanism. It can deform in the manner indicated by the broken lines in Fig. 3.4 (a). This is why, in 'real' structures, diagonal cross-bracing must often be provided to ensure stability.

If we look at the frame shown in Fig. 3.4 (b), we see that it is a square which is diagonally cross-braced twice. Applying the $m = 2j - 3$ formula we find that it is statically indeterminate, which means that it contains at least one redundant member. On further investigation we find that we can remove any one of the six members without affecting the stability of the structure.



(a) Standard square (unstable) (b) An over-braced square

(c) Perfect frame, mechanism or over-braced?

Figure 3.4 Frame stability general cases

Consider the frame shown in Fig. 3.4(c). It contains nine members and six joints, so $m = 9$ and $j = 6$ and it can thus readily be shown that $m = 2j - 3$ in this case, which suggests that the framework is a perfect frame. In fact, an inspection of the frame shows that this is not, in fact, the case. The left hand part of the frame is an un-braced square, which is a mechanism and can deform in the same manner as the frame shown in Fig. 3.4 (a). But the right-hand part of the frame has double diagonal cross-bracing, which suggests that it is ‘over-stable’ and contains redundant members in the same way as the frame shown in Fig. 3.4 (b). So, part of the frame shown in Fig. 3.4 (c) is a mechanism and the other part is statically indeterminate, but this does not make an overall perfect frame, as predicted by the formula! The lesson to be learned from this is that the formula $m = 2j - 3$ should be regarded as a guide only – it doesn’t always work. A given frame should always be inspected to see whether there are any signs of either (a) mechanism or (b) over-stability.

3.4 Frames on Supports

Up till now we have conveniently ignored the fact that, in practice, frames have to be supported. We therefore need to consider the effects of supports on the overall stability of frames. we knew about the three different types of support (roller, pinned and fixed). We also saw that:

- a roller support provides one restraint ($r = 1$);
- a pinned support provides two restraints ($r = 2$);
- a fixed support provides three restraints ($r = 3$).

The $m = 2j - 3$ used above is now modified to $m + r = 2j$ where supports are present. As before, m is the number of members and j is the number of joints. The letter r represents the total number of restraints (one for each roller support, two for each pinned support and three for each fixed support).

(1). If $m + r = 2j$, then the frame is a perfect frame and is statically determinate (SD), which means it can be analysed by various methods.

(2) If $m + r < 2j$, then the frame is a mechanism – it is unstable and should not be used as a structure.

(3) If $m + r > 2j$, then the frame contains redundant members and is statically indeterminate (SI), which means it cannot be analysed without resorting to advanced methods of structural analysis.

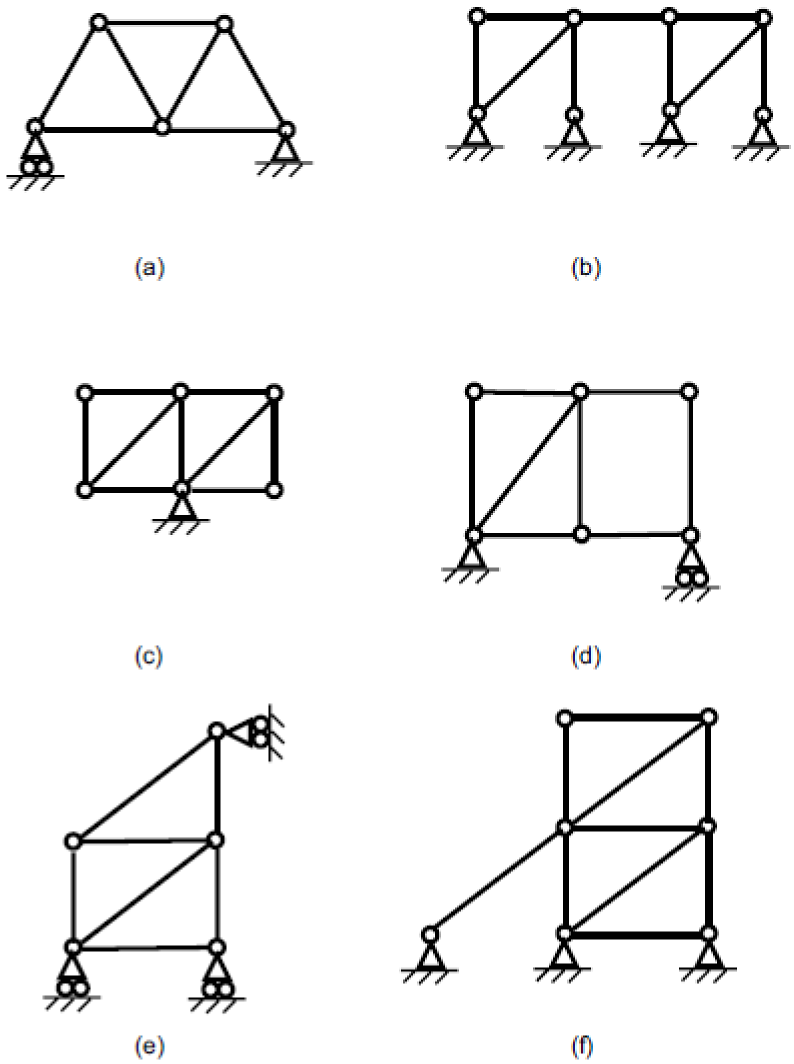


Figure 3.5 Are these structures stable ?

Examples

For each of the frames shown in Fig. 3.5, use the equation $m + r = 2j$ to determine whether the frame is (a) statically determinate, (b) a mechanism or (c) statically indeterminate. Where the frame is a mechanism, indicate the manner in which the frame could deform. Where the frame is statically determinate, consider which members could be removed without affecting the stability of the structure. The answers are given in Table 3.3.

Table 3.3 Stability of structures shown in Fig. 3.5							
	m	j	$2j$	r	$m + r$	Is $m + r = 2j$? (or > or <)	**Stability type**
Figure 3.5 (a)	7	5	10	3	10	=	SD
Figure 3.5 (b)	9	8	16	8	17	>	SI
Figure 3.5 (c)	9	6	12	2	11	<	Mech
Figure 3.5 (d)	8	6	12	3	11	<	Mech
Figure 3.5 (e)	7	5	10	3	10	=	SD
Figure 3.5 (f)	10	7	14	6	16	>	SI

The frames shown in Figs 3.5 (b) and (f) are statically indeterminate. This means they are over-stable and that one or more members may be removed. In the case of Fig. 3.5 (b), one of the diagonal members may be removed (but not both of them!) and the structure would still be stable. In Fig. 3.5 (f), the 'lean-to' diagonal member may be removed without compromising stability. The frames shown in Figs 3.5 (c) and (d) are mechanisms. The structure in Fig. 3.5 (c) is obviously unstable, being free to rotate about its single central support. In Fig. 3.5 (d), the square part of the frame is free to deform in the manner indicated in Fig. 3.4 (a).

3.5 Stability of real structures

In practice, the stability of a structure is assured in one of three ways:

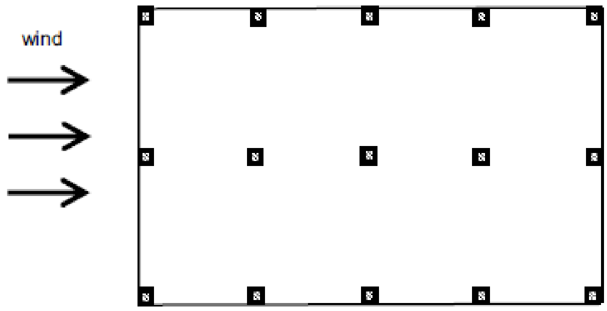
(1) Shear walls/stiff core.
(2) Cross-bracing.
(3) Rigid joints.

Let's look at each of these in more detail.

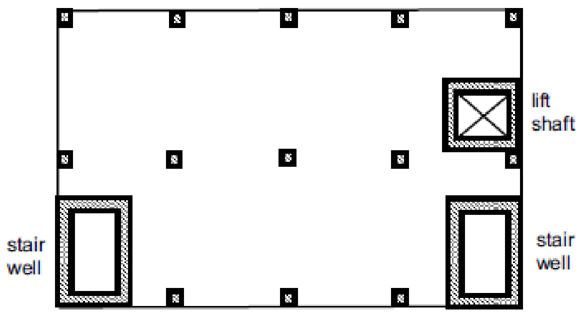
3.5.1 Shear walls/stiff core

This form of stability is usually (but not exclusively) used in concrete buildings. Consider the structural plan of an upper floor of a typical concrete office building, as shown in Fig. 3.6 (a). The structure comprises a grid layout of columns, which support beams and slabs at each floor level. The wind blows horizontally against the building from any direction. It is obviously important that the building doesn't collapse in the manner of a 'house of cards' under the effects of this horizontal wind force. We could design each individual column to resist the wind forces, but for various reasons this is not the way it is normally done.

Instead, shear walls are used. These walls are designed to be stiff and strong enough to resist all the lateral forces on the building. Since most buildings have staircases and many have lift shafts, the walls that surround the staircases and lift shafts are often designed and constructed to perform this role, as shown in Fig. 3.6 (b). On larger buildings, the shear walls may be constructed in such a way as to comprise an inner core to the building, which often contains stairwells, lift shafts, toilets and ducts for services.



(a) Typical floor plan of reinforced concrete office building



(b) Same floor plan with shear walls added

Figure 3.6 Provision of stability using shear walls

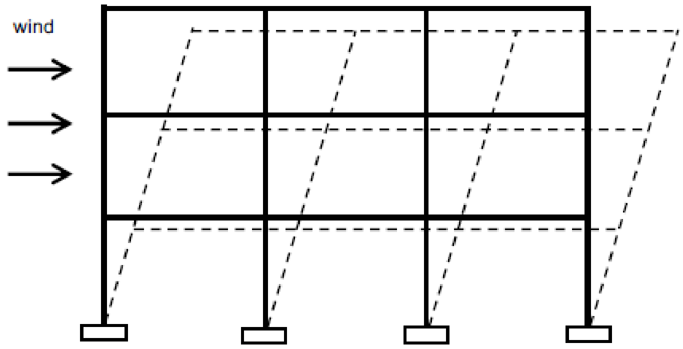
3.5.2 Cross bracing

This form of stability is common in steel-framed buildings. Figure 3.7 (a) shows the elevation of a three-storey steel-framed building, on which the wind is blowing. There is nothing to stop the building tilting over and collapsing in the manner indicated by the broken lines in Fig. 3.7 (a).

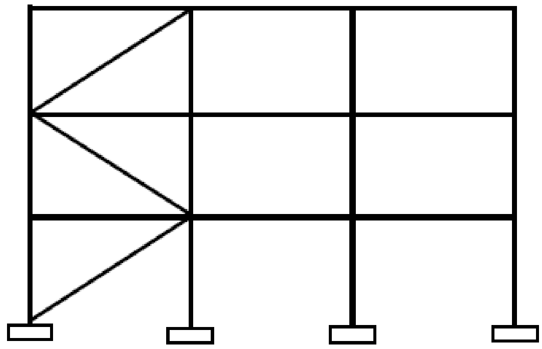
One way of ensuring stability is to stop the 'squares' in the building elevation from becoming trapeziums. Earlier in this chapter we saw that (a) a triangle is the most basic stable structure and (b) a diagonal member can stop a square from deforming (illustrated in Figs 3.1 (b) and (d) respectively). So diagonal cross-bracing is used to ensure stability, as shown in Fig. 3.7 (b).

3.5.3 Rigid Joints

A third method of providing lateral stability is simply to make the joints strong and stiff enough that movement of the beams relative to the columns is not possible. The black blobs in Fig. 3.8 indicate stiff joints that stop the action depicted in Fig. 3.7 (a) from happening.



(a) Section through three-storey steel framed building



(b) Same section with diagonal bracing added

Figure 3.7 Provision of stability using cross bracing

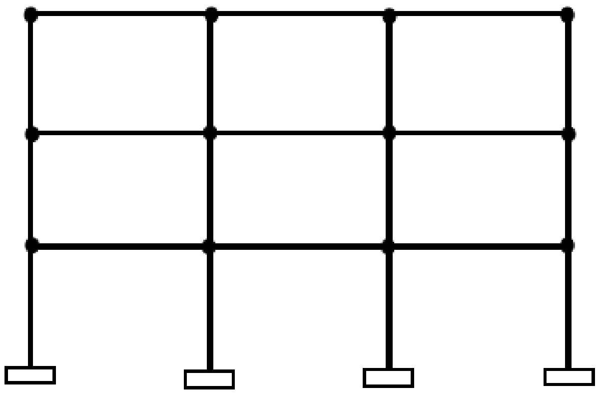


Figure 3.8 Provision of stability using rigid frames

3.6 Space Frames

Normally the centre lines of bars, forces applied and support reactions in the case of plane trusses lie in a plane. When all these lie in different planes i.e in three dimensional space, such a structure is called a space truss or space frame which is nothing but an assemblage of bars in three dimensional space. Tetrahedron is the simplest space frame consisting of six members. Antenna towers, transmission line towers, guyed masts, derricks, offshore structures etc are some of the common examples of space frames. We can construct a space frame from the basic tetrahedron by adding three new members and a joint. To get a stable space frame, we have to arrange adequate number of bars in a suitable manner starting with a basic tetrahedron. There are six bars and four joints in the basic tetrahedron. For each joint added, we have now three additional members. Therefore, we can have a reaction between the member of bars (b) and the number of joints (j) as given below

b-6=3(j-4)

b=3j-6

The above expression gives the minimum number of bars required to construct a stable space truss or space frame. If the number of bars in the space truss is less than that required by the above expression, then we consider the space frame as unstable. In Contrast, if the number of bars is more than the minimum number required then the space frame is considered internally indeterminate.

Sample Problems

Problem 1:

(1) For each of the examples shown in Fig. 3.9, determine whether the frame is (a) a perfect frame (SD), (b) unstable (a mechanism) or (c) over-stable (containing redundant members). If the framework is unstable, state where a member could be added to make it stable. If the frame is over-stable, determine which members could be removed and the structure would still be stable.

(2) Select a framed structure near where you live. Determine how lateral stability is provided to the structure and state the reasons why the designer may have chosen that particular method of ensuring stability.

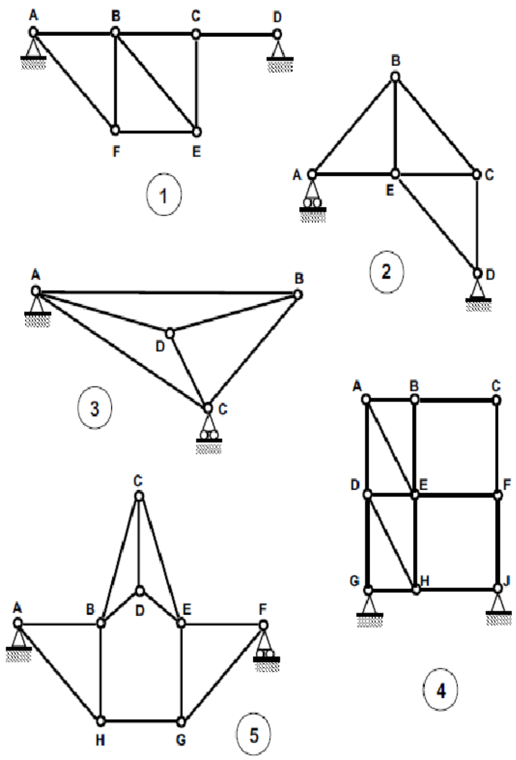


Figure 3.9 Sample problems

Lecture 4 Introduction to **kinematically determinate/indeterminate** structures with reference to 2D and 3D structures. Degree of freedom.

4.1 Degrees of Freedom

The degrees of freedom (DOF) can be defined as asset of independent displacements that specify completely the deformed position and orientation of the body or system under loading. Hence, displacements include deflections and rotations as well. A rigid body that moves in 3D space in linear directions has three translational displacement components as DOFs. The rigid body can also undergo angular motion, which is called rotation. So the body has three rotational DOFs. Altogether a rigid body can have at least six DOFs, three translations and three rotations. Translation refers to the ability of a body to move without rotating whereas rotation refers to its angular motion about some axis. When a structure is loaded, the joints also called nodes will undergo unknown displacements. These displacements are referred to as the DOF for the structures.

4.2 Kinematics

We have discussed in the previous sections that loads applied on structural systems in turn induce internal forces in the system. As a consequence of this the system undergoes deformation which generically is called as motion. The study relating to forces and motions constitutes an applied science which is a branch of mechanics. The cardinal principle underlying this body is the equilibrium. It is a condition which describes a state of balance of a system when forces applied on it. As the structural system is initially at rest and in equilibrium too under a system of forces acting in it, we call that part of mechanics concerned with relations between these forces as **statics**. There is another part of mechanics called **dynamics** which refers to the other part of mechanics dealing with rigid bodies of motion. Dynamics is divided into two parts, namely, **kinematics** and **kinetics**. Kinematics is the study of the geometry of motion. It is used to relate displacement, velocity, acceleration and time without any reference to the forces causing the motion. **Kinetics** is the study of the relation existing between the forces acting on a body, the mass of the body and the motion of the body. It is used to predict the motion caused by the given forces or to determine the forces required to produce a given motion.

In structural analysis, kinematics refers to quantities associated with geometry, the position changes or the deformation of the geometry. This term is used in opposition to the term statics.

Displacement refers to a translation or a rotation of a specific point in a structure. For example, we consider a simple beam as shown in Figure 4.1.

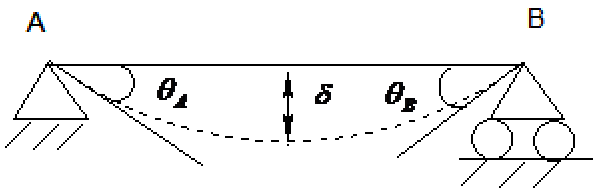


Figure 4.1 Displacement of simple beam

It is free to undergo displacement in the form of translation in the direction perpendicular to its own axis as shown in Figure 4.1. which is called deflection as well as rotate at its supports. The quantity δ is the vertical translation of the beam and is called deflection of the beam. The rotation at support A is θ_A and at support B is θ_B . These rotations are called slopes.

A joint in a truss can translate in two mutually perpendicular directions as shown in Figure 4.2. The joint C can displace along x and y directions only. The joint cannot rotate.

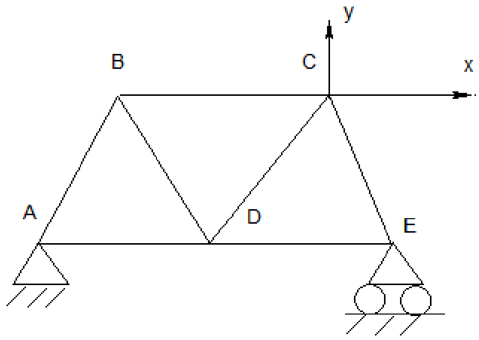


Figure 4.2 Displacement in a truss

A rigid frame can undergo translation and rotation at joints as shown in Figure 4.3. The joint B in Figure 4.3 undergoes horizontal translation Δ_B and a rotation θ_B .

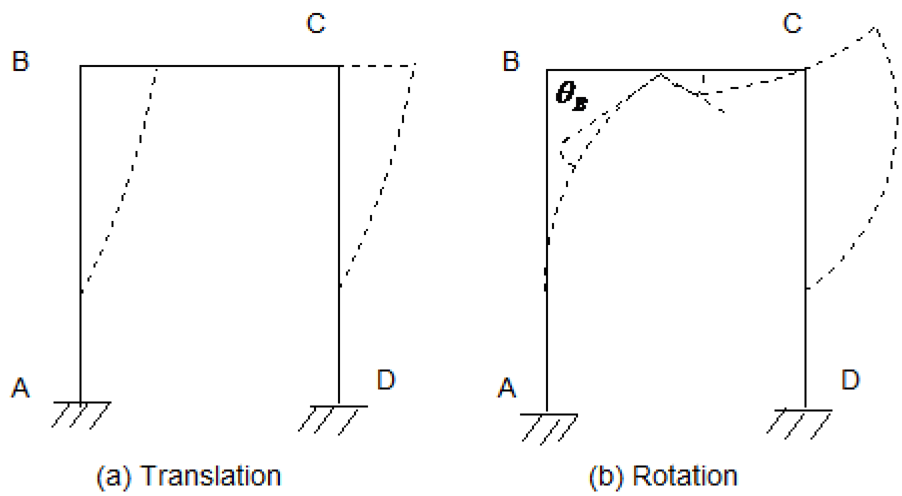


Figure 4.3 Displacement in a frame

These translations and rotations constitute the degrees of freedom of a structural system. In structural analysis, these displacements other than that at the supports are in general not known. Therefore, the objective of the analysis is to determine their values. The number of the independent joint displacement in a structure is called the degree of kinematic indeterminancy or the number of degrees of freedom. This number is a sum of the degree of freedom in rotation and translation. For example, in a two span beam as shown in Figure 44, the degree of kinematic indeterminancy is 2 since the structure can undergo rotations at joints B and C and these are indeterminates. Rotation D1 at joint B and rotation D2 at joint C are the two unknowns. Because support A is fixed, the rotation D3 is zero which is a known quantity and hence determinate.

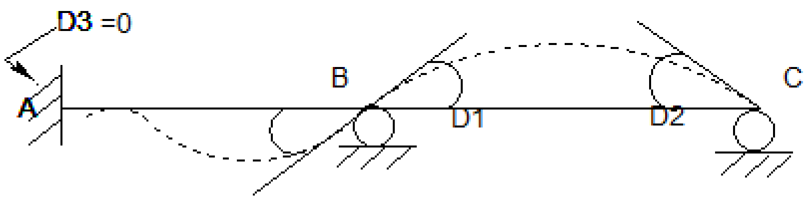


Figure 4.4 : Degree of kinematic indeterminancy

Lecture 5 B.M. and S.F. diagrams for **different loading** on simply supported beam, cantilever and overhanging beams.

5.1. Introduction

Beams are usually straight horizontal members used primarily to carry vertical loads. Quite often they are classified according to the way they are supported, as indicated in Fig. 5.1. In particular, when the cross section varies the beam is referred to as tapered or hunched. Beam cross sections may also be "built up" by adding plates to their top and bottom.

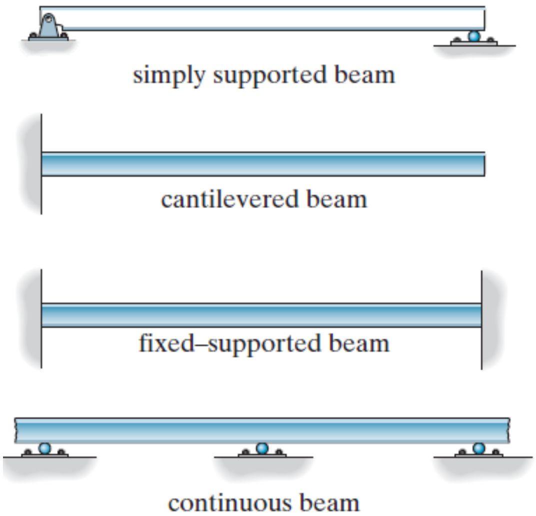


Figure 5.1 Types of beams

Beams are primarily designed to resist bending moment; however, if they are short and carry large loads, the internal shear force may become quite large and this force may govern their design. When the material used for a beam is a metal such as steel or aluminium, the cross section is most efficient when it is shaped as shown in Fig. 5.2. Here the forces developed in the top and bottom flanges of the beam form the necessary couple used to resist the applied moment M, whereas the web is effective in resisting the applied shear V. This cross section is commonly referred to as a "wide flange," and it is normally formed as a single unit in a rolling mill in lengths up to 23 m. If shorter lengths are needed, a cross section having tapered flanges is sometimes selected. When the beam is required to have a very large span and the loads applied are rather large, the cross section may take the form of a plate girder. This member is fabricated by using a large plate for the web and welding or bolting plates to its ends for flanges. The girder is often transported to the field in segments, and the segments are designed to be spliced or joined together at points where the girder carries a small internal moment.

Concrete beams generally have rectangular cross sections, since it is easy to construct this form directly in the field. Because concrete is rather weak in resisting tension, steel "reinforcing rods" are cast into the beam within regions of the cross section subjected to tension. Precast concrete beams or girders are fabricated at a shop or yard in the same manner and then transported to the job site. Beams made from timber may be sawn from a solid piece of wood or laminated. Laminated beams are constructed from solid sections of wood, which are fastened together using high-strength glues.

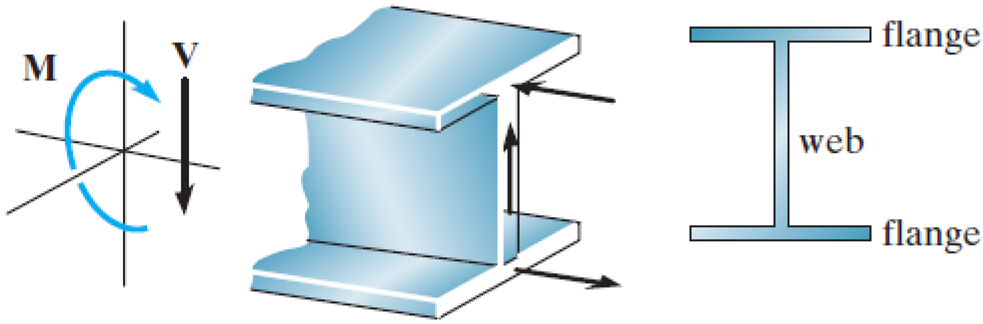


Figure 5.2 Cross section of a beam

5.2 Loads

Once the dimensional requirements for a structure have been defined, it becomes necessary to determine the loads the structure must support. Often, it is the anticipation of the various loads that will be imposed on the structure that provides the basic type of structure that will be chosen for design. For example, high-rise structures must endure large lateral loadings caused by wind, and so shear walls and tubular frame systems are selected, whereas buildings located in areas prone to earthquakes must be designed having ductile frames and connections.

Once the structural form has been determined, the actual design begins with those elements that are subjected to the primary loads the structure is intended to carry, and proceeds in sequence to the various supporting members until the foundation is reached. Thus, a building floor slab would be designed first, followed by the supporting beams, columns, and last, the foundation footings. In order to design a structure, it is therefore necessary to first specify the loads that act on it.

Since a structure is generally subjected to several types of loads, a brief discussion of these loadings will now be presented to illustrate how one must consider their effects in practice.

5.2.1 Dead load

Dead loads consist of the weights of the various structural members and the weights of any objects that are permanently attached to the structure. Hence, for a building, the dead loads include the weights of the columns, beams, and girders, the floor slab, roofing, walls, windows, plumbing, electrical fixtures, and other miscellaneous attachments.

In some cases, a structural dead load can be estimated satisfactorily from simple formulas based on the weights and sizes of similar structures. Through experience one can also derive a “feeling” for the magnitude of these loadings.

For example, the average weight for timber buildings is 1.9-2.4 KN/m^2 for steel framed buildings it is 2.9-3.6 KN/m^2 and for reinforced concrete buildings it is 5.3-6.2 KN/m^2 Ordinarily, though, once the materials and sizes of the various components of the structure are determined, their weights can be found from tables that list their densities.

Although calculation of dead loads based on the use of tabulated data is rather straightforward, it should be realized that in many respects these loads will have to be estimated in the initial phase of design. These estimates include non-structural materials such as prefabricated facade panels, electrical and plumbing systems, etc. Furthermore, even if the material is specified, the unit weights of elements reported in codes may vary from those given by manufacturers, and later use of the building may include some changes in dead loading. As a result, estimates of dead loadings can be in error by 15% to 20% or more.

Normally, the dead load is not large compared to the design load for simple structures such as a beam or a single-story frame; however, for multi-story buildings it is important to have an accurate accounting of all the dead loads in order to properly design the columns, especially for the lower floors.

5.2.2 Live loads

Live Loads can vary both in their magnitude and location. They may be caused by the weights of objects temporarily placed on a structure, moving vehicles, or natural forces. The minimum live loads specified in codes are determined from studying the history of their effects on existing structures. Usually, these loads include additional protection against excessive deflection or sudden overload.

5.2.3 Building Loads

The floors of buildings are assumed to be subjected to uniform live loads, which depend on the purpose for which the building is designed. These loadings are generally tabulated in local, state, or national codes. The values are determined from a history of loading various buildings. They include some protection against the possibility of overload due to emergency situations, construction loads, and serviceability requirements due to vibration. In addition to uniform loads, some codes specify minimum concentrated live loads, caused by hand carts, automobiles, etc., which must also be applied anywhere to the floor system. For example, both uniform and concentrated live loads must be considered in the design of an automobile parking deck.

For some types of buildings having very large floor areas, many codes will allow a reduction in the uniform live load for a floor, since it is unlikely that the prescribed live load will occur simultaneously throughout the entire structure at any one time.

5.2.4 Highway Bridge loads

The primary live loads on bridge spans are those due to traffic, and the heaviest vehicle loading encountered is that caused by a series of trucks.

5.2.5 Impact Loads

Moving vehicles may bounce or sidesway as they move over a bridge, and therefore they impart an impact to the deck.The percentage increase of the live loads due to impact is called the impact factor, I.This factor is generally obtained from formulas developed from experimental evidence.

5.2.6 Wind loads

When structures block the flow of wind, the wind's kinetic energy is converted into potential energy of pressure, which causes a wind loading. The effect of wind on a structure depends upon the density and velocity of the air, the angle of incidence of the wind, the shape and stiffness of the structure, and the roughness of its surface. For design purposes, wind loadings can be treated using either a static or a dynamic approach.

5.2.7 Snow loads

In some parts of the country, roof loading due to snow can be quite severe, and therefore protection against possible failure is of primary concern. Design loadings typically depend on the building's general shape and roof geometry, wind exposure, location, its importance, and whether or not it is heated.

5.2.8 Earthquake loads

Earthquakes produce loadings on a structure through its interaction with the ground and its response characteristics. These loadings result from the structure's distortion caused by the ground's motion and the lateral resistance of the structure. Their magnitude depends on the amount and type of ground accelerations and the mass and stiffness of the structure.

5.2.9 Hydrostatic and Soil Pressure

When structures are used to retain water, soil, or granular materials, the pressure developed by these loadings becomes an important criterion for their design. Examples of such types of structures include tanks, dams, ships, bulkheads, and retaining walls. Here the laws of hydrostatics and soil mechanics are applied to define the intensity of the loadings on the structure.

5.3 Support Conditions

Structural members are joined together in various ways depending on the intent of the designer. The three types of joints most often specified are the pin connection, the roller support, and the fixed joint. A pin-connected joint and a roller support allow some freedom for slight rotation, whereas a fixed joint allows no relative rotation between the connected members and is consequently more expensive to fabricate. Examples of these joints, fashioned in metal and concrete, are shown in Figs. 5.3 and 5.4, respectively. For most timber structures, the members are assumed to be pin connected, since bolting or nailing them will not sufficiently restrain them from rotating with respect to each other.

Idealized models used in structural analysis that represent pinned and fixed supports and pin-connected and fixed-connected joints are shown in Figs. 5.5*a* and 5.5*b*. In reality, however, all connections exhibit some stiffness toward joint rotations, owing to friction and material behavior. In this case a more appropriate model for a support or joint might be that shown in Fig. 5.5*c*. If the torsional spring constant the joint is a pin, and if k : q, the joint is fixed.

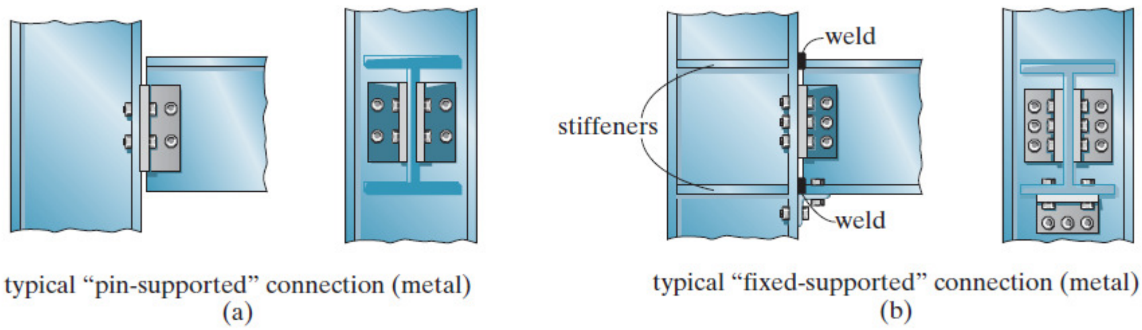


Figure 5.3 Typical pin and fixed supported connections

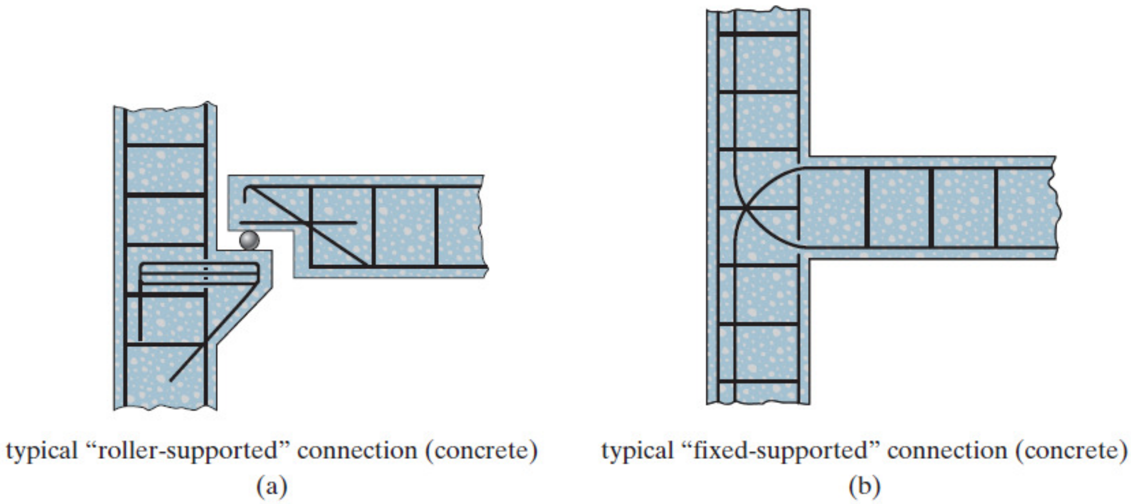


Figure 5.4 typical roller and fixed supported connections

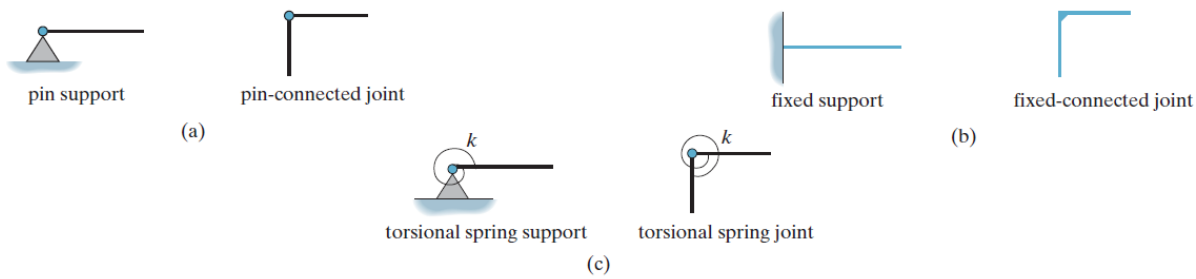


Figure 5.5 Various support conditions

When selecting a particular model for each support or joint, the engineer must be aware of how the assumptions will affect the actual performance of the member and whether the assumptions are reasonable for the structural design. For example, consider the beam shown in Fig. 5.6a, which is used to support a concentrated load P. The angle connection at support A is like that in Fig. 5.2a and can therefore be idealized as a typical pin support. Furthermore, the support at B provides an approximate point of smooth contact and so it can be idealized as a roller. The beam's thickness can be neglected since it is small in comparison to the beam's length, and therefore the idealized model of the beam is as shown in Fig. 5.6b.The analysis of the loadings in this beam should give results that closely approximate the loadings in the actual beam.

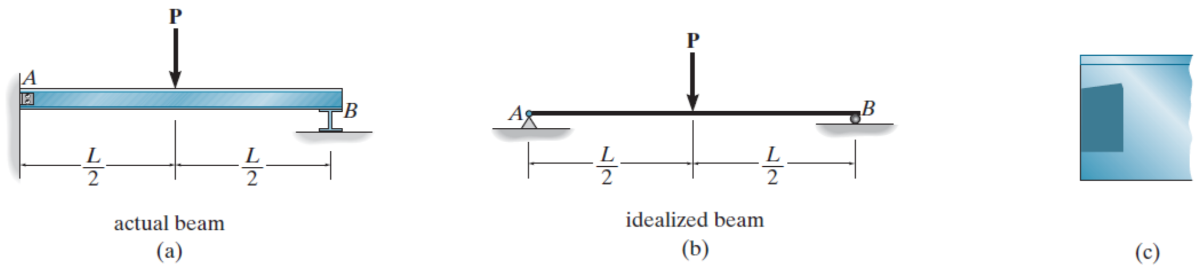


Figure 5.6 Actual and idealised beams

5.4 **Calculation of Reactions**

We found out earlier that if a body or object of any sort is stationary, then the forces on it balance, as follows:

Total force upwards = Total force downwards
Total force to the left = Total force to the right

Next we will find out how to use this information to calculate reactions – that is, the upward forces that occur at beam supports in response to the forces on the beam.

Moment Equilibrium

we found that if an object or body is stationary, it doesn't rotate and the total clockwise moment about any point on the object is equal to the total anticlockwise moment about the same point. This is the third rule of equilibrium. The three rules of equilibrium are expressed in Fig. 5.7

The three rules of equilibrium can be used to calculate reactions. A reaction is a force (usually upwards) that occurs at a support of a beam or similar structural element. A reaction counteracts the (usually downward) forces in the structure to maintain equilibrium. It is important to be able to calculate these reactions. If the support is a column, for example, the reaction represents the force in the column, which we would need to know in order to design the column.

Consider the example shown in Fig. 5.8. The thick horizontal line represents a beam of span 6 metres which is simply supported at its two ends, A and B. The only load on the beam is a point load of 18 kN, which acts vertically downwards at a position 4 metres from point A. We are going to calculate the reactions *R*A and *R*B (that is, the support reactions at points A and B respectively).

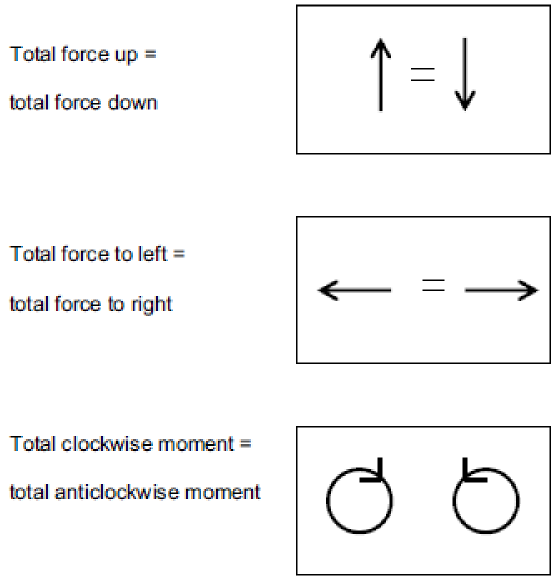


Figure 5.7 The rules of equilibrium.

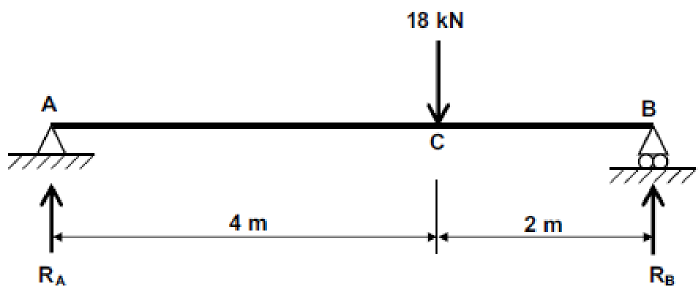


Figure 5.8 Calculation of reactions for point loads.

From ***vertical equilibrium***, we know that:

Total force upwards = Total force downwards

Applying this to the example shown in Fig. 5.8, we can see that: *R*A + *R*B = 18 kN

Of course, this doesn't tell us the value of *R*A and it doesn't tell us the value of *R*B. It merely tells us that the sum of *R*A and *R*B is 18 kN. To evaluate *R*A and *R*B then, we clearly have to do something different.

Let's use our new-found knowledge of moment equilibrium. We found out above that if any structure is stationary, then at any given point in the structure:

Total clockwise moment = Total anticlockwise moment

The above applies at any point in a structure. So, taking moments about point A:

(18 kN × 4 m) = (*R*B × 6 m)

Therefore RB = 12 kN. Note that there is no moment due to force RA. This is because force RA passes straight through the point (A) about which we are taking moments.

Similarly, taking moments about point B:

Total clockwise moment = Total anticlockwise moment

(RA × 6 m) = (18 kN × 2 m)

Therefore RA = 6 kN.
\
As a check, let's add RA and RB together:

RA + RB = 6 + 12 = 18 kN

which is what we would expect from the first equation above.

Calculation of reactions when uniformly distributed loads (UDLs) are present

In practice, most loads in 'real' buildings and other structures are uniformly distributed loads – or can be represented as such – so we need to know how to calculate end reactions for such cases. The main problem we encounter is in taking moments. For point loads it is straightforward – the appropriate moment is calculated by multiplying the load (in kN) by the distance from it to the point about which we're taking moments. However, with a uniformly distributed load, how do we establish the appropriate distance?

Figure 5.9 represents a portion of uniformly distributed load of length x. The intensity of the uniformly distributed load is w kN/m. The chain-dotted line in Fig. 5.9 represents the centre line of the uniformly distributed load. Let's suppose we want to calculate the moment of this piece of UDL about a point A, which is located a distance a from the centre line of the UDL. In this situation, the moment of the UDL about A is the total load multiplied by the distance from the centre line of the UDL to the point about which we're taking moments. The total UDL is $w \times x$, the distance concerned is x, so:

moment of UDL about A = wax.

Apply this principle whenever you're working with uniformly distributed loads.

Calculate the end reactions for the beam shown in Fig. 5.10. Use the same procedure as before.

Vertical equilibrium:

RA + RB = (3 kN/m × 2 m) = 6 kN

Taking moments about A:

(3 kN/m × 2 m) × 1 m = RB × 4 m

Therefore:

RB = 1.5 kN

Taking moments about B:

(3 kN/m × 2 m) × 3 m = RA × 4 m

Therefore:

RA = 4.5 kN

Check:

RA + RB = 4.5 + 1.5 = 6 kN

(as expected from the first equation).

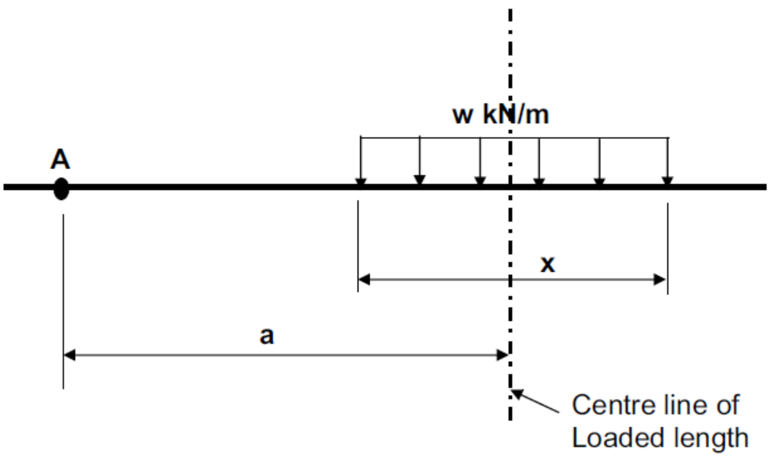


Figure 5.9 Bending moment calculation for uniformly distributed load (UDL): general case.

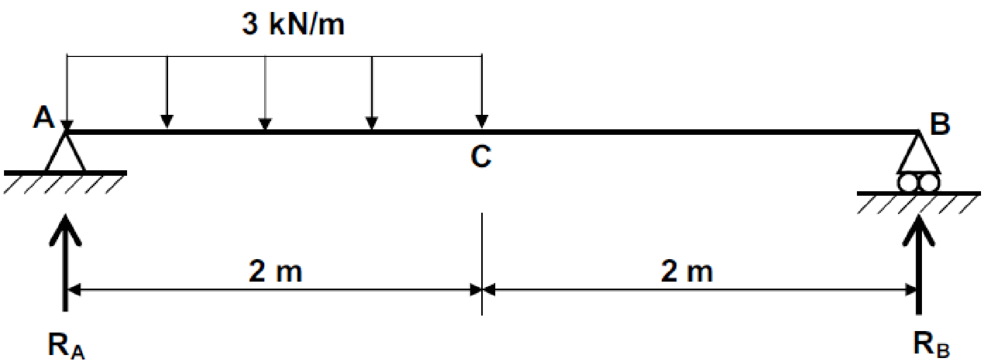


Figure 5.10 Calculation of reactions for uniformly distributed loads.

Sample problems

Calculate the reactions for the problems shown in Figure 5.11.

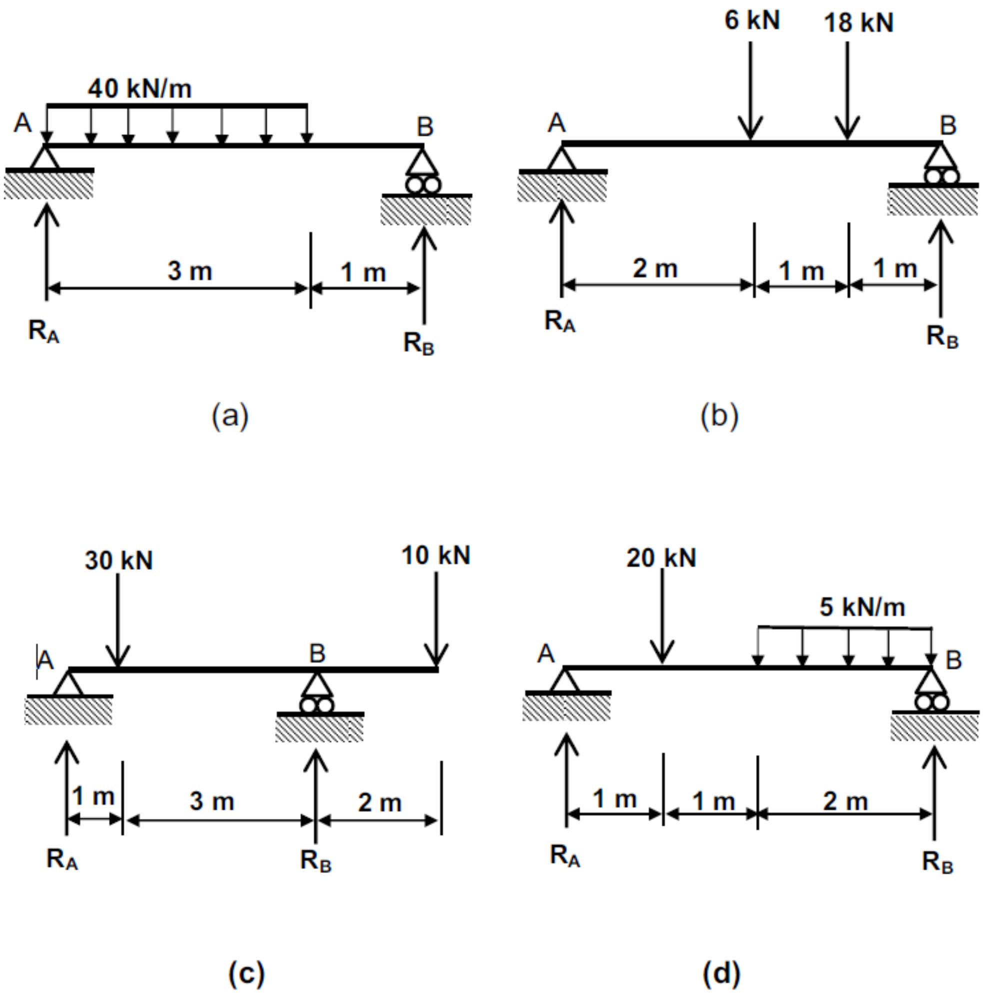


Figure 5.11 Reactions: for sample problems

Lecture 6 **B.M. and S.F. diagrams** for different loading on simply supported beam, cantilever and overhanging beams

6.1.. Deformations in Structures

Imagine that the beams indicated by the thick solid horizontal lines in Fig. 6.1 are quite flexible but not particularly strong, so will readily deform under the loads shown. The lines in Fig. 6.2 indicate the deformed (or deflected) forms of the corresponding beams in Fig. 6.1.

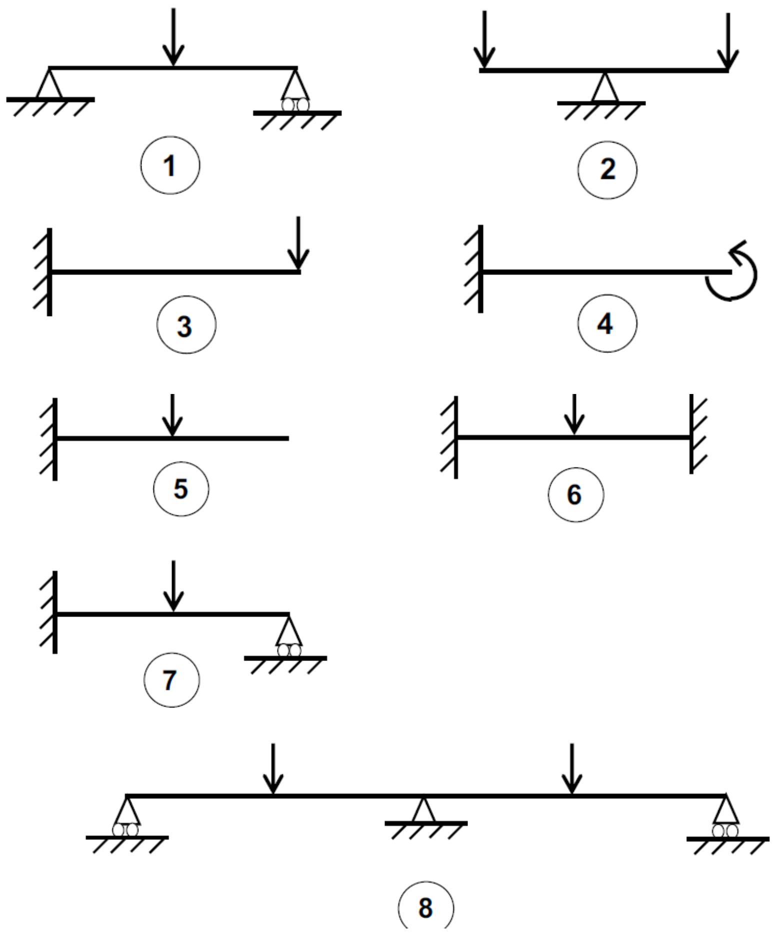


Figure 6.1 Typical beams

Hogging and sagging

Let us discuss the deformations shown in Fig. 6.2, but before we do so let’s define two important terms. You have probably already encountered the term sagging – for example, you may have a bed that sags, or dips, in the middle (in which case, my advice is: get a better bed – it’s well worth the investment). Sagging, or downward deformation, is illustrated in Fig. 6.3 (a). Hogging – an upward deformation – is the opposite (or mirror image) of sagging. The concept of hogging is illustrated in Fig. 6.3 (b).

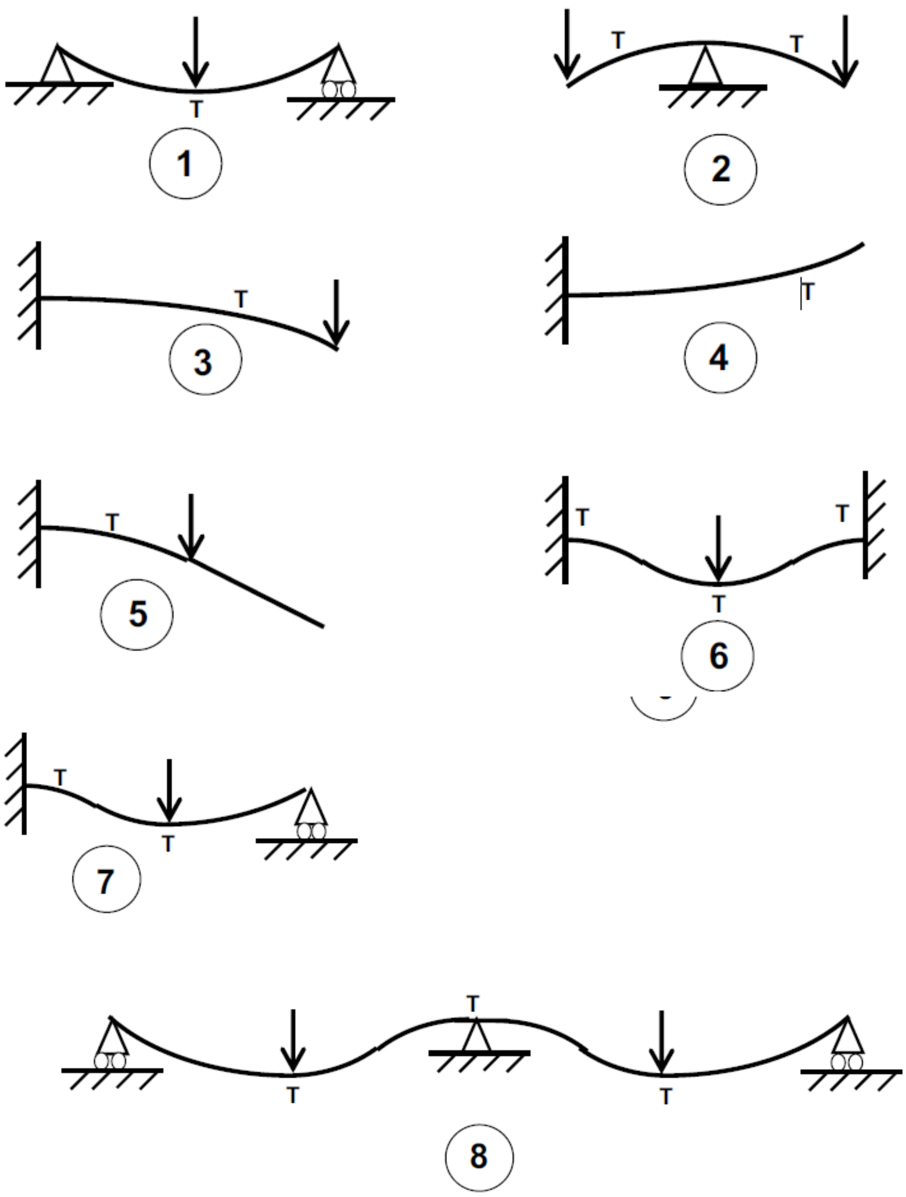


Figure 6.2 Deformations in beams

Consider, as an example, beam number 1 in Fig. 6.1, which is simply supported at either end and is subjected to a central point load. Clearly, the beam will tend to sag under that load, as indicated by the line in the corresponding diagram in Fig. 6.2. When the beam has sagged, the fibres in the very top of the beam will be squashed together; in other words, they will be compressed. Similarly, the fibres in the bottom part of the beam will have stretched, which indicates that the bottom of the beam is in tension.

The fact that the bottom of the beam is in tension is indicated by the letter T (for tension) placed underneath the line in beam number 1 in Fig. 6.2. Beam number 2 in Fig. 6.1 will tend to hog (or ‘break its back’) over the central support as a result of the point loads at either end. This hogging profile is indicated by the line in the corresponding diagram in Fig. 6.2. In this case, we will see that the top of the beam will be in tension and therefore we’ve indicated tension (letter T) above the line at the support position. We can analyse the remaining beams in Fig. 6.1 in a similar fashion and obtain the deformed profiles and tension positions for each one (indicated by the lines and letter T respectively in Fig. 6.2).

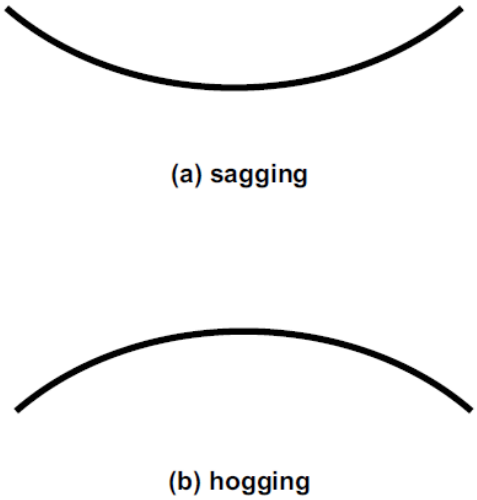


Figure 6.3 Hogging and sagging.

If you have difficulty visualising the deformation of the beam shown in beam number 4, replicate the situation by holding a standard-length ruler horizontal by gripping it firmly with your left hand at its left-hand end and applying an anticlockwise twist with your right hand at the right-hand end. You will then see the ruler deform in the manner depicted for beam number 4 in Fig. 6.2 and tension will occur on the underside. When examining the deformed shapes of the beams indicated in Fig. 6.2 for beams 6 and 7, remember that a fixed support firmly grips a beam, while a pinned (or simple) support permits rotation to take place.

6.2.. Shear and Bending

(1) Shear is a cutting or slicing action which causes a beam to simply break or snap. A heavy load located near the support of a weak beam might cause a shear failure to occur.

(2) If a beam is subjected to a load it will bend. The more load that is applied, the more the beam will bend. The more the beam bends, the greater will be the tensile and compressive stresses induced in the beam. Eventually, these stresses will increase beyond the stresses the material can bear and failure will occur – in other words the beam will break. In short, if you increase the bending in a beam, eventually it will break.

So, a beam can fail in shear or it can fail in bending. A natural question at this stage is: which will occur first? Unfortunately, there is no general answer to that question. In some circumstances, a beam will fail in shear; in other cases, a beam will fail in bending. Which happens first depends on the longitudinal profile of the beam: its spans, the position and nature of its supports and the positions and magnitudes of the loading on it. Only by calculation can we tell whether a shear or a bending failure will occur first. The first thing we need to do is develop a system of quantifying shear and bending effects. These quantifications are called shear force and bending moment respectively and are defined in the following paragraphs.

Shear Force

A shear force is the force tending to produce a shear failure at a given point in a beam. The value of shear force at any point in a beam = the algebraic sum of all upward and downward forces to the left of the point. (The term 'algebraic sum' means that upward forces are regarded as being positive and downward forces are considered to be negative.)

Consider the example shown in Fig. 6.4, in which the end reactions have already been calculated as 25 kN and 15 kN as shown. To calculate the shear force at point A, ignore everything to the right of A and examine all the forces that exist to the left of A. Remember, upward forces are positive and downward forces are negative.

Adding the forces together:

Shear force at A = + 25 – 30 – 10 = –15 kN

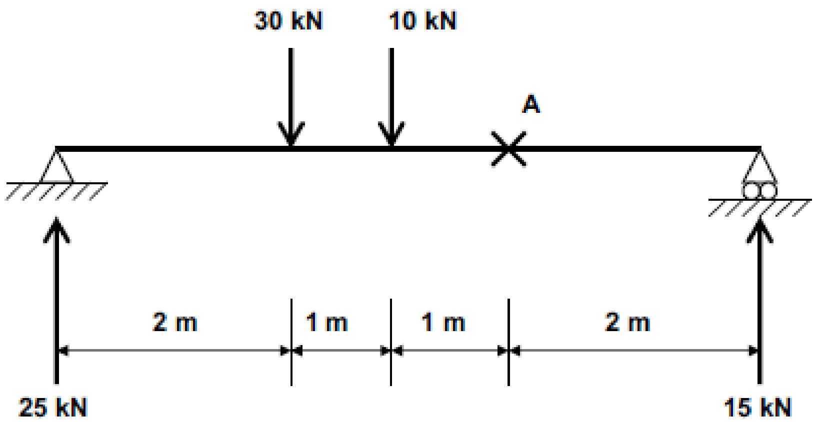


Figure 6.4 Shear force and bending moment at a point

Bending moment

The bending moment is the magnitude of the bending effect at any point in a beam. We knew that a moment is a force multiplied by a perpendicular distance, it's either clockwise or anticlockwise and is measured in kN.m or N.mm. The value of bending moment at any point on a beam = the sum of all bending moments to the left of the point. (Regard clockwise moments as being positive and anticlockwise moments as being negative.)

Consider – again – the beam shown in Fig. 6.4. To calculate the bending moment at point A, ignore everything to the right of A and examine the forces (and hence moments) that exist to the left of A. You should realise that, as we are calculating the moment at A, all distances should be measured from point A to the position of the relevant force. See Fig. 6.5 for clarification.

Bending moment at A = (25 kN × 4 m) – (30 kN × 2 m) – (10 kN × 1 m) = 100 – 60 – 10 = 30 kN.m

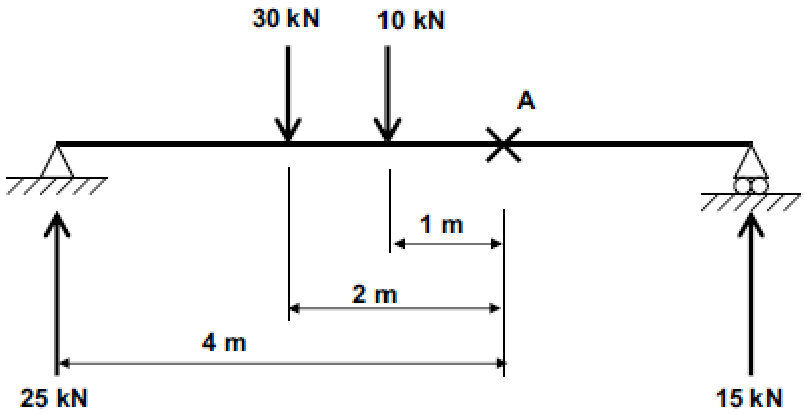


Fig. 6.5 Bending moment at point A.

Figure 6.6 shows a more generalised case. Beam AB supports two point loads, M and N, located at the positions shown. The end reactions at A and B are RA and RB respectively. Suppose we are interested in finding the shear force at position X, which is located a distance x1 from the support A, x2 from point load M and x3 from point load N. The shear force and bending moment at X are calculated as follows:

Shear force at X = RA – M – N

Bending moment at X = (RA × x1) – (M × x2) – (N × x3)

(Remember: clockwise moments are positive, anticlockwise moments are negative.)

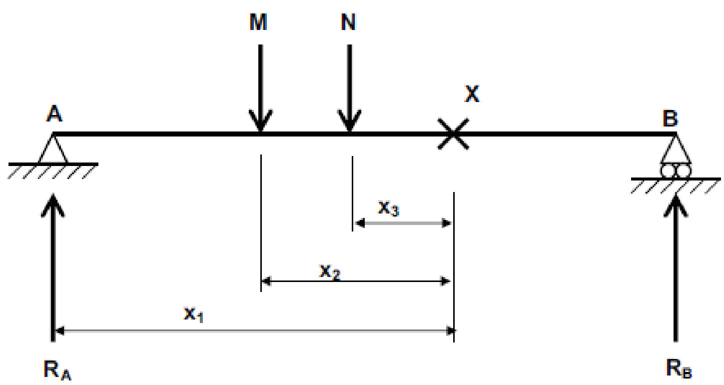


Fig. 6.6 Shear forces and bending moments: general case.

Sample problems

Calculate the shear force and bending moments for the cases shown in Figure 6.7

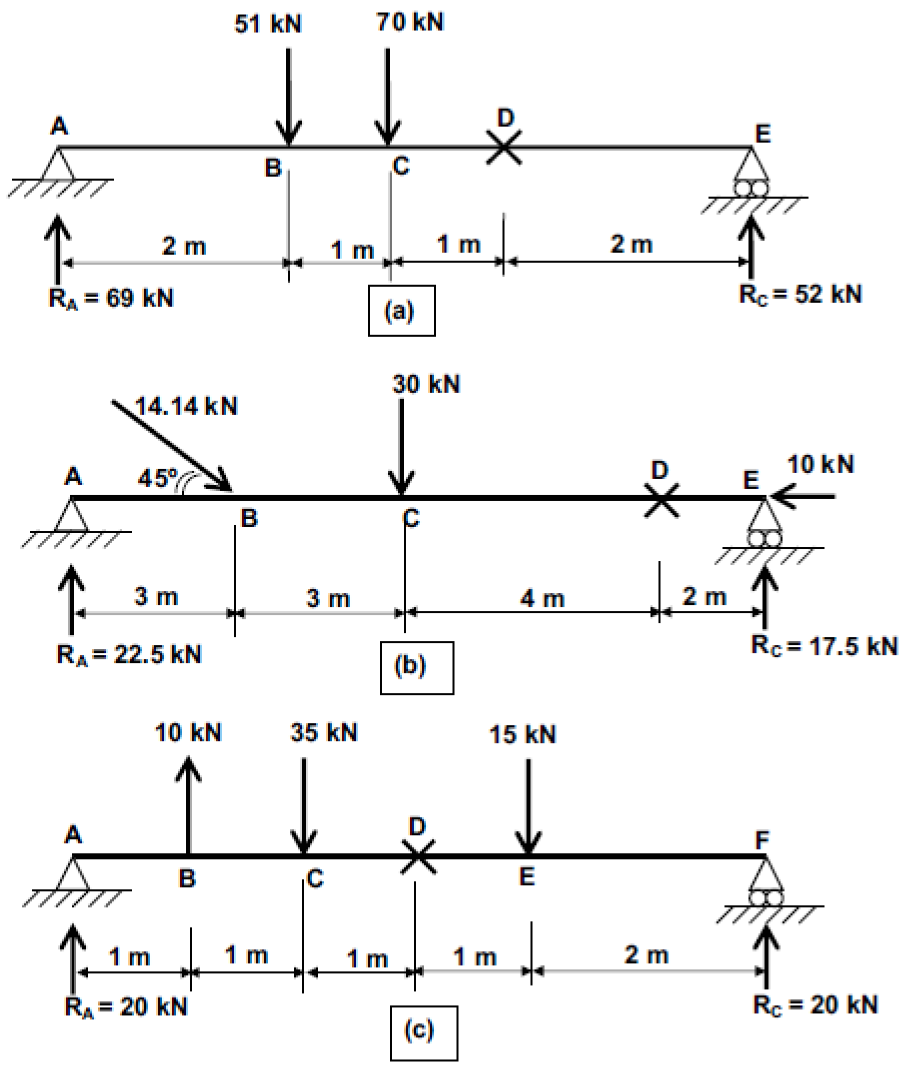


Figure 6.7 Shear forces and bending moments at a point: sample problems

Lecture 7 B.M. and S.F. diagrams for different loading on **simply supported beam**, cantilever and overhanging beams

Up till now we've discussed how to calculate values of shear force and bending moment at a specific point in a beam. As architects, we're not interested so much in the values at a specific point as in how shear force and bending moment vary along the entire length of a beam. Accordingly, we can calculate and draw graphical representations of shear force and bending moment and their variation along a beam. These are called shear force and bending moment diagrams.

Problem: 1

Draw the shear and moment diagrams for a simply supported beam of span 6m carrying a concentrated load of 200 KN at its centre

Solution:

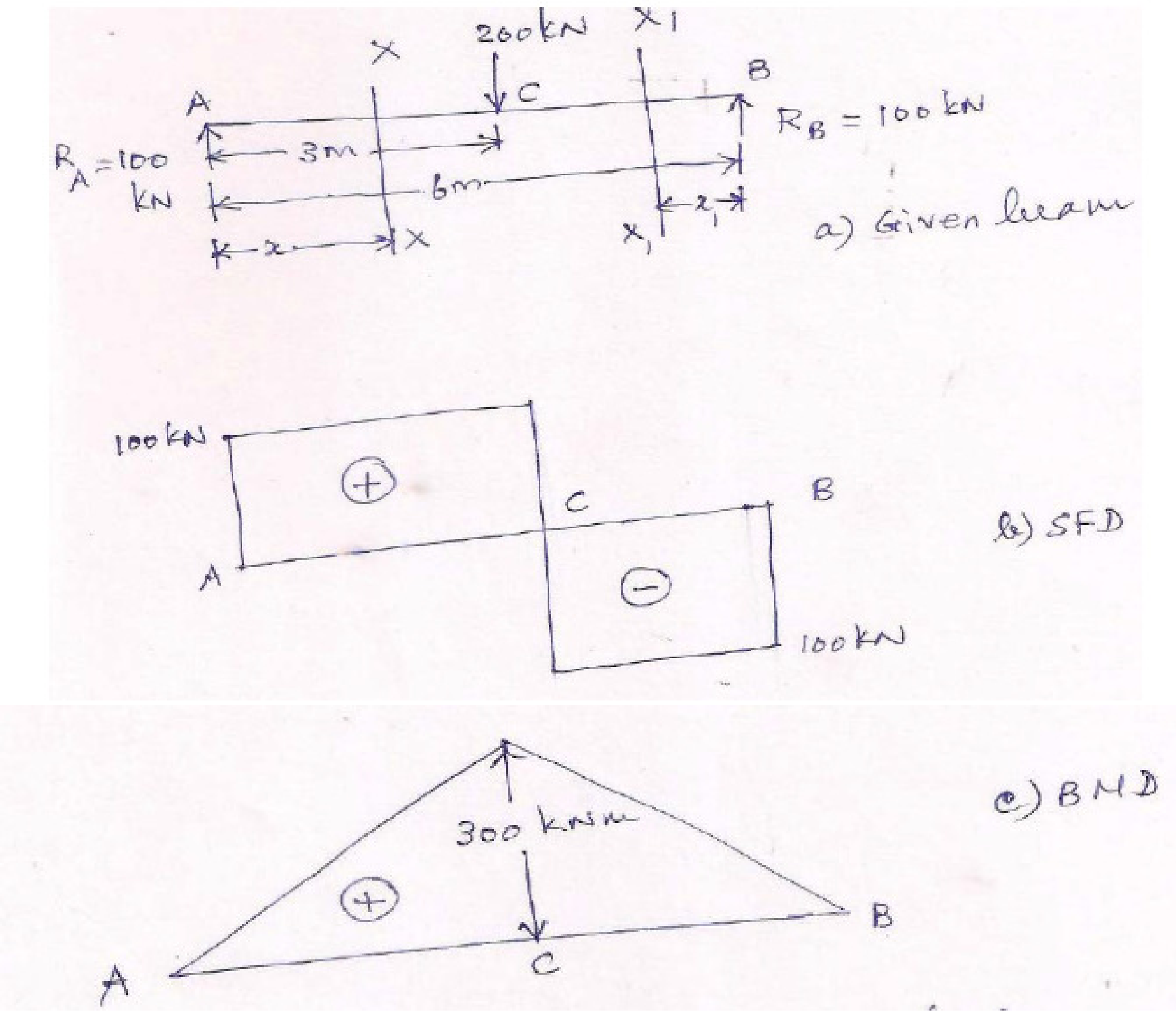


Figure 7.1 SFD and BMD of simple beam

The simple beam is shown in Fig. 7.1(a).

The load is symmetrical and hence the reactions are equal. RA = RB = 100 kN. We consider a section XX at a distance x from left support A as shown in Fig. 7.1(a). The shear force (SF) at this section is Vx = 100 kN by considering forces acting on the left of the section.

The shear force is positive and is independent of x. Therefore it remains constant as x increases up to the point C.

We consider a section X1X1 at a distance x1 from the right support B. The shear force at this section is Vx = - 100 kN by considering forces acting on the right of the section.

The shear force is negative and remains constant till the section reaches the point C. The final shear force diagram (SFD) is shown in Fig. 7.1(b).

The bending moment (BM) at section XX is

$Mx = 100\,x = 100x$

The BM is positive and depends on x. As x increases the BM increases and reaches a maximum value at C.

Therefore MC = 100 x 3 = 300 kNm.

The BM at section X1X1 is MX1 = 100 $x1$

The BM is positive and increases with $x1$. It reaches a maximum value at C. The final BM diagram (BMD) is shown in Fig. 7.1(c).

Problem 2:
Draw the shear and moment diagrams for a simply supported beam of span 6m carrying UDL of 45 KN/m

Solution:

The beam with loading is shown in Fig. 7.2(a). The UDL on the beam is symmetrical. The support reactions are (45x6/2) = 135 kN. We consider a section XX at a distance x from the left support A. The SF at this section XX is

$Vx = 135 – 45x$

The SF is dependent on x and it varies linearly. At support where $x = 0$, VA = 135 kN. At mid-span where $x = 3$ m, SF VC = 135 – 45x3 = 0. We now consider a section XX from right support B. The SF is

$Vx = - 135 + 45x$

When $x = 0$, VB = - 135 + 45x0 = - 135 kN. When x = 3 m, VC = - 135 + 45x3 = 0.

The SFD is shown in Fig. 7.2(b). The BM on the left section is

Mx = 135*x* – (45*x*2/2)

At x = 0, MA = 0. At mid-span where x = 3 m, MC = 135x3 – (45x32/2) = 202.5 kNm.

The BM is maximum at mid-span. If we take a section from the right support the BM expression is the same. The BM at right support is zero and maximum at mid-span. The BMD is shown in Fig. 7.2(c). It is symmetrical about mid-span and also positive throughout.

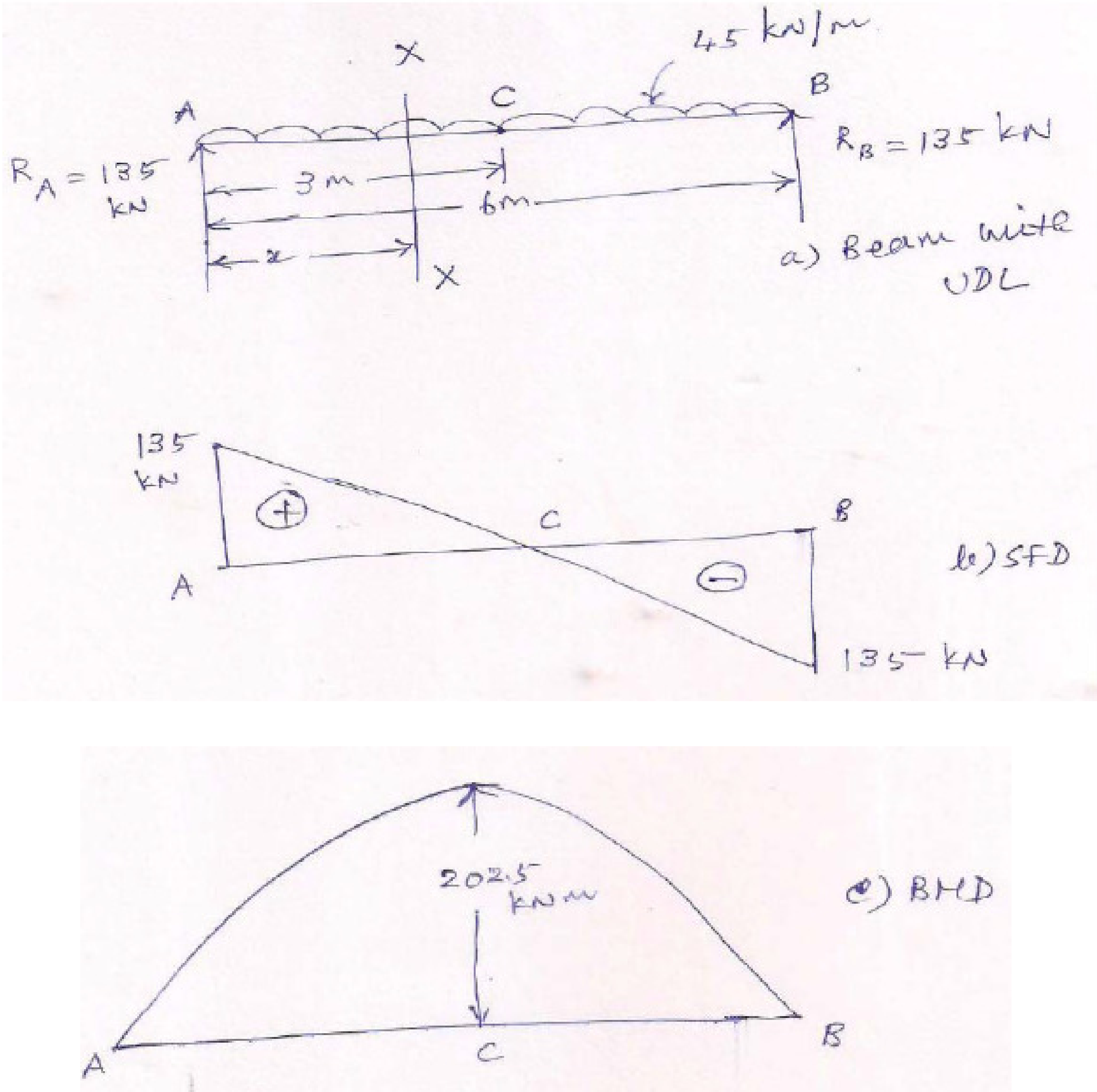


Figure 7.2 SFD and BMD of beam with udl

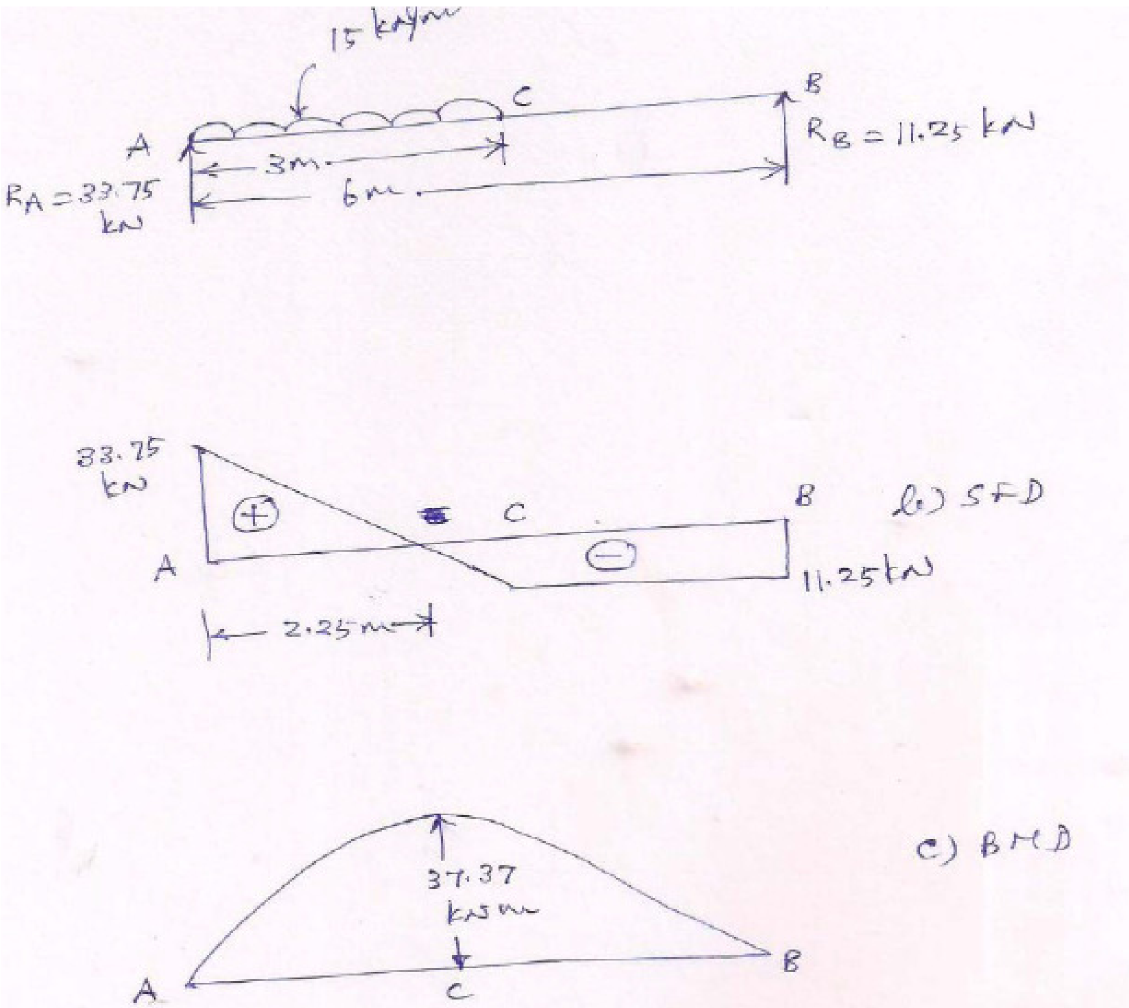


Figure 7.3 Beam carrying udl half of the span

Problem 3:
Sketch the shear and moment diagrams of a simply supported beam of 6 m. The load on the beam consists of UDL of 15 KN/m over the left half of the span

Solution;

The beam with loading is shown in Fig. 7.3(a). The total load on the beam is 45 kN. It acts at its CG of 1.5 m from left support A. Now the reaction $R_A = (45 \cdot 4.5/6) = 33.75$ kN. The reaction $R_B = (45 \cdot 1.5/6) = 11.25$ kN. At a section XX distance *x* from A, SF is

$V_x = 33.75 - 15x$

At $x = 0$, $V_A = 33.75$ kN; at $x = 3$ m, $V_C = 33.75 - 15 \cdot 3 = -11.25$ kN At a section from B, SF is

$V_x = -11.25$ kN

The SF remains constant up to point C. The SFD is shown in Fig. 7.3(b). The SF is zero when 33.75 – 15x = 0, i.e., x = 2.25 m from A. The BM at section XX from A is

$M_x = 33.75x - 15\ (x^2/2)$

At x = 0, $M_A = 0$; The BM is maximum where SF is zero. Therefore

$M_{max} = 33.75 \cdot 2.25 - 15(2.25^2/2) = 37.37$ kNm

The BMD is shown in Fig. 7.3(c).

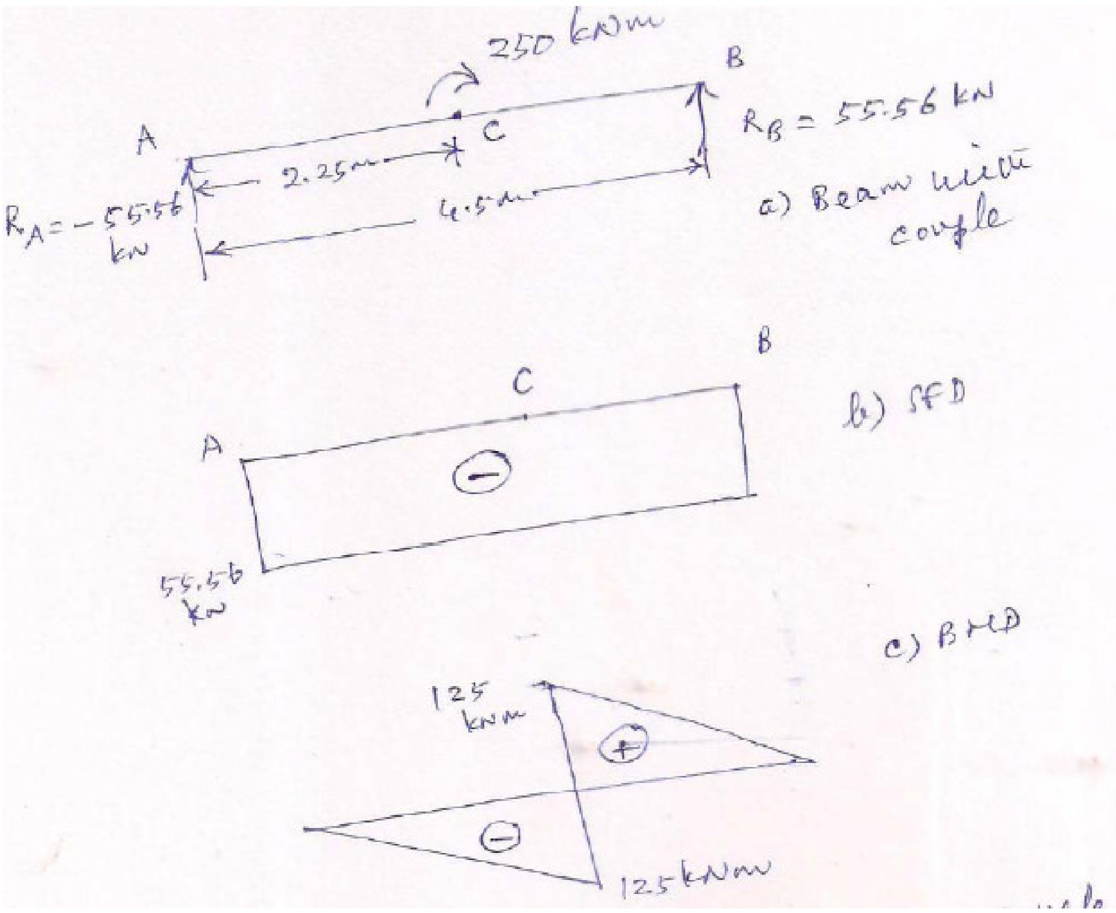


Figure 7.4 SFD and BMD of a beam carrying couple

Problem 4 A simply supported beam carries a moment of 250 KNm at its centre. If the span of the beam is 4.5 m, draw the shear and moment diagrams of the beam.

Solution:

The beam with the couple is shown in Fig. 7.4(a).
We assume the applied moment 250 kNm act clockwise as shown in Fig. 7.4(a). From Statics ($R_A = -(250/4.5) = -55.56$ kN and $R_B = (250/4.5) = 55.56$ kN. The SFD is shown in Fig. 7.4(b).

The SF is negative throughout the span and is a rectangle. From Fig. 3.29(g) the moment at C in the segment AC is

M_C = - (55.56 · 2.25) = - 125 kNm. The BM is negative.

Similarly, the moment at C in the segment BC is
M_C = (55.56 · 2.25) = - 125 kNm. The BM is positive.

The BMD is shown in Fig. 7.4 (c).

Lecture 8 B.M. and S.F. diagrams for different loading on simply supported beam, **cantilever and overhanging beams**: Numerical Problems

Problem 5 A cantilever beam of span 5 m carried a concentrated load of 50 KN at 2.5 m from the fixed end. Draw the shear and bending moment diagrams.

Solution:

The cantilever beam with load is shown in Fig. 8.1(a).

The reaction $R_A = 50$ kN. The moment at fixed end $M_A = -50 \cdot 2.5 = -125$ kNm. The SF at a section from A between A and C is

$V_x = 50$ kN. This is positive and constant throughout.

The SFD is shown in Fig. 8.1(b).

The BM at a section between A and C is

$M_x = -125 + 50x$

At x = 0, $M_A = -125$ kNm and at $x = 2.5$ m, $M_C = -125 + 50 \cdot 2.5 = 0$

The BMD is shown in Fig. 8.1(c).

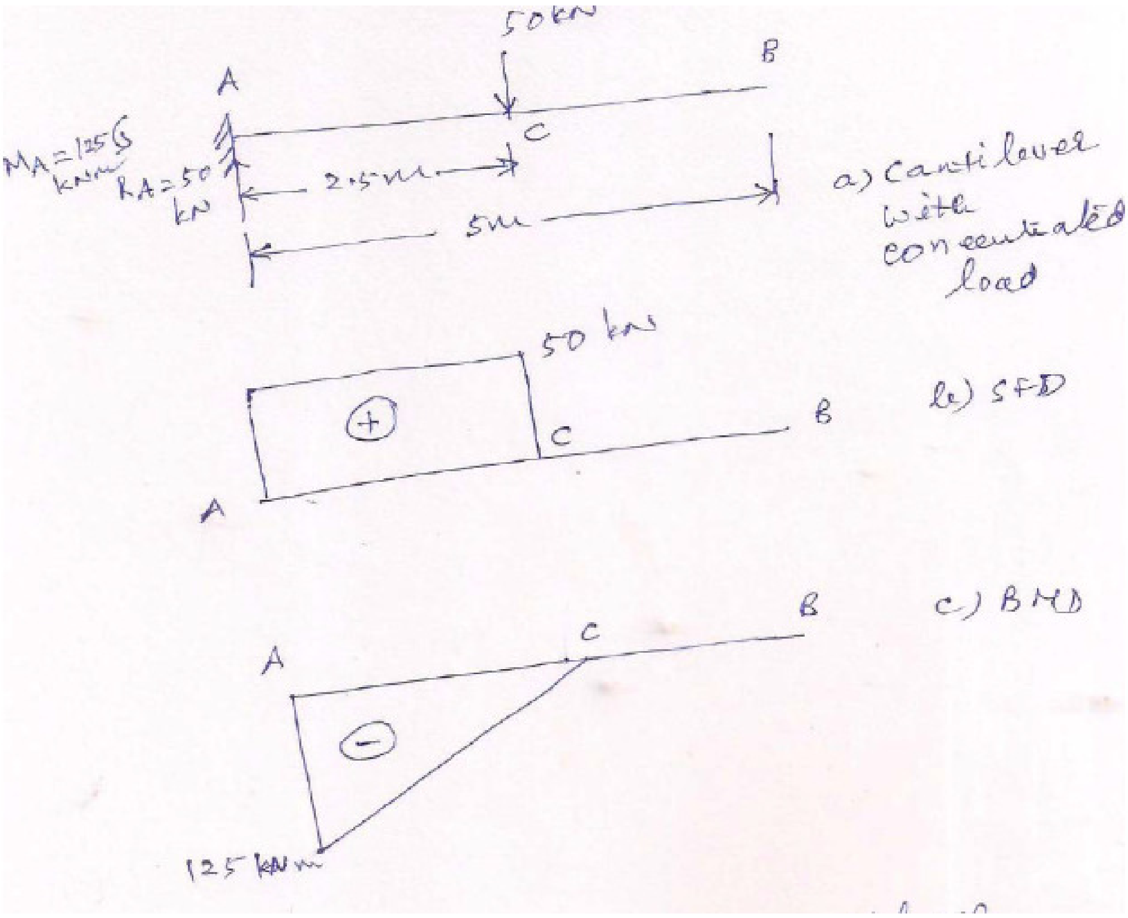


Figure 8.1 SFD and BMD of a cantilever beam

Problem 6: Draw the shear and moment diagrams for a cantilever beam of span 5 m carrying UDL of 15 KN/m for the entire span.

Solution:

The cantilever beam with UDL is shown in Fig. 8.2(a).

From Fig. 3.30(c), R_A = 15・ 5 = 75 kN. The SFD is shown in Fig. 8.2(b).

It is positive throughout. From Fig. 8.2(c), the BM at fixed end is – (15・ 5^2/2) = - 187.5 kNm. The BMD is shown in Fig. 8.2(c) and it is negative all along the span.

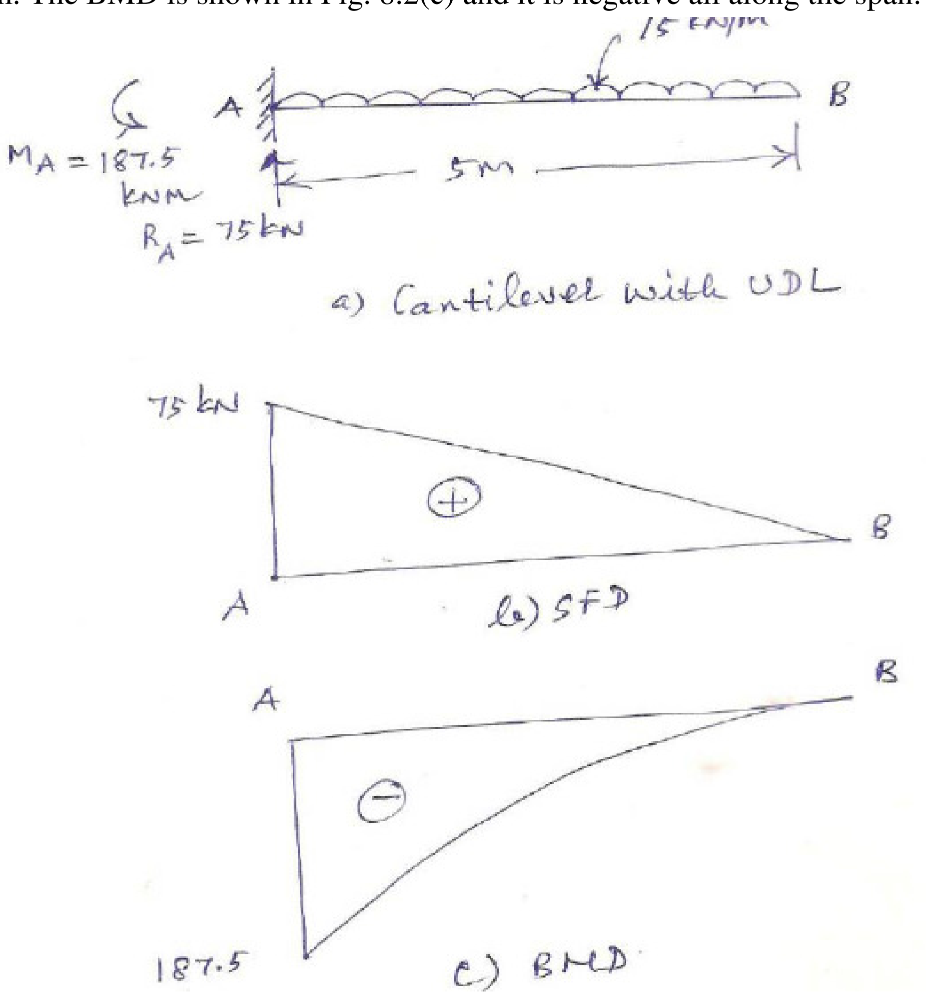


Figure 8.2 SFD and BMD of a cantilever for UDL

Problem 7: The span of a cantilever beam is 5.5 m. It carries UDL of 25 KN/m over a distance of 3m from the free end. Sketch the shear and moment diagrams of the beam.

Solution:

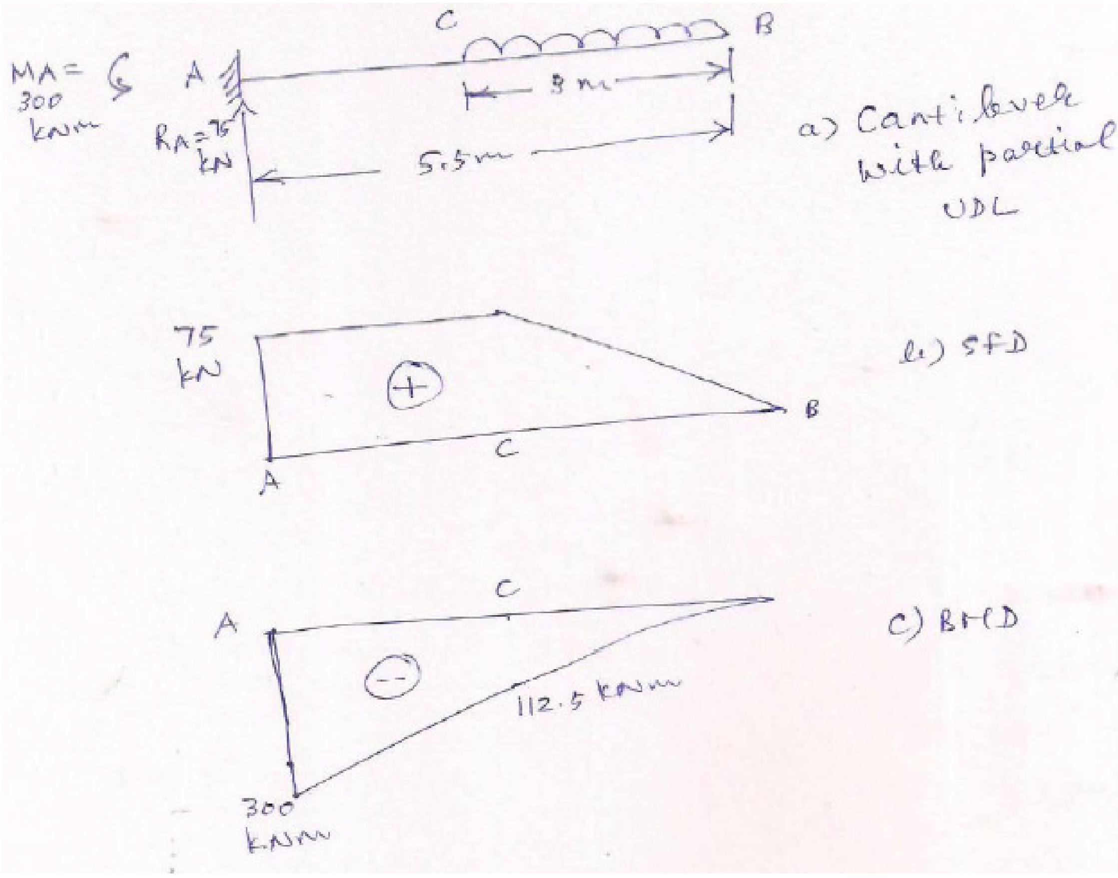


Figure 8.3 SFD and BMD of a cantilever

Problem 8: A beam of span 6m has a overhang of 1.5 m on its right. It carries a central concentrated load of 150 KN on the 6 m span and UDL of 5 KN/m on its overhang. Draw the shear and moment diagrams of the over-hanging beam.

Solution:

The overhanging beam is shown in Fig. 8.4(a).
Taking moment of forces about A

R_Bx6 = 150x3 + 5x1.5x6.75; R_B = 83.44 kN

R_A = 150 + 5x1.5 – 83.44; R_A = 74.06 kN

The SF between A and C is

Vx = 74.06 kN. It is positive and constant

The SF between C and B is

Vx = 74.06 – 150 = - 75.94 kN. It is negative and constant

The SF between B and D

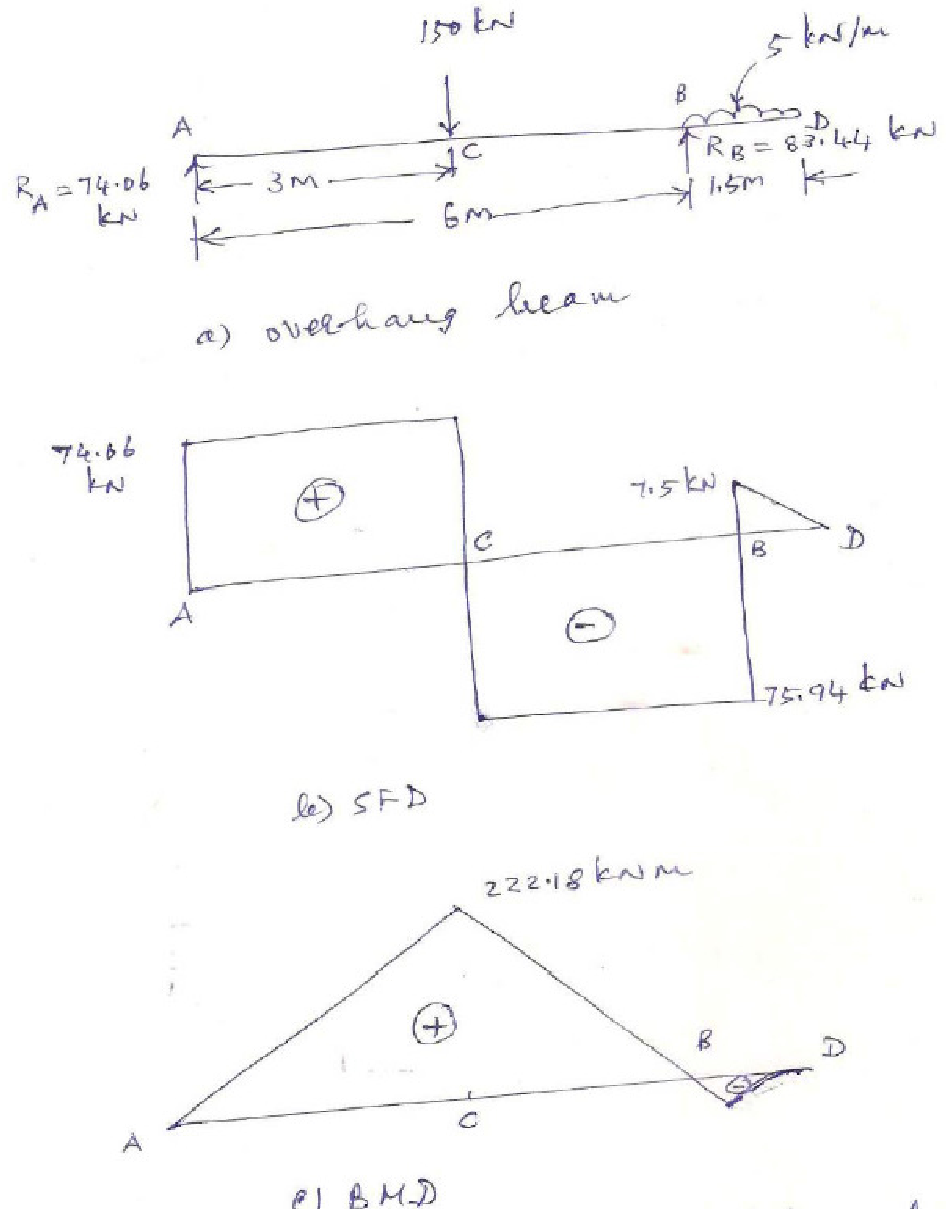


Figure 8.4 SFD and BMD of overhang beam

$V_x = 74.06 - 150 + 83.44 - 5(x - 6)$

At x = 6 m, $V_B = 74.06 - 150 + 83.44 - 5(6 - 6) = 7.5$ kN

At x = 7.5 m, $V_D = 74.06 - 150 + 83.44 - 5(7.5 - 6) = 0$

The SFD is shown in Fig. 8.4(b).

The BM between A and C

$M_x = 74.06x$

When x = 0, MA = 0; x = 3 m, MC = 74.063 = 222.18 kNm

The BM at B

MB = - 5x1.5x0.75 = - 5.625 kNm
The BMD is shown in Fig. 8.4(c).

Use of MATLAB in Structural Analysis

The procedure for obtaining the shear and moment diagrams is described through the solution of a sample beam and loading condition shown in Figure 8.5. Prior to developing the MATLAB script file for this problem, the following preliminary task involving the determination of the algebraic expressions for the shear and moment needs to be performed.

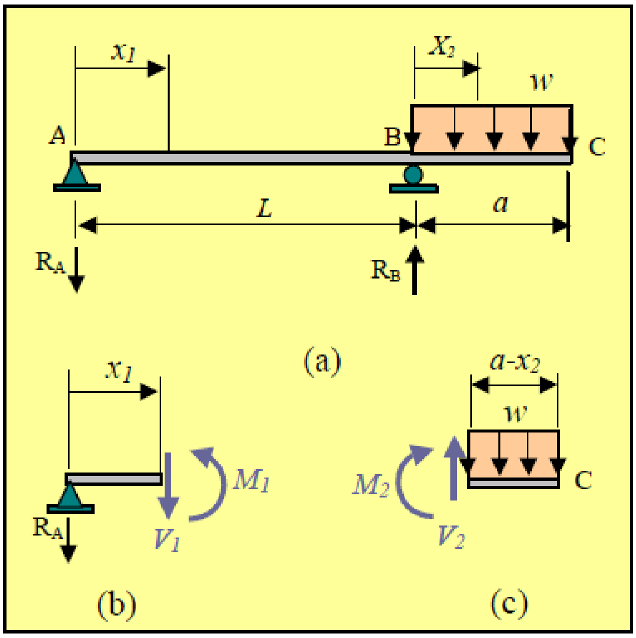


Figure 8.5 Free body diagram of the beam

Writing the force and moment equilibrium equations for the free body diagrams of the two sections of the beam shown at the bottom of Figure 8.5, the following expressions for the shear force V and bending moment M can be established for each of the beam segments AB and BC.

$$0 \le x_1 \le L$$

$$V_1 = \frac{-wa^2}{2L} \qquad M_1 = \frac{-wa^2}{2L}x_1$$

$$0 \le x_2 \le a$$

$$V_1 = w(a - x_2) \qquad M_2 = \frac{-w(a - x_2)^2}{2}$$

Now that the theoretical formulation of the problem is complete, a MATLAB script file can be created. The script file can be developed in a form which prompts the user to input the values for the parameters w, E, I, L, and a. Then, a MATLAB loop can be employed to compute the values for the shear and moment along the length of the beam for a series of values of x, measured from the left support at A, starting from x = 0 and ending at x = L. Note that it is necessary to include a conditional statement within the loop, so that the proper expressions for the determination of unknowns is selected and used in the computations. The MATLAB script file for the given beam is provided in Figure 8.6, along with the generated plots in Figure 8.7. These plots are for the case when w = 0.2 kip/ft, E = 29000 ksi, I = 100 in4, L = 20 ft, and a = 10 ft. Using the powerful MATLAB plotting commands and tools, the users can control and create the plots in any format they desire.

MATLAB SCRIPT TO PLOT SHEAR FORCE AND BENDING MOMENTS

```
w=0.2;
E=29000;
I=100;
L=20;
a=10;
w=w/12;L=L*12;a=a*12;
fprintf(' x(in.) Shear(kip) Moment(kip.in)\n')
%
________________________________________________________________________
__________________
% Computing the shear, moment
x=linspace(0,L+a,(L+a)/6+1);
for k=1:1:(L+a)/6+1
if x(k)<=L
x1(k)=x(k);
V(k)=-w*a^2/(2*L);
M(k)=-w*a^2*x1(k)/(2*L);
fprintf('%4.0f %12.3f %14.3f\n',x(k),V(k),M(k))
else
x2(k)=x(k)-L;
V(k)=w*(a-x2(k));
M(k)=-w*(a-x2(k))^2/2;
fprintf('%4.0f %12.3f %14.3f\n',x(k),V(k),M(k))
end
end
%
________________________________________________________________________
__________________
% plotting the shear, moment
subplot(2,1,1),plot(x,V),title('Shear'),xlabel('x (in)'),ylabel('Shear
Force (kip)'),...
axis([0 360 -1 3]),set(gca,'XTick',[0:60:L+a]),text(60,0,'V_1 =-
wa^2/(2L)'),...
text(250,2.2,'V_2 =w(a-x_2)')

subplot(2,1,2),plot(x,M),title('Moment'),xlabel('x (in)'),ylabel('Moment
(kip.in)'),...
axis([0 360 -150 50]),set(gca,'XTick',[0:60:L+a]),text(30,-90,'M_1 =-
wa^2x_1/(2L)'),...
text(210,20,'M_2 =-w(a-x_2)^2/2')
```

Figure 8.6 MATLAB Script for BM and SF

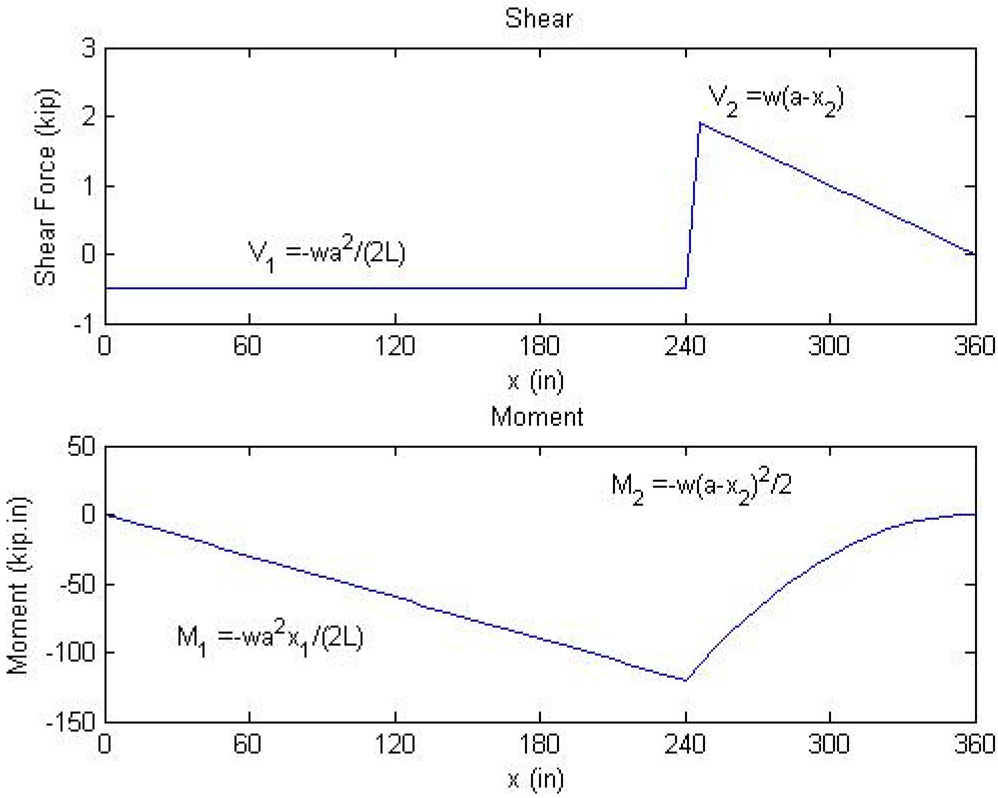


Figure 8.7 Shear force and bending moment diagram of a overhanging beam

Lecture 9 B.M. shear and normal thrust of **three hinged arches**: Theory

9.1 Introduction

Arches also offer a potential synthesis of architectural and structural form. At Ludwig Erhard House, Berlin (Fig. 9.1) repeated arches structure a vault-like building form. Varying arch spans respond to an irregularly shaped site. Suspended floors either hang from tension hangers under the arches, or as on the street frontage, are propped off them. This is an example of reasonably conventional arch usage where arches are regularly spaced and aligned vertically. But at the Great Glasshouse, Carmarthenshire, arches form a toroidal dome (Fig. 9.2). The dome's two constant orthogonal radii of curvature require that the arches distant from the building's centreline lean over in response to the three-dimensional surface curvature. Clarity of the arched structural form is undiminished by the small diameter tubes that run longitudinally to tie the arches back at regular intervals to a perimeter ring beam. Apart from supporting the roof glazing they also prevent the arches from buckling laterally and deflecting from their inclined planes.

Figure 9.1 Ludwig Erhard House, Berlin, Germany, Nicholas Grimshaw & Partners, 1998. Arched end of building as seen from the rear.

Figure 9.2 The Great Glasshouse, Carmarthenshire, Wales, Foster and Partners, 1998. Arched roof.

9.2 Three Hinged Arches

The three-hinged arch has hinged supports at each abutment that provide four external restraints. The introduction of another hinge in the arch member provides a moment release. The extra hinge provides an additional equation of equilibrium that, together with the three basic equations of equilibrium, makes the solution of the arch possible. Typical examples of three-hinged arches are shown in Figure 9.3.

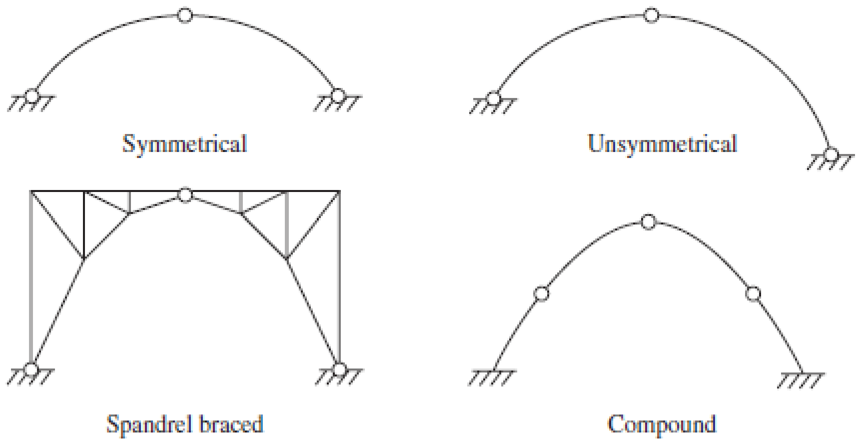


Figure 9.3 Three hinged arches

For a three-hinged arch subjected to vertical loads only, the horizontal support reactions at the arch springings are equal and opposite and act inward. The vertical support reactions at the arch springings are equal to those of a simply supported beam of identical length with identical loads.

For the symmetrical three-hinged arch shown in Figure 9.4 with a vertical applied load *W*, the unknown horizontal thrust at the springings is *H*, and the unknown vertical reactions are *V*1 and *V* 2. The vertical reaction at support 2 is obtained by considering moment equilibrium about support 1. Hence:

$$M_1 = 0$$

$$M_1 = lV_2 - Wa = 0$$
$$V_2 = Wa/l$$
$$V_1 = W - V_2 = W(1 - a/l)$$

These values for V_1 and V_2 are identical to the reactions of a simply supported beam of the same span as the arch with the same applied load W.

The horizontal thrust at the springing is determined from a free body diagram of the right half of the arch as shown at (ii). Considering moment equilibrium about the crown hinge at 3;

$$M_3 = lV_2/2 - Hc = 0$$

$H = lV_2 / 2c$

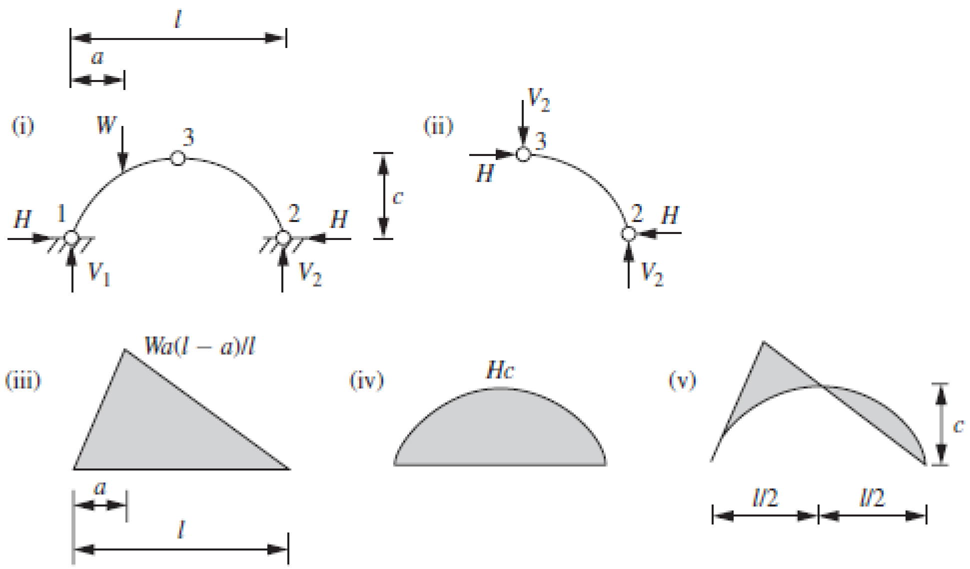


Figure 9.4 Three hinged arch

This value for H is identical to the bending moment at the center of a simply supported beam of the same length with the same applied load W multiplied by 1/c.

The bending moment in the arch at any point a distance x from the left support is given by the expressions:

$$M_x = V_1 x - Hy \cdots for x \leq a$$

$$M_x = V_2 (l - x) - Hy \cdots for a \prec x \leq l$$

Where y is the height of the arch at a distance x from the left support.

At x=l/2, y=c, and the bending moment at the crown hinge is:

$$M_{l/2} = 0$$

The expressions for bending moment may be considered as the superposition of the bending moment of a simply supported beam of the same span with the same applied load W plus the bending moment due to the horizontal thrust H . The bending moment due to the applied load on a simply supported beam is shown at (iii) and the bending moment due to the horizontal thrust is shown at (iv); this is identical to the shape of the arch. Since the bending moment is zero at the crown hinge, the combined bending moment for the arch is obtained by adjusting the scale of the free bending moment to give an ordinate of magnitude c at x= l/2 and superimposing this on a drawing of the arch. This is shown at (v), drawn on the compression side of the arch. In the case of a three hinged parabolic arch with a uniformly distributed applied load, no bending moment is produced in the arch rib.

Lecture 10 B.M. shear and normal thrust of **three hinged arches**: Numerical Examples

Problem 1: A parabolic arch has span 60 m and rise 5 m carries a concentrated load of 10 KN at a distance of 45 m from the left abutment. Find the reactions at the supports and the resultant moment at the load.

Solution:

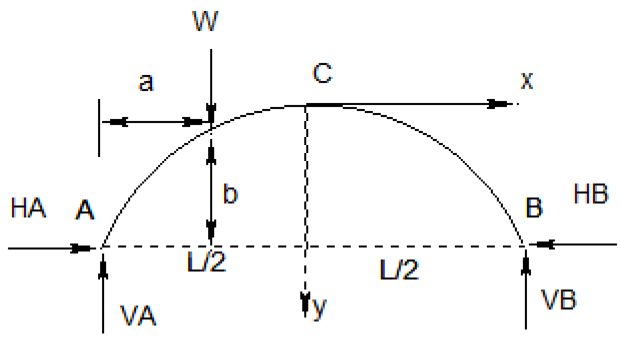


Figure 10.1 Three hinged arch under single concentrated load

The arch is parabolic. We assume it is a three-hinged arch.

$$V_A = \frac{10\times 15}{60} = 2.5KN \quad V_B = \frac{10\times 45}{60} = 7.5KN$$

$$H = \frac{10\times 15}{2\times 5} = 15KN$$

The BM at the load as a simple beam is

$$M_o = \frac{10\times 45\times 15}{60} = 112.5KNm$$

We assume the equation of the arch with the origin at the crown and x axis directed to the right and y axis directed downwards is

$$y = \frac{4hx^2}{L^2}$$

The height of the arch axis at the load above the springing line is (h –y).

For x=15m

$$y = \frac{4\times 5\times 15^2}{60^2} = 1.25m$$

$$\therefore (h-y) = 5-1.25 = 3.75m$$

Moment due to thrust is $H(h-y) = 15\times 3.75 = 56.25KNm$

The resultant moment at the load is $(112.5-56.25) = 56.25KNm$

Problem 2: A three hinged parabolic arch is as shown below. Determine the vertical and horizontal reactions at supports A and B.

Solution:

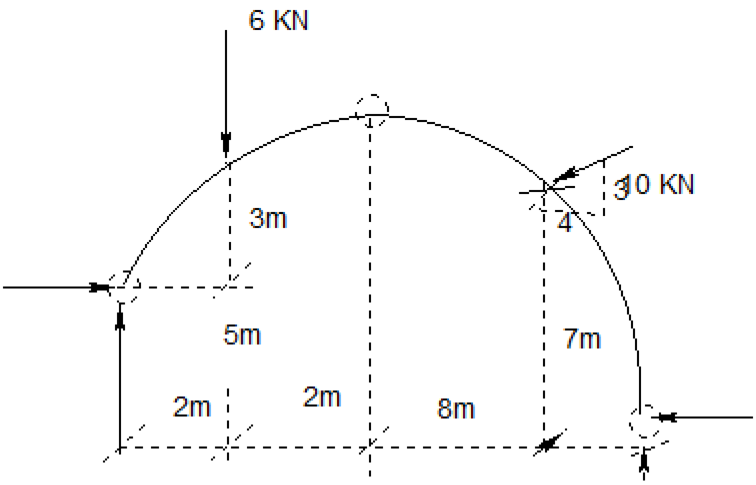


Figure 10.2 Three hinged arch

The inclined load of 10 kN is situated at a distance of 4 m from C horizontally and 7 m vertically above B. We can resolve this load into horizontal and vertical components.

The vertical component is

$$V_W = \frac{3\times10}{5} = 6KN$$

The horizontal component is

$$H_W = \frac{4\times10}{5} = 8KN$$

Taking moment of forces about A

$$R_B \times 12 - H_B \times 7 = 6\times2 + 6\times8 - 8\times2$$
$$R_B \times 12 - H_B \times 7 = 44KN \qquad (1)$$

Taking moment of forces about C

$$R_B \times 8 - H_B \times 8 = 6\times4 + 8\times1$$
$$R_B - H_B = 4 \qquad (2)$$

From Equation (2), $R_B = 4 + H_B$ (3)

Substituting Eq (3) into Eq(1) and solving $H_B = -0.8KN$ (4)

Substituting Eq. (4) into Eq(2). And solving $R_B = 8.8KN$ (5)

From $\sum F_V = 0 \quad R_A + R_B = 6 + 6$

Substituting for R_B from Eq. (5) and solving we get $R_A = 8.8KN$

From $\sum F_H = 0 \quad H_A + H_B = 8$

Substituting for H_B from Eq. (4) and solving we get $H_A = 7.2KN$

Sample Problems

Problem 1: A parabolic arch hinged at the springing and crown has a span of 20 m. The central rise of the arch is 4m. It is loaded with a uniformly distributed load of intensity 2KN/m on the left 8 m length. Calculate a) the direction and magnitude of reaction at the hinges and b) the bending moment, normal thrust and shear at 4m and 15 m from the left end. and c) maximum positive and negative bending moments.

Problem 2: A symmetrical parabolic arch with a central hinge, of rise r and span L is supported at its ends on pins at the same level. What is the value of the horizontal thrust when a load W which is uniformly distributed horizontally covers the whole span ?

Lecture 11 Deflection of statically determinate beams: **Double Integration method**,

11.1.. Introduction

Deflections of structures can occur from various sources, such as loads, temperature, fabrication errors, or settlement. In design, deflections must be limited in order to provide integrity and stability of roofs, and prevent cracking of attached brittle materials such as concrete, plaster or glass. Furthermore, a structure must not vibrate or deflect severely in order to “appear” safe for its occupants. More important, though, deflections at specified points in a structure must be determined if one is to analyze statically indeterminate structures.

Under linear elastic condition, a structure subjected to a load will return to its original un-deformed position after the load is removed. The deflection of a structure is caused by its internal loadings such as normal force, shear force, or bending moment. For *beams* and *frames,* however, the greatest deflections are most often caused by *internal bending*, whereas *internal axial forces* cause the deflections of a *truss*. Before the slope or displacement of a point on a beam or frame is determined, it is often helpful to sketch the deflected shape of the structure when it is loaded in order to partially check the results. This *deflection diagram* represents the *elastic curve* or locus of points which defines the displaced position of the centroid of the cross section along the members. For most problems the elastic curve can be sketched without much difficulty.When doing so, however, it is necessary to know the restrictions as to slope or displacement that often occur at a support or a connection. Supports that *resist a force,* such as a pin, *restrict displacement;* and those that *resist moment,* such as a fixed wall, *restrict rotation*. Note also that deflection of frame members that are fixed connected causes the joint to rotate the connected members by the same amount . On the other hand, if a pin connection is used at the joint, the members will each have a *different slope* or rotation at the pin, since the pin cannot support a moment.
If the elastic curve seems difficult to establish, it is suggested that the moment diagram for the beam or frame be drawn first. By the sign convention for moments, a *positive moment* tends to bend a beam or horizontal member *concave upward,* Likewise, a *negative moment* tends to bend the beam or member *concave downward*. Therefore, *if the shape of the moment diagram is known, it will be easy to construct the elastic curve and vice versa*. For example, consider the beam in Fig. 11.1 with its associated moment diagram. Due to the pin-and-roller support, the displacement at *A* and *D* must be zero. Within the region of negative moment, the elastic curve is concave downward; and within the region of positive moment, the elastic curve is concave upward. In particular, there must be an *inflection point* at the point where the curve changes from concave down to concave up, since this is a point of zero moment.

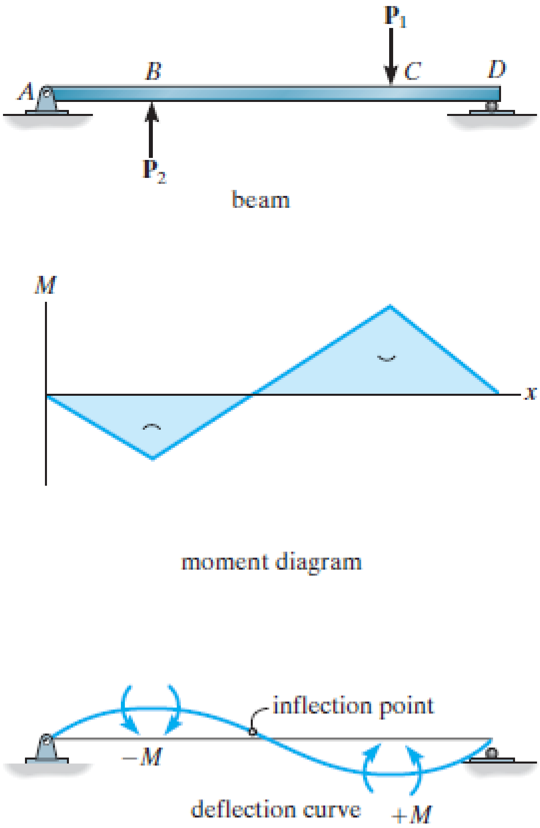


Figure 11.1 BMD and elastic curve of a simply supported beam under varying loading conditions

11.2 Elastic Beam Theory

Let us discuss the most common case of an initially straight beam that is elastically deformed by loads applied perpendicular to the beam's *x* axis and lying in the x-v plane of symmetry for the beam's cross-sectional area, Fig. 11.2*a*. Due to the loading, the deformation of the beam is caused by both the internal shear force and bending moment. If the beam has a length that is much greater than its depth, the greatest deformation will be caused by bending, and therefore we will direct our attention to its effects.

When the internal moment *M* deforms the element of the beam, each cross section remains plane and the angle between them becomes $d\theta$, Fig. 11.2*b*. The arc *dx* that represents a portion of the elastic curve intersects the neutral axis for each cross section.The *radius of curvature* for this arc is defined as the distance , which is measured from the *center of curvature O'* to *dx*. Any arc on the element other than *dx* is subjected to a normal strain. For example, the strain in arc *ds*, located at a position *y* from the neutral axis, is $\varepsilon = \dfrac{ds' - ds}{ds}$.

However, $ds = dx = \rho d\theta$ and $ds' = (\rho - y)d\theta$,and so $\varepsilon = \dfrac{(\rho - y)d\theta - \rho d\theta}{\rho d\theta}$ or $\dfrac{1}{\rho} = \dfrac{-\varepsilon}{y}$

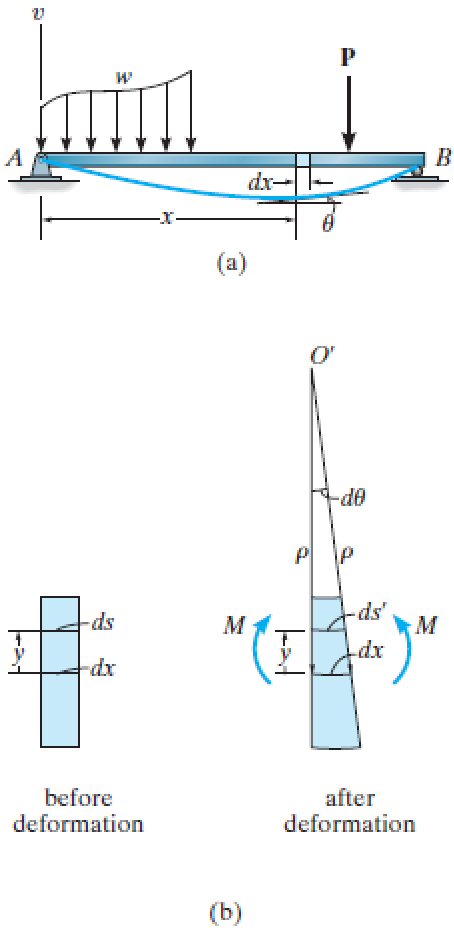


Figure 11.2 Simple beam bending

If the material is homogeneous and behaves in a linear elastic manner, then Hooke's law applies, $\varepsilon = \sigma / E$ Also, since the flexure formula applies, $\sigma = -My / I$ Combining these equations and substituting into the above equation, we have

$$\frac{1}{\rho} = \frac{M}{EI} \tag{11.1}$$

Here

ρ = the radius of curvature at a specific point on the elastic curve $1/\rho$ (is referred to as the curvature)

M= the internal moment in the beam at the point where ρ is to be determined

E= the material's modulus of elasticity

I = the beam's moment of inertia computed about the neutral axis

The product *EI* in this equation is referred to as the *flexural rigidity,* and it is always a positive quantity. Since $dx = \rho d\theta$ then from Eq. 11.1,

$$d\theta = \frac{M}{EI} dx \tag{11.2}$$

If we choose the axis positive upward, Fig. 11–2*a*, and if we can express the curvature $1/\rho$ in terms of *x* and , we can then determine the elastic curve for the beam. In most calculus books it is shown that this curvature relationship is

$$1/\rho = \frac{d^2 v / dx^2}{\left[1 + (dv/dx)^2\right]^{3/2}}$$

Therefore

$$\frac{M}{EI} = \frac{d^2 v / dx^2}{\left[1 + (dv/dx)^2\right]^{3/2}} \tag{11.3}$$

This equation represents a nonlinear second-order differential equation. Its solution $v = f(x)$, gives the exact shape of the elastic curve assuming of course that beam deflections occur only due to bending. In order to facilitate the solution of a greater number of problems, Eq. 11–3 will be modified by making an important simplification. Since the slope of the elastic curve for most structures is very small, we will use small deflection theory and assume $dv/dx \approx 0$ Consequently its square will be negligible compared to unity and therefore Eq. 11–3 reduces to

$$\frac{d^2 v}{dx^2} = \frac{M}{EI} \tag{11.4}$$

Once M is expressed as a function of position x, then successive integrations of Eq. 11–4 will yield the beam's slope, $\theta \approx \tan\theta = dv/dx = \int (M/EI)dx$ (Eq. 11–2), and the equation of the elastic curve, $v = f(x) = \iint (M/EI)dx$ respectively. For each integration it is necessary to introduce a "constant of integration" and then solve for the constants to obtain a unique solution for a particular problem. If the loading on a beam is discontinuous—that is, it consists of a series of several distributed and concentrated loads—then several functions must be written for the internal moment, each valid within the region between the discontinuities. For example, consider the beam shown in Fig. 11–3. The internal moment in regions AB, BC, and CD must be written in terms of the and coordinates. Once these functions are integrated through the application of Eq. 11–4 and the constants of integration determined, the functions will give the slope and deflection (elastic curve) for each region of the beam for which they are valid.

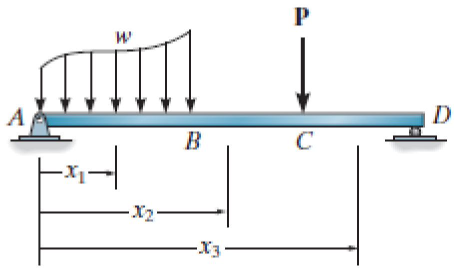


Figure 11.3 Beam with general loading

Sign Convention

When applying Eq. 11–4, it is important to use the proper sign for M as established by the sign convention that was used in the derivation of this equation, Fig. 11–4a. Furthermore, recall that positive deflection, v , is upward, and as a result, the positive slope angle θ will be measured counter-clockwise from the x axis. The reason for this is shown in Fig. 11.4b. Here, positive increases dx and dv in x and v create an increase $d\theta$ that is counter-clockwise. Also, since the slope angle θ will be very small, its value in radians can be determined directly from $\theta \approx \tan\theta = dv/dx$

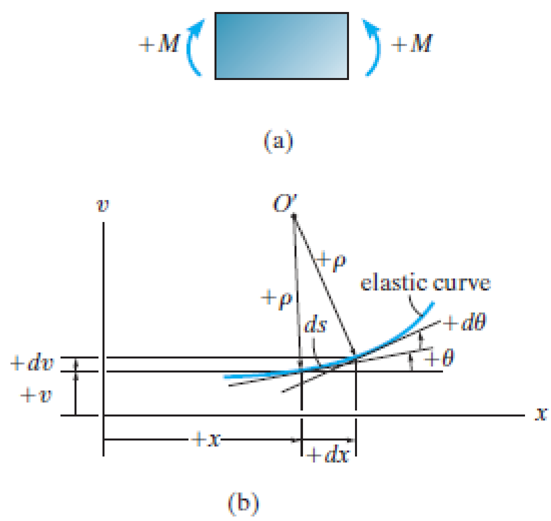


Figure 11.4 Beam with notations

Boundary continuity conditions

The constants of integration are determined by evaluating the functions for slope or displacement at a particular point on the beam where the value of the function is known. These values are called boundary conditions. For example, if the beam is supported by a roller or pin, then it is required that the displacement be zero at these points. Also, at a fixed support the slope and displacement are both zero. If a single x coordinate cannot be used to express the equation for the beam's slope or the elastic curve, then continuity conditions must be used to evaluate some of the integration constants. Consider the beam in Fig. 11–5. Here the x_1 and x_2 coordinates are valid only within the regions AB and BC, respectively. Once the functions for the slope and deflection are obtained, they must give the same values for the slope and deflection at point B $x_1+x_2 = a$, so that the elastic curve is physically continuous. Expressed mathematically, this requires $\theta_1(a) = \theta_2(\theta)$ and v_1 (a)=v_2 (a). These equations can be used to determine two constants of integration.

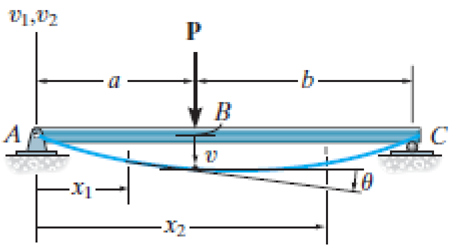


Figure 11.5

Example 1: The cantilevered beam shown in Fig. 11–6a is subjected to a couple moment $M_o$at its end. Determine the equation of the elastic curve. EI is constant.

Solution:

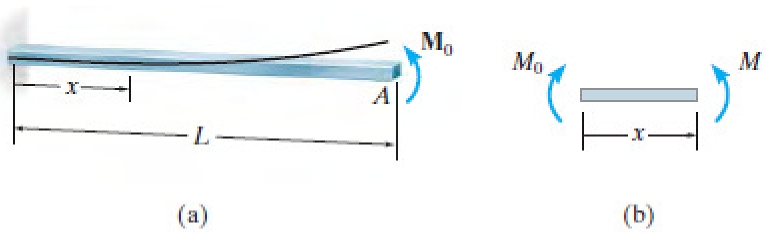


Figure 11.6

Elastic Curve. The load tends to deflect the beam as shown in Fig. 11.6a. By inspection, the internal moment can be represented throughout the beam using a single x coordinate.

Moment Function. From the free-body diagram, with M acting in the positive direction, Fig. 11–6b, we have $M=M_o$

Slope and Elastic Curve

Applying Eq (11.4), and integrating twice yields

$$EI\frac{d^2v}{dx^2} = M_o \tag{1}$$

$$EI\frac{dv}{dx} = M_o x + C_1 \tag{2}$$

$$EIv = \frac{M_o x^2}{2} + C_1 x + C_2 \tag{3}$$

Using the boundary conditions $dv/dx = 0$ at x=0 and v=0 at x=0 , then $C_1=C_2 = 0$. Substituting these results in Eqs (2) and (3) with $\theta = dv/dx$, we get

$$\theta = \frac{M_o x}{EI}$$

$$v = \frac{M_o x^2}{2EI}$$

Maximum slope and displacement occur at A x=L, for which

$$\theta = \frac{M_o L}{EI} \tag{4}$$

$$v = \frac{M_o L^2}{2EI} \tag{5}$$

The positive result for slope indicates counter-clockwise rotation and the positive result for deflection indicates that is upward. This agrees with the results sketched in Fig. 11.6a.

USE OF MATLAB FOR SLOPE AND DEFLECTION

The procedure for obtaining the slope and deflection is described through the solution of a sample beam and loading condition shown in Figure 11.7. Prior to developing the MATLAB script file for this problem, the following preliminary task involving the determination of the algebraic expressions for the slope and deflection needs to be performed.

Writing the force and moment equilibrium equations for the free body diagrams of the two sections of the beam shown at the bottom of Figure 11.7, the following expressions for the shear force V and bending moment M can be established for each of the beam segments AB and BC.

$0 \le x_1 \le L$

$$V_1 = \frac{-wa^2}{2L} \quad M_1 = \frac{-wa^2}{2L} x_1$$

$$0 \le x_2 \le a$$

$$V_1 = w(a - x_2) \quad M_2 = \frac{-w(a - x_2)^2}{2}$$

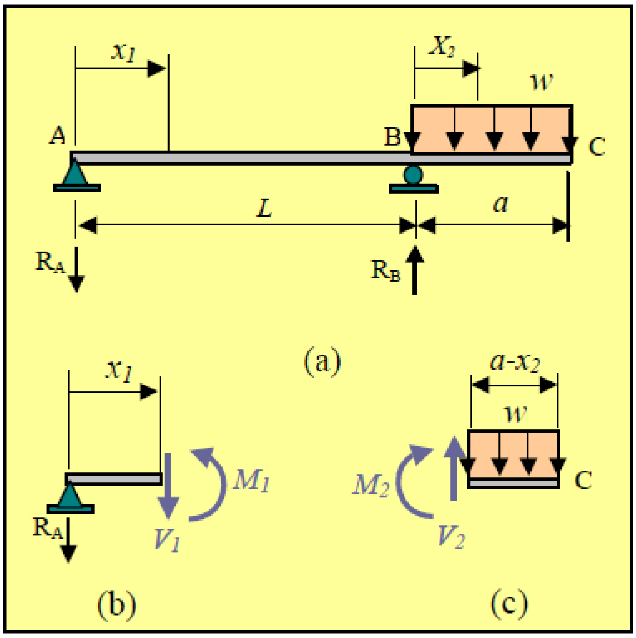


Figure 11.7 Free body diagram of the beam

Upon substituting for the moments M_1 and M_2 in the differential equations of the two regions of the beam shown below:

$$0 \le x_1 \le L$$

$$EIv_1'' = M_1$$

$$0 \le x_2 \le a$$

$$EIv_2'' = M_2$$

the following two differential equations are obtained for the beam.

$$0 \le x_1 \le L$$

$$EIv_1'' = \frac{-wa^2}{2L} x_1$$

$$0 \le x_2 \le a$$

$$EIv_2'' = \frac{-w(a - x_2)^2}{2}$$

In these expressions E, and I are respectively the modulus of elasticity and the moment of inertia of the beam. All other parameters are as defined in Figure 11.7. Upon utilizing the method of successive integration and applying the boundary conditions:

$v_1(x_1 = 0) = 0 \quad v_2(x_2 = 0) = 0$

stating that the deflection of the beam at the supports A and B are zero, and the continuity equations:

$v_1'(x_1 = L) = v_2'(x_2 = 0) \qquad v_1(x_1 = L) = v_2(x_2 = 0)$

stating that there should only be a single value for the slope and a single value for the deflection at point B, the following expressions are obtained for the slope v' and deflections v for the two beam segments AB and BC.

$$0 \le x_1 \le L$$

$$v_1' = \frac{wa^2}{12EIL}\left(L^2 - 3x_1^2\right)$$

$$v_1 = \frac{wa^2 x_1}{12EIL}\left(L^2 - x_1^2\right)$$

$$0 \le x_2 \le a$$

$$v_2' = \frac{-w}{6EI}\left(a^2L + 3a^2x_2 - 3ax_2^2 + x_2^3\right)$$

$$v_2 = \frac{-wx_2}{24EI}\left(4a^2L + 6a^2x_2 - 4ax_2^2 + a_2^3\right)$$

Now that the theoretical formulation of the problem is complete, a MATLAB script file can be created. The script file can be developed in a form which prompts the user to input the values for the parameters w, E, I, L, and a. Then, a MATLAB loop can be employed to compute the values for the slope and deflection along the length of the beam for a series of values of x, measured from the left support at A, starting from x = 0 and ending at x = L. Note that it is necessary to include a conditional statement within the loop, so that the proper expressions for the determination of unknowns is selected and used in the computations. The MATLAB script file for the given beam is provided in Figure 11.8, along with the generated plots in Figure 11.9. These plots are for the case when w = 0.2 kip/ft, E = 29000 ksi, I = 100 in4, L = 20 ft, and a = 10 ft. Using the powerful MATLAB plotting commands and tools, the users can control and create the plots in any format they desire.

MATLAB SCRIPT FOR SLOPE AND DEFLECTION

```
w=0.2;
E=29000;
I=100;
L=20;
a=10;
w=w/12;L=L*12;a=a*12;
fprintf(' x(in.) Slope(rad) Deflection(in)\n')
%
```

```
% Computing the slope, and deflection.
x=linspace(0,L+a,(L+a)/6+1);
for k=1:1:(L+a)/6+1
if x(k)<=L
x1(k)=x(k);
theta(k)=w*a^2*(L^2-3*x1(k)^2)/(12*E*I*L);
delta(k)=w*a^2*x1(k)*(L^2-x1(k)^2)/(12*E*I*L);
fprintf('%4.0f %19.2e %17.2e\n',x(k),theta(k),delta(k))
else
x2(k)=x(k)-L;
theta(k)=-w*(a^2*L+3*a^2*x2(k)-3*a*x2(k)^2+x2(k)^3)/(6*E*I);
delta(k)=-w*x2(k)*(4*a^2*L+6*a^2*x2(k)-4*a*x2(k)^2+x2(k)^3)/(24*E*I);
fprintf('%4.0f %19.2e %17.2e\n',x(k),theta(k),delta(k))
end
end
% plotting the slope, and deflection.
subplot(2,1,1),plot(x,theta),title('Slope'),xlabel('x (in)'),ylabel('Slope
(rad)'),...
axis([0 360 -0.006 .004]),set(gca,'XTick',[0:60:L+a]),text(10,-0.003,...
'\theta_1 =wa^2(L^2-3x_1^2)/(12EIL)'),text(40,0.002,'\theta_2 =-
w(a^2L+3a^2x_2-3ax_2^2+x_2^3)/(6EI)')
subplot(2,1,2),plot(x,delta),title('Deflection'),xlabel('x
(in)'),ylabel('Deflection (in)'),...
axis([0 360 -0.6 0.4]),set(gca,'XTick',[0:60:L+a]),,text(10,-0.1,'v_1
=wa^2x_1(L^2-x_1^2)/(12EIL)'),...
text(5,0.25,'v_2 =-wx_2(4a^2L+6a^2x_2-4ax_2^2+x_2^3)/(24EI)')
```

Figure 11.8 Matlab Script

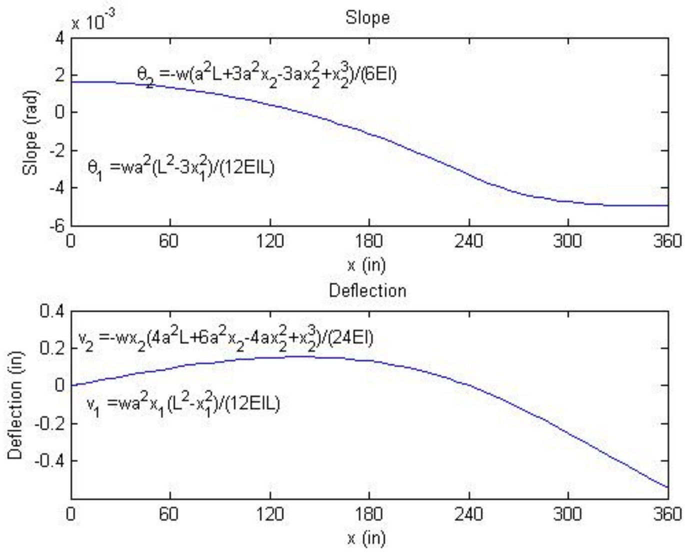


Figure 11.9 Variation of slope and deflection along the length of the beam

Sample Problems

Problem 1: A cantilever beam carries a load of 15 KN. Its span is 2m. The cross section is a circle of diameter 200 mm. The Young's modulus of elasticity is 200 KN/m^2 . Find the deflection and slope at the free end.

Lecture 12 Deflection of statically determinate beams: **Moment Area method,**

12.1 Introduction

The initial ideas for the two moment-area theorems were developed by Otto Mohr and later stated formally by Charles E. Greene in 1873.These theorems provide a semigraphical technique for determining the slope of the elastic curve and its deflection due to bending. They are particularly advantageous when used to solve problems involving beams, especially those subjected to a series of concentrated loadings or having segments with different moments of inertia.

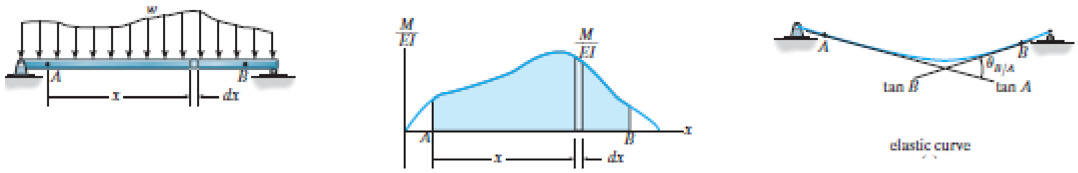


Figure 12.1 a, b and c

To develop the theorems, reference is made to the beam in Fig. 12–1*a*. If we draw the moment diagram for the beam and then divide it by the flexural rigidity, *EI*, the "*M/EI* diagram" shown in Fig. 12–1*b* results.

$$d\theta = \left(\frac{M}{EI}\right)dx \qquad (12.1)$$

Thus it can be seen that the change $d\theta$ in the slope of the tangents on either side of the element *dx* is equal to the lighter-shaded *area* under the *M/EI* diagram. Integrating from point *A* on the elastic curve to point *B*, Fig. 12–1*c*, we have

$$\theta_{B/A} = \int_A^B \frac{M}{EI}dx \qquad (12.2)$$

Theorem 1: The change in slope between any two points on the elastic curve equals the area of the M/EI diagram between these two points.

The notation $\theta_{B/A}$ is referred to as the angle of the tangent at *B* measured with respect to the tangent at *A*. From the proof it should be evident that this angle is measured *counterclockwise* from tangent *A* to tangent *B* if the area of the *M/EI* diagram is *positive,* Fig. 12–1*c*. Conversely, if this area is *negative,* or below the *x* axis, the angle $\theta_{B/A}$ is measured *clockwise* from tangent *A* to tangent *B*. Furthermore, from the dimensions of Eq. 12.2, $\theta_{B/A}$ is measured in radians.

The second moment-area theorem is based on the relative deviation of *tangents* to the elastic curve. Shown in Fig. 12–2*c* is a greatly exaggerated view of the *vertical deviation dt* of the tangents on each side of the differential element *dx*. This deviation is measured along a vertical line passing through point *A*. Since the slope of the elastic curve and its deflection are assumed to be very small, it is satisfactory to approximate the length of each tangent line by *x* and the arc ds' by *dt*. Using the circular-arc formula $s = \theta r$ where *r* is of length *x*, we can write $dt = x d\theta$ $d\theta = \left(M / EI \right) dx$ the vertical deviation of the tangent at *A with respect to the tangent* at *B* can be found by integration, in which case

$$t_{A/B} = \int_A^B x \frac{M}{EI} dx \tag{12.3}$$

Recall from statics that the centroid of an area is determined from $\bar{x} \int dA = \int x dA$ Since $\int M / EI \, dx$ represents an area of the *M/EI* diagram, we can also write

$$t_{A/B} = \bar{x} \int_A^B \frac{M}{EI} dx \tag{12.4}$$

Here $\bar{x}$ is the distance from the vertical axis through *A* to the *centroid* of the area between *A* and *B*, Fig. 12–2*b*.

The second moment-area theorem can now be stated as follows:

Theorem 2: The vertical deviation of the tangent at a point (*A*) on the elastic curve with respect to the tangent extended from another point (*B*) equals the "moment" of the area under the *M/EI* diagram between the two points (*A* and *B*). This moment is computed about point *A* (the point on the elastic curve), where the deviation $t_{A/B}$ is to be determined.

Provided the moment of a *positive M/EI* area from *A* to *B* is computed, as in Fig. 12.2 *b*, it indicates that the tangent at point *A* is *above* the tangent to the curve extended from point *B*, Fig. 12.2 *c*. Similarly, *negative M/EI* areas indicate that the tangent at *A* is *below* the tangent extended from *B*. Note that in general $t_{A/B}$ is not equal to $t_{B/A}$ which is shown in Fig. 12.2 *d*. Specifically, the moment of the area under the *M/EI* diagram between *A* and *B* is computed about point *A* to determine $t_{A/B}$ Fig. 12.2*b*, and it is computed about point *B* to determine $t_{B/A}$.

It is important to realize that the moment-area theorems can only be used to determine the angles or deviations between two tangents on the beam's elastic curve. In general, they *do not* give a direct solution for the slope or displacement at a point on the beam. These unknowns must first be related to the angles or vertical deviations of tangents at points on the elastic curve. Usually the tangents at the supports are drawn in this regard since these points do not undergo displacement and/or have zero slope. Specific cases for establishing these geometric relationships are given in the example problems.

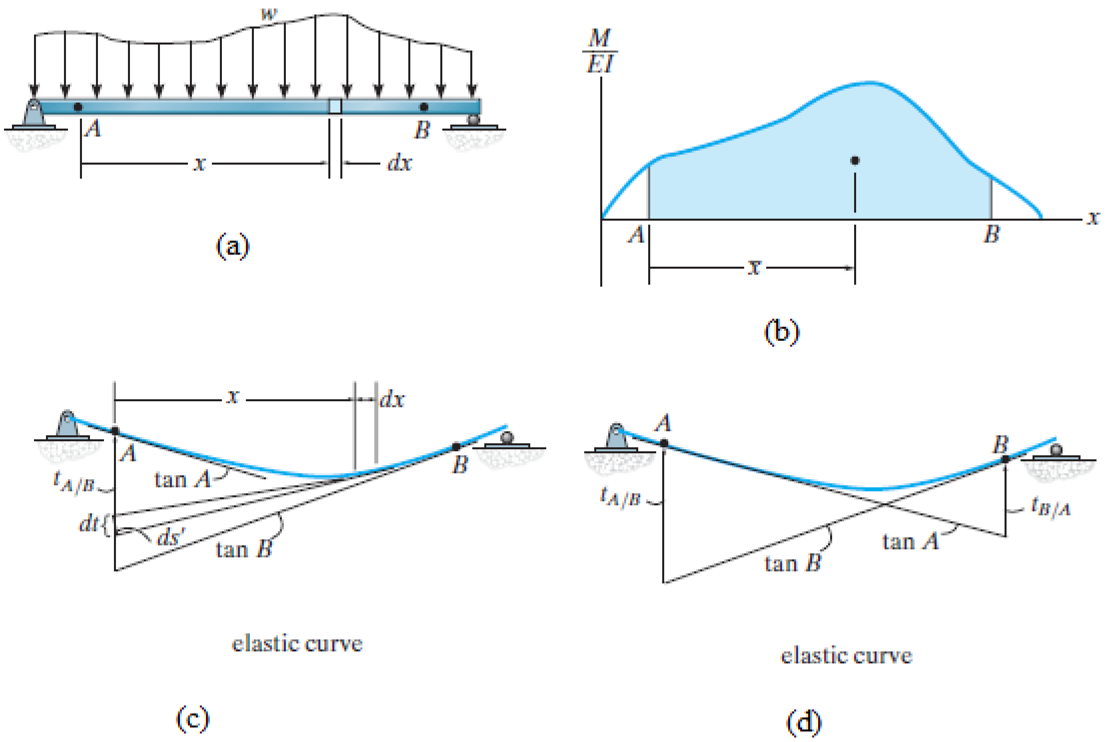


Figure 12.2 a,b,c and d

Problem 1: Apply the moment area method to find the slope and deflection at the free end of a cantilever beam subject to a concentrated load P applied at the free end. Assume the flexural rigidity EI to be constant.

Solution: Applying the moment area theorem I and noting that the tangent at the fixed end A is horizontal i.e $\theta_A = 0$

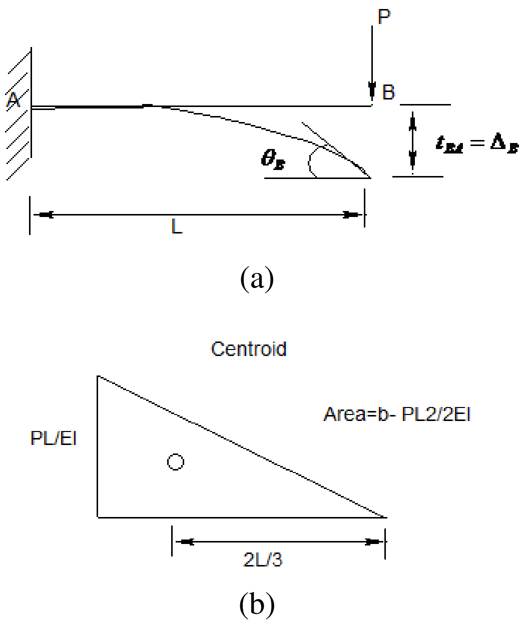


Figure 12.3 Cantilever beam with concentrated load

$\theta_B - \theta_A$ is the area of curvature diagram between A and B

$$\therefore \quad \theta_B - 0 = \frac{1}{2}\left(\frac{P}{EI}\right)(L)$$

$$\therefore \quad \theta_B = -\left(\frac{PL^2}{2EI}\right)$$

The negative sign indicates a clockwise rotation
Next, applying theorem II, and noting that $t_{BA} = \Delta_B$

$$t_{BA} = \Delta_B = \bar{x}_B \times Area = \left(\frac{2L}{3}\right)\left(\frac{-PL^2}{2EI}\right) = \frac{-PL^3}{3EI}$$

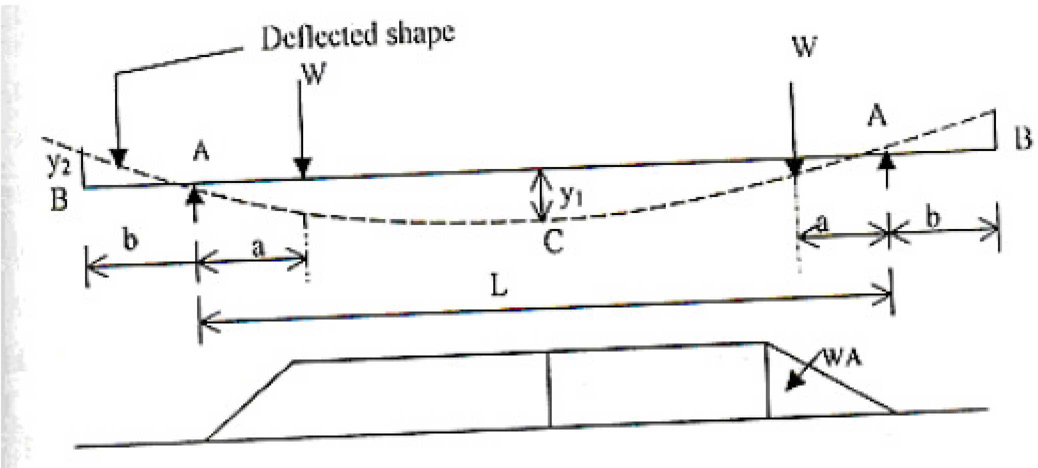
The negative sign indicates that B lies below the tangent at A i.e the deflection is downward.

Laboratory Experiment on Moment Area Method

Aim: - To verify the moment area theorem regarding the slopes and deflections of the beam.

Apparatus: - Moment of area theorem apparatus.

Diagram;



Theory : -
According to moment area theorem

1. The change of slope of the tangents of the elastic curve between any two points of the deflected beam is equal to the area of M/EI diagram between these two points.

2. The deflection of any point relative to tangent at any other point is equal to the moment of the area of the M/EI diagram between the two point at which the deflection is required.

Slope at B = Y_2 / b

Since the tangent at C is horizontal due to symmetry,

Slope at B = shaded area/EI= 1/EI [$Wa^2/2$+WA(L/2-a)]

Displacement at B with respect to tangent at C

=(y_1 + y_2) = Moment of shaded area about B/EI

=1/EI[$Wa^2/2$(b+2a/3)+Wa(L/2-a)(b+a/2+L/2)]

Procedure

1.. Measure a, b and L of the beam

2.. Place the hangers at equal distance from the supports A and load them with equal loads

3. . Measure the deflection by dial gauges at the end B (y_2) and at the center C (y_1)

4.. Repeat the above steps for different loads

Observation Table:-

Length of main span, L (cm) =

Length of overhang on each side a(cm) =

Modulus of elasticity , E (kg/cm^2) = 2 x 10^6

Sl No	Load at each Hanger (kg)	Central deflection Y_1 (cm)	Deflection at free end Y_2 (cm)	Slope at B Y_2/b	Deflection at C = Deflection at B (Y_1)

Calculation:-
1. Calculate the slope at B as y2 / b (measured value).
2. Compute slope and deflection at B theoretically from B.M.D. and compare with experimental values.
3. Deflection at C = y1(measured value).
4. Deflection at C = Average calculated value

Result :- The slope and deflection obtained is close to the slope and deflection obtained by using moment area method.

Sample problems:

Problem 2: A simply supported beam of span 6 m is carrying a load of 12 KN/m. If E = 200, 000 N/mm^2 and I is 5.5 x 10^{-4} m^4. Find the maximum slope at the support and maximum deflection at mid span

Lecture 13 Deflection of statically determinate beams: **Conjugate Beam method**,

The conjugate-beam method was developed by H. Müller-Breslau in 1865. Essentially, it requires the same amount of computation as the moment-area theorems to determine a beam's slope or deflection; however, this method relies only on the principles of statics, and hence its application will be more familiar.

Let us write down the following equations which have been discussed before.

$$\frac{dV}{dx} = w \qquad \frac{d^2M}{dx^2} = w$$
$$\frac{d\theta}{dx} = \frac{M}{EI} \qquad \frac{d^2v}{dx^2} = \frac{M}{EI}$$

On integrating

$$V = \int w dx \qquad M = \int \left[\int w dx\right] dx$$
$$\theta = \int \left(\frac{M}{EI}\right) dx \quad v = \int \left[\int \left(\frac{M}{EI}\right) dx\right] dx$$

Here the shear V compares with the slope θ, the moment M compares with the displacement v and the external load w compares with the M/EI diagram. To make use of this comparison we will now consider a beam having the same length as the real beam, but referred to here as the "conjugate beam," Fig. 13–1. The conjugate beam is "loaded" with the M/EI diagram derived from the load w on the real beam. From the above comparisons, we can state two theorems related to the conjugate beam, namely,

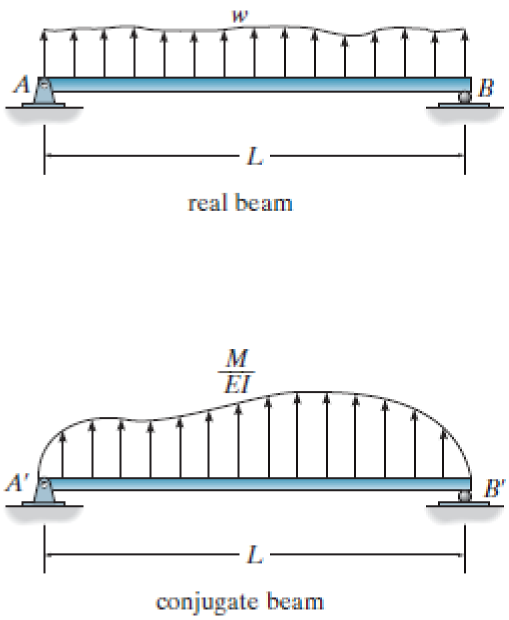


Figure 13.1 Real and Conjugate Beam

Theorem 1: The slope at a point in the real beam is numerically equal to the shear at the corresponding point in the conjugate beam.

Theorem 2: The displacement of a point in the real beam is numerically equal to the moment at the corresponding point in the conjugate beam.

Conjugate Beam Supports

When drawing the conjugate beam it is important that the shear and moment developed at the supports of the conjugate beam account for the corresponding slope and displacement of the real beam at its supports, a consequence of Theorems 1 and 2. For example, as shown in Table 13.1, a pin or roller support at the end of the real beam provides zero displacement, but the beam has a nonzero slope. Consequently, from Theorems 1 and 2, the conjugate beam must be supported by a pin or roller, since this support has zero moment but has a shear or end reaction. When the real beam is fixed supported (3), both the slope and displacement at the support are zero. Here the conjugate beam has a free end, since at this end there is zero shear and zero moment. Corresponding real and conjugate-beam supports for other cases are listed in the table. Examples of real and conjugate beams are shown in Fig. 13.2. Note that, as a rule, neglecting axial force, statically determinate real beams have statically determinate conjugate beams; and statically indeterminate real beams, as in the last case in Fig. 13.2, become unstable conjugate beams. Although this occurs, the M/EI loading will provide the necessary "equilibrium" to hold the conjugate beam stable.

Table 13.1

	Real Beam		Conjugate Beam	
1)	θ, $\Delta = 0$	pin	V, $M = 0$	pin
2)	θ, $\Delta = 0$	roller	V, $M = 0$	roller
3)	$\theta = 0$, $\Delta = 0$	fixed	$V = 0$, $M = 0$	free
4)	θ, Δ	free	V, M	fixed
5)	θ, $\Delta = 0$	internal pin	V, $M = 0$	hinge
6)	θ, $\Delta = 0$	internal roller	V, $M = 0$	hinge
7)	θ, Δ	hinge	V, M	internal roller

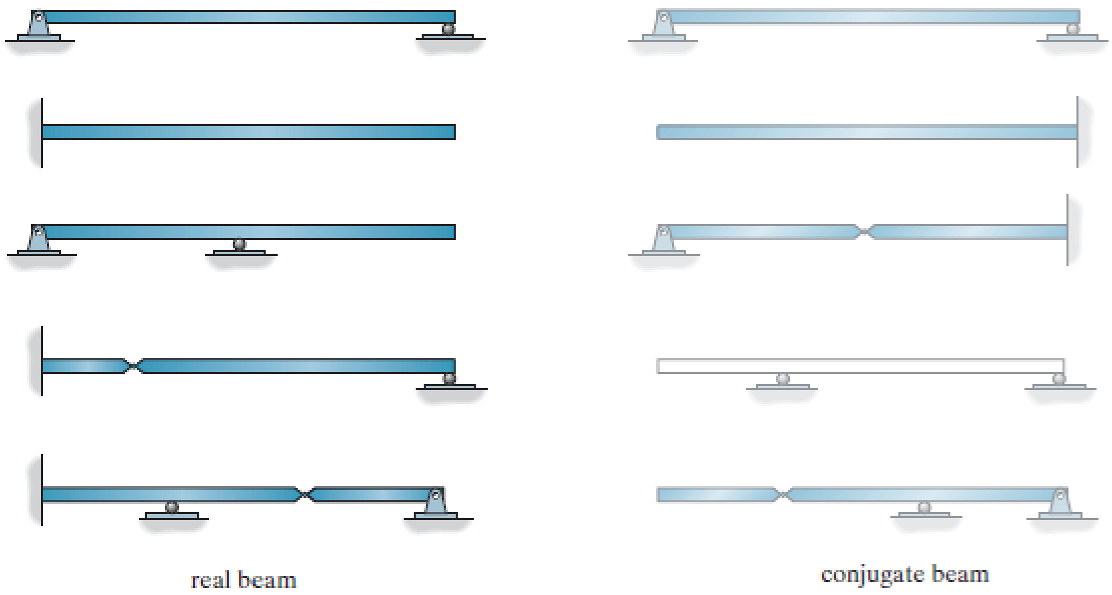


Figure 13.2

Example 1: Determine the maximum deflection of the steel beam shown in Fig. 13.3 a. The reactions have been computed. E=200 GPa, I=60 x 10^6 mm^4

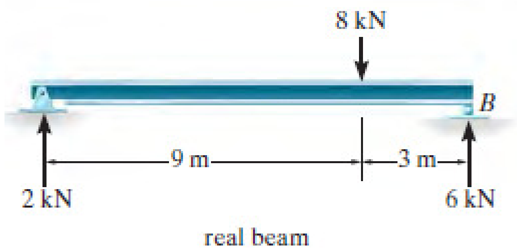


Figure 13.3 a Real Beam

Solution:

The conjugate beam loaded with the M/EI diagram is shown in Fig. 13.3 b. Since the M/EI diagram is positive, the distributed load acts upward (away from the beam).

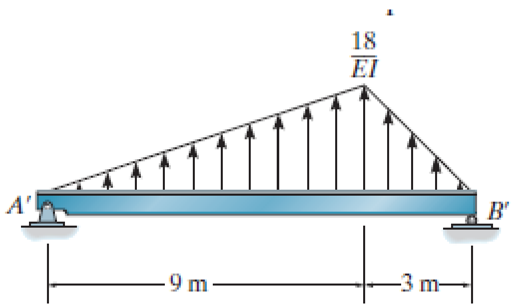


Figure 13.3 b Conjugate beam

The external reactions on the conjugate beam are determined first and are indicated on the free-body diagram in Fig. 13.3 c. Maximum deflection of the real beam occurs at the point where the slope of the beam is zero. This corresponds to the same point in the conjugate beam where the shear is zero. Assuming this point acts within the region $0 \le x \le 9\,\text{m}$ from A' we can isolate the section shown in Fig. 13.3 d. Note that the peak of the distributed loading was determined from proportional triangles, that is, $w/x = (18/EI)/9$. We require V'=0. So that $+\uparrow \sum F_y = 0;\quad -\frac{45}{EI} + \frac{1}{2}\left(\frac{2x}{EI}\right)x = 0$ x=6.71 m $0 \le x \le 9\,\text{m}$ OK

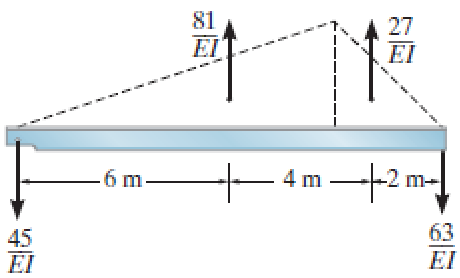


Figure 13.3 c external reaction

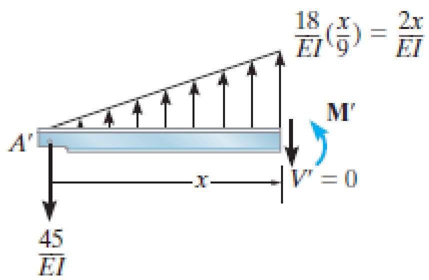


Figure 13.3 d internal reactions

Using this value for x, the maximum deflection in the real beam corresponds to the moment M'. Hence

$$+\uparrow \sum M = 0;\quad \frac{45}{EI}(6.71) - \left[\frac{1}{2}\left(\frac{2(6.71)}{EI}\right)6.71\right]\frac{1}{3}(6.71) + M' = 0$$

$$\Delta_{\max} = M' = -\frac{201.2KNm^3}{EI} = \frac{-201.2KNm^3}{[200(10^6)KN/m^2][60(10^6)mm^4(1m^4/(10^3)^4 mm^4)]}$$ =-0.0168m=-16.8 mm

The negative sign indicates the deflection is downward.

Sample Problems

Problem 1: Find the deflection at the free end of a cantilever of span L carrying a concentrated load of W at a section L/2 from the free end

Lecture 14 Deflection of statically determinate beams by energy methods- **strain energy method**, castiglianos theorems, reciprocal theorem, unit load method. Deflection of pin-jointed trusses, Williot-Mohr diagram

14.1 Introduction

The semigraphical methods presented in the previous lectures are very effective for finding the displacements and slopes at points in beams subjected to rather simple loadings. For more complicated loadings or for structures such as trusses and frames, it is suggested that energy methods be used for the computations. Most energy methods are based on the conservation of energy principle, which states that the work done by all the external forces acting on a structure, U_e is transformed into internal work or strain energy, U_i which is developed when the structure deforms. If the material's elastic limit is not exceeded, the elastic strain energy will return the structure to its undeformed state when the loads are removed. The conservation of energy principle can be stated mathematically as

$$U_e = U_i$$

14.2 External Work

The work of a moment is defined by the product of the magnitude of the moment M and the angle $d\theta$ through which it rotates, that is, $U_e = Md\theta$ as shown in Figure 14.1.

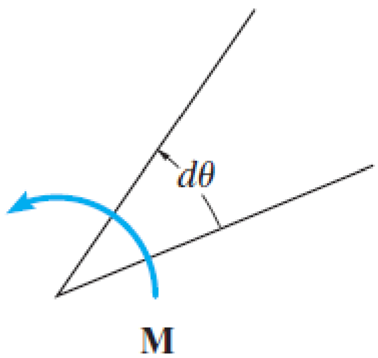


Figure 14.1

If the total angle of rotation is θ radians, the work becomes

$$U_e = \int_0^{d\theta} Md\theta$$

As in the case of force, if the moment is applied gradually to a structure having linear elastic response from zero to M, the work is then

$$U_e = \frac{1}{2}M\theta$$

However, if the moment is already applied to the structure and other loadings further distort the structure by an amount θ' and M rotates θ' and the work is

$$U_e^{'} = M\theta^{'}$$

14.3 Strain energy, Bending

Consider the beam shown in Fig. 14.2 a, which is distorted by the gradually applied loading P and w.These loads create an internal moment M in the beam at a section located a distance x from the left support. The resulting rotation of the differential element dx, Fig. 14.2 b, can be found from the relationship $d\theta = (M / EI)dx$, Consequently, the strain energy, or work stored in the element, is determined from the earlier relationship since the internal moment gradually developed. Hence,

$$dU_i = \frac{M^2 dx}{2EI}$$

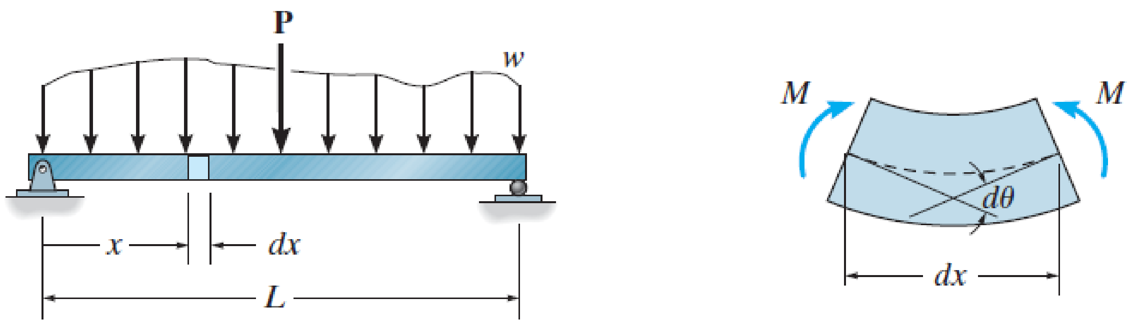


Figure 14.2

The strain energy for the beam is determined by integrating this result over the beam's entire length L. The result is

$$U_i = \int_0^L \frac{M^2 dx}{2EI}$$

14.4 Principle of Work and Energy

Now that the work and strain energy for a force and a moment have been formulated, we will illustrate how the conservation of energy or the principle of work and energy can be applied to determine the displacement at a point on a structure. To do this, consider finding the displacement at the point where the force **P** is applied to the cantilever beam in Figure 14.3

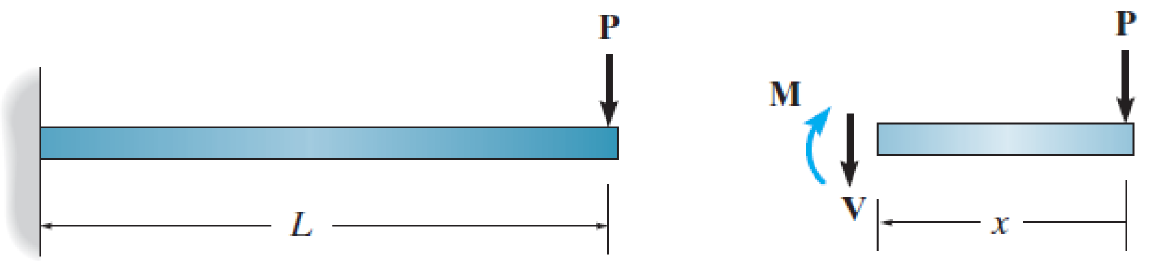


Figure 14.3

The external work done by the force as shown in Figure 14.4 is $U_e = \frac{1}{2} P\Delta$

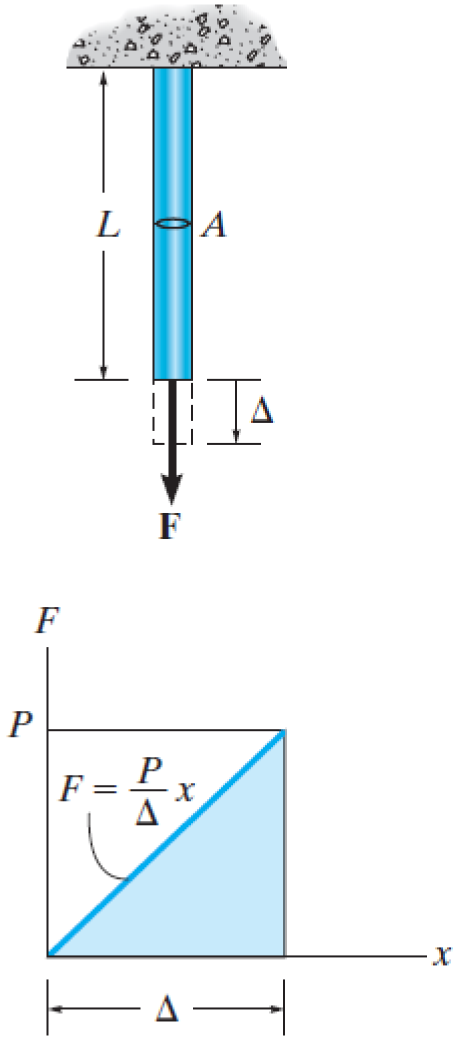


Figure 14.4

To obtain the resulting strain energy, we must first determine the internal moment as a function of position x in the beam $M = -Px$ so that

$$U_i = \int_0^L \frac{M^2 dx}{2EI} = \int_o^L \frac{(-Px)^2 dx}{2EI} = \frac{P^2 L^3}{6EI}$$

Equating the external work to internal strain energy and solving for the unknown displacement Δ, we have

$$U_i = U_i$$

$$\frac{1}{2} P\Delta = \frac{P^2 L^3}{6EI}$$

$$\Delta = \frac{PL^3}{3EI}$$

Although the solution here is quite direct, application of this method is limited to only a few select problems. It will be noted that only one load may be applied to the structure, since if more than one load were applied, there would be an unknown displacement under each load, and yet it is possible to write only one “work” equation for the beam. Furthermore, only the displacement under the force can be obtained, since the external work depends upon both the force and its corresponding displacement. One way to circumvent these limitations is to use the method of virtual work or Castigliano's theorem

Lecture 15 Deflection of statically determinate beams by energy methods- strain energy method, **castiglianos theorems**, reciprocal theorem, unit load method. Deflection of pin-jointed trusses, Williot-Mohr diagram

15.1 Castigliano's Theorem

In 1879 Alberto Castigliano, an Italian railroad engineer, published a book in which he outlined a method for determining the deflection or slope at a point in a structure, be it a truss, beam, or frame. This method, which is referred to as Castigliano's second theorem, or the method of least work, applies only to structures that have constant temperature, unyielding supports, and linear elastic material response. If the displacement of a point is to be determined, the theorem states that it is equal to the first partial derivative of the strain energy in the structure with respect to a force acting at the point and in the direction of displacement. In a similar manner, the slope at a point in a structure is equal to the first partial derivative of the strain energy in the structure with respect to a couple moment acting at the point and in the direction of rotation.

To derive Castigliano's second theorem, consider a body (structure) of any arbitrary shape which is subjected to a series of n forces P_1, P_2, ..., P_n
Since the external work done by these loads is equal to the internal strain energy stored in the body, we can write

$U_i = U_e$

The external work is a function of the external loads $\left(U_e = \sum \int P dx\right)$, thus

$$U_i = U_e = f\left(P_1, P_2, ..., P_n\right)$$

Now, if any one of the forces, say P_i is increased by a differential amount dP_i the internal work is also increased such that the new strain energy becomes

$$U_i + dU_i = U_i + \frac{\partial U_i}{\partial P_i} dP_i$$

This value, however, should not depend on the sequence in which the n forces are applied to the body. For example, if we apply dP_i to the body first, then this will cause the body to be displaced a differential amount $d\Delta_i$ in the direction of dP_i. By Equation $\left(U_e = \frac{1}{2} P\Delta\right)$, the increment of strain energy would be $\frac{1}{2} dP_i d\Delta_i$. This quantity, however, is a second-order differential and may be neglected. Further application of the loads P_1, P_2, ..., P_n which displace the body $\Delta_1, \Delta_2, ..., \Delta_n$ yields the strain energy

$$U_i + dU_i = U_i + dP_i \Delta_i$$

Here, as before, U_i is the internal strain energy in the body, caused by the loads P_1, P_2, ..., P_n and $dU_i = dP_i \Delta_i$ is the additional strain energy caused by dP_i ($U_e = P\Delta'$).

In Summary then the expression $U_i + dU_i = U_i + \frac{\partial U_i}{\partial P_i} dP_i$ represents the strain energy in the body determined by first applying the loads P_1, P_2, ..., P_n then dP_i and the expression $U_i + dU_i = U_i + dP_i \Delta_i$ represents the strain energy determined by first applying dP_i and then the loads P_1, P_2, ..., P_n . Since these two expressions must be equal, we require

$$\Delta_i = \frac{\partial U_i}{\partial P_i}$$

which proves the theorem; i.e., the displacement Δ_i in the direction of P_i is equal to the first partial derivative of the strain energy with respect to P_i.

It should be noted that above expression for displacement is a statement regarding the structure's compatibility. Also, the above derivation requires that only conservative forces be considered for the analysis. These forces do work that is independent of the path and therefore create no energy loss. Since forces causing a linear elastic response are conservative, the theorem is restricted to linear elastic behavior of the material.

Lecture 16 Deflection of statically determinate beams by energy methods- strain energy method, **castiglianos theorems**, reciprocal theorem, unit load method. Deflection of pin-jointed trusses, Williot-Mohr diagram

16.1 Castigliano's Theorem for Beams

The internal bending strain energy for a beam or frame is given by the expression $U_i = \int_0^L \frac{M^2 dx}{2EI}$. Substituting the above expression into $\Delta_i = \frac{\partial U_i}{\partial P_i}$ and omitting the subscript i, we have

$$\Delta = \frac{\partial}{\partial P}\int_0^L \frac{M^2 dx}{2EI}$$

Rather than squaring the expression for internal moment M, integrating, and then taking the partial derivative, it is generally easier to differentiate prior to integration. Provided E and I are constant, we have

$$\Delta = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI}$$

where

Δ = external displacement of the point caused by the real loads acting on the beam or frame.

P= external force applied to the beam or frame in the direction of Δ

M= internal moment in the beam or frame, expressed as a function of x and caused by both the force P and the real loads on the beam.

E= modulus of elasticity of beam material.

I= moment of inertia of cross-sectional area computed about the neutral axis.

If the slope θ at a point is to be determined, we must find the partial derivative of the internal moment M with respect to an external couple moment M' acting at the point, i.e.,

$$\theta = \int_0^L M\left(\frac{\partial M}{\partial M'}\right)\frac{dx}{EI}$$

Example 1: Determine the displacement of point B of the beam as shown in Figure 16.1 below, Take E=200 GPa and I=500 (10^6) mm^4.

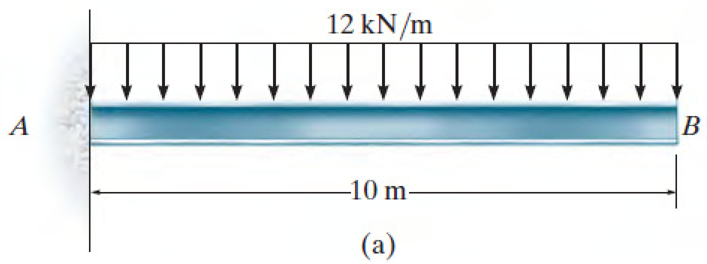
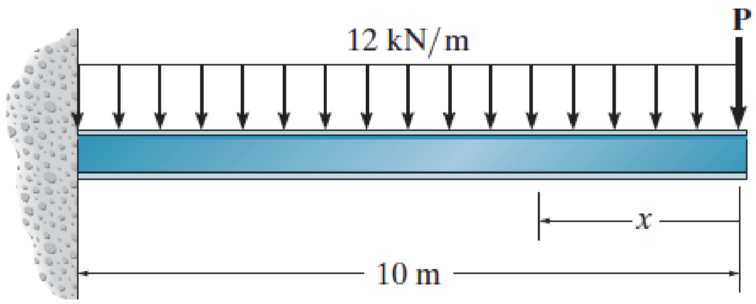


Figure 16.1 a

Solution:

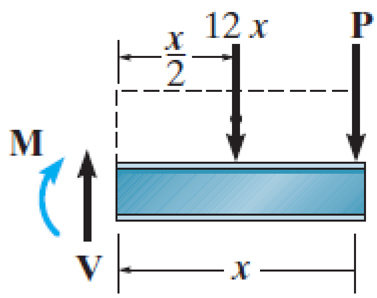


(b)

Figure 16.1 b

A vertical force P is placed on the beam at B as shown in Figure 16.1 b

A single x coordinate is needed for the solution, since there are no discontinuities of loading between A and B. Using the method of sections, Fig. 16.1 c, we have



(c)

Figure 16.1 c

$$\circlearrowleft + \sum M = 0 \quad -M - (12x)\left(\frac{x}{2}\right) - Px = 0$$

$$M = -6x^2 - Px \qquad \frac{\partial M}{\partial P} = -x$$

Setting P = 0, its actual value, yields

$$M = -6x^2 \qquad \frac{\partial M}{\partial P} = -x$$

The deflection at B is given by

$$\Delta_B = \int_0^L M\left(\frac{\partial M}{\partial P}\right)\frac{dx}{EI} = \int_0^{10}\frac{(-6x^2)(-x)dx}{EI} = \frac{15(10^3)KNm^3}{EI}$$

Or

$$\Delta_B = \frac{15(10^3)KNm^3}{200(10^6)KN/m^2\left[500(10^6)mm^4\right](10^{-12}m^4/mm^4)} = 0.150m = 150mm$$

Lecture 17 Deflection of statically determinate beams by energy methods- strain energy method, castiglianos theorems, **reciprocal theorem**, unit load method. Deflection of pin-jointed trusses, Williot-Mohr diagram

17.1 Reciprocal Theorem

Maxwell's law of Reciprocal Deflection

As applied to beam deflections and rotations, Maxwell's theorem of reciprocal deflections has the following three versions:

(1) The deflection at A due to unit force at B is equal to the deflection at B due to unit force at A as shown in Figure 17.1 a
$\delta_{AB} = \delta_{BA}$

(2) The slope at A due to unit couple at B is equal to the slope at B due to unit couple at A as shown in Figure 17.1 b
$\phi_{AB} = \phi_{BA}$

(3) The slope at A due to unit load at B is equal to the deflection at B due to unit couple at A as shown in Figure 17.1 c
$\phi_{AB'} = \delta_{BA'}$

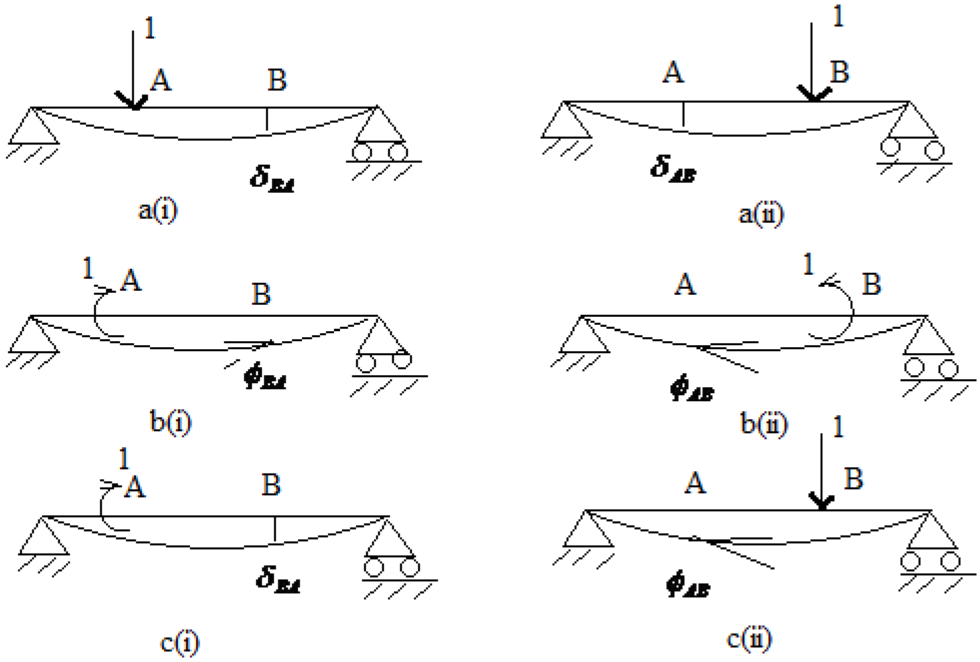


Figure 17.1

17.2 Generalised Maxwell's Theorem: Betti's Reciprocal Theorem

Generalised Statement: If an elastic system is in equilibrium under one set of forces with their corresponding displacements and if the same system is also in equilibrium under second set of forces acting through the same points with their corresponding displacements then the product of the first group of forces and the corresponding displacements caused by second group is equal to the product of the second group of forces and the corresponding displacements caused by the first group

$$P_A \Delta_{A'} + P_B \Delta_{B'} = P'_A \Delta_A + P'_B \Delta_B$$

Where P and Δ constitute first group of forces and their corresponding displacements and P' and Δ' constitute second group of forces and displacements.

That is the virtual work done by the first set of forces acting through the second set of displacements is equal to the virtual work done by the second set of forces acting through the first set of displacements.

In Betti's theorem, the symbols P and Δ can also denote couples and rotations respectively as well as forces and linear deflections i.e

$$M_A \theta_{A'} + M_B \theta_{B'} = M'_A \theta_A + M'_B \theta_B$$

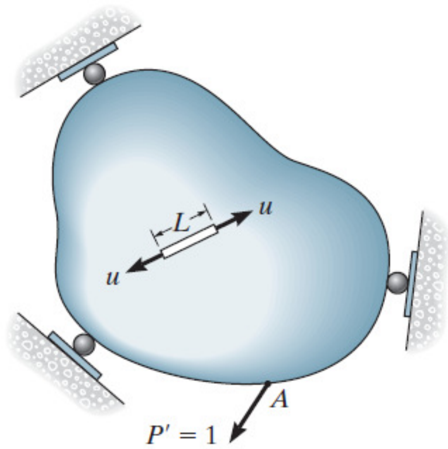
Thus according to Betti's law, we have in general

$$\sum P\Delta' = \sum M\theta' = \sum P'\Delta + \sum M'\theta$$

Lecture 18 Deflection of statically determinate beams by energy methods- strain energy method, castiglianos theorems, reciprocal theorem, **unit load method**. Deflection of pin-jointed trusses, Williot-Mohr diagram

18.1 Principle of Virtual Work

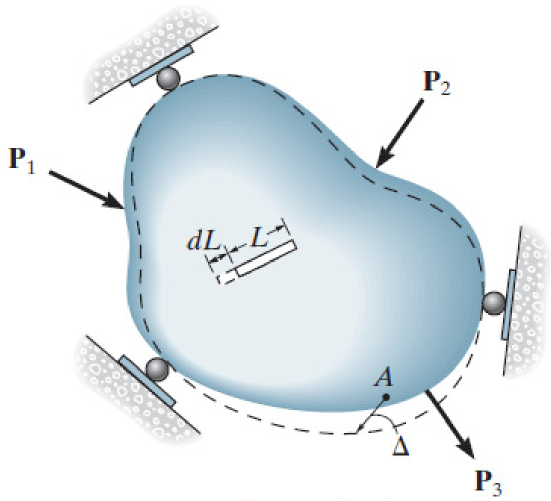
The principle of virtual work was developed by John Bernoulli in 1717 and is sometimes referred to as the unit-load method. It provides a general means of obtaining the displacement and slope at a specific point on a structure, be it a beam, frame, or truss.



Apply virtual load $P' = 1$

(a)

Figure 18.1 a



Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$

(b)

Figure 18.1 (b)

Before developing the principle of virtual work, it is necessary to make some general statements regarding the principle of work and energy, which was discussed in the previous section. If we take a deformable structure of any shape or size and apply a series of external loads P to it, it will cause internal loads u at points throughout the structure. It is necessary that the external and internal loads be related by the equations of equilibrium. As a consequence of these loadings, external displacements Δ will occur at the P loads and internal displacements will occur at each point of internal load u. In general, these displacements do not have to be elastic, and they may not be related to the loads; however, the external and internal displacements δ must be related by the compatibility of the displacements. In other words, if the external displacements are known, the corresponding internal displacements are uniquely defined.

In general, then, the principle of work and energy states: $\sum P\Delta = \sum u\delta$ i.e Work of external loads = work of internal loads

Based on this concept, the principle of virtual work will now be developed. To do this, we will consider the structure (or body) to be of arbitrary shape as shown in Fig. 18.1b. Suppose it is necessary to determine the displacement Δ of point A on the body caused by the "real loads" P_1, P_2 and P_3. It is to be understood that these loads cause no movement of the supports; in general, however, they can strain the material beyond the elastic limit. Since no external load acts on the body at A and in the direction of Δ the displacement Δ can be determined by first placing on the body a "virtual" load such that this force P' acts in the same direction as Δ Fig. 18.1 a. For convenience, which will be apparent later, we will choose P' to have a "unit" magnitude, that is, $P'=1$ The term "virtual" is used to describe the load, since it is imaginary and does not actually exist as part of the real loading. The unit load P' does, however, create an internal virtual load u in a representative element or fiber of the body, as shown in Fig. 9–6a. Here it is required that P' and u be related by the equations of equilibrium Once the virtual loadings are applied, then the body is subjected to the real loads P_1 , P_2 and P_3 . Figure 18.1 b. Point A will be displaced an amount Δ causing the element to deform an amount dL. As a result, the external virtual force P' and internal virtual load u "ride along" by Δ and dL. respectively, and therefore perform external virtual work of 1. Δ on the body and internal virtual work of u.dL on the element. Realizing that the external virtual work is equal to the internal virtual work done on all the elements of the body, we can write the virtual-work equation as

virtual loadings (1, u)

$$1 \cdot \Delta = \Sigma u \cdot dL$$

real displacements (Δ, dL)

Where

P' = 1 = external virtual unit load acting in the direction of Δ

u= internal virtual load acting on the element in the direction of dL.

Δ = external displacement caused by the real loads.

dL = internal deformation of the element caused by the real loads.

By choosing P' = 1 it can be seen that the solution for Δ follows directly, Since $\Delta = \sum udL$

In a similar manner, if the rotational displacement or slope of the tangent at a point on a structure is to be determined, a virtual couple moment M' having a "unit" magnitude is applied at the point. As a consequence, this couple moment causes a virtual load u_θ in one of the elements of the body. Assuming that the real loads deform the element an amount dL, the Rotation θ can be found from the virtual-work equation

virtual loadings

$$1 \cdot \theta = \Sigma u_\theta \cdot dL$$

real displacements

where

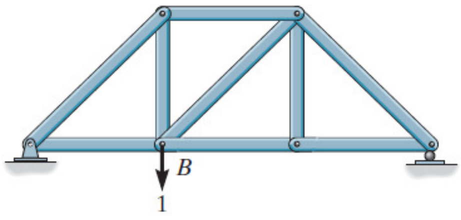
M' = 1= external virtual unit couple moment acting in the direction of θ
u_θ = internal virtual load acting on an element in the direction of dL.
θ = external rotational displacement or slope in radians caused by the real loads.
dL = internal deformation of the element caused by the real loads.

This method for applying the principle of virtual work is often referred to as the method of virtual forces, since a virtual force is applied resulting in the calculation of a real displacement. The equation of virtual work in this case represents a compatibility requirement for the structure. Although not important here, realize that we can also apply the principle of virtual work as a method of virtual displacements. In this case virtual displacements are imposed on the structure while the structure is subjected to real loadings. This method can be used to determine a force on or in a structure, so that the equation of virtual work is then expressed as an equilibrium requirement.

Lecture 19 Deflection of statically determinate beams by energy methods- strain energy method, castiglianos theorems, reciprocal theorem, unit load method. **Deflection of pin-jointed trusses**, Williot-Mohr diagram

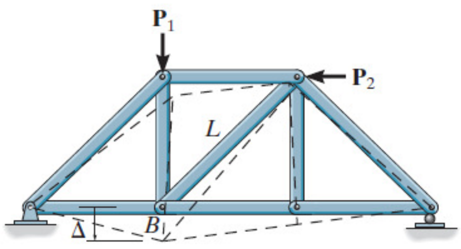
19.1 Method of Virtual Work: Trusses

We can use the method of virtual work to determine the displacement of a truss joint when the truss is subjected to an external loading, temperature change, or fabrication errors. Each of these situations will now be discussed.



Apply virtual unit load to B

(a)



Apply real loads $\mathbf{P}_1$, $\mathbf{P}_2$

(b)

Figure 19.1

External Loading. For the purpose of explanation let us consider the vertical displacement Δ of joint B of the truss in Fig. 19–1a. Here a typical element of the truss would be one of its members having a length L, Fig.19–1b. If the applied loadings P_1 and P_2 and cause a linear elastic material response, then this element deforms an amount $\Delta = \frac{NL}{AE}$ where N is the normal or axial force in the member, caused by the loads. Applying the expression $1.\Delta = \sum udL$, the virtual-work expression for the truss is therefore

$$1.\Delta = \sum \frac{nNL}{AE}$$

where

1= external virtual unit load acting on the truss joint in the stated direction of Δ

n= internal virtual normal force in a truss member caused by the external virtual unit load.

Δ = external joint displacement caused by the real loads on the truss.

N= internal normal force in a truss member caused by the real loads.

L= length of a member.

A= cross sectional area of a member.

E = modulus of elasticity of a member.

Here the external virtual unit load creates internal virtual forces n in each of the truss members. The real loads then cause the truss joint to be displaced Δ in the same direction as the virtual unit load, and each member is displaced nL/AE in the same direction as its respective n force. Consequently, the external virtual work $1.\Delta$ equals the internal virtual work or the internal (virtual) strain energy stored in all the truss members, that is, $\sum nL/AE$

Temperature. In some cases, truss members may change their length due to temperature. If α is the coefficient of thermal expansion for a member and ΔT is the change in its temperature, the change in length of a member is $dL = \alpha \Delta T L$ Hence, we can determine the displacement of a selected truss joint due to this temperature change is written as

$$1.\Delta = \sum n\alpha \Delta T L$$

where

1= external virtual unit load acting on the truss joint in the stated direction of Δ

n= internal virtual normal force in a truss member caused by the external virtual unit load.

Δ = external joint displacement caused by the temperature change.

α = coefficient of thermal expansion of member.

ΔT = change in temperature of member.

L= length of member.

Fabrication Errors and Camber. Occasionally, errors in fabricating the lengths of the members of a truss may occur. Also, in some cases truss members must be made slightly longer or shorter in order to give the truss a camber. Camber is often built into a bridge truss so that the bottom cord will curve upward by an amount equivalent to the downward deflection of the cord when subjected to the bridge's full dead weight. If a truss member is shorter or longer than intended, the displacement of a truss joint from its expected position can be determined from the earlier expression, written as

$$1.\Delta = \sum n\Delta L$$

Where

1= external virtual unit load acting on the truss joint in the stated direction of Δ

n= internal virtual normal force in a truss member caused by the external virtual unit load.

Δ = external joint displacement caused by the fabrication errors.

ΔL = difference in length of the member from its intended size as caused by a fabrication error.

A combination of the right sides of the above three expressions will be necessary if both external loads act on the truss and some of the members undergo a thermal change or have been fabricated with the wrong dimensions.

Example 1:

The cross-sectional area of each member of the truss shown in Fig. 19.2 *a* is A=400 mm^2 and E = 200 GPa. (a) Determine the vertical displacement of joint *C* if a 4-kN force is applied to the truss at *C*. (b) If no loads act on the truss, what would be the vertical displacement of joint *C* if member *AB* were 5 mm too short?

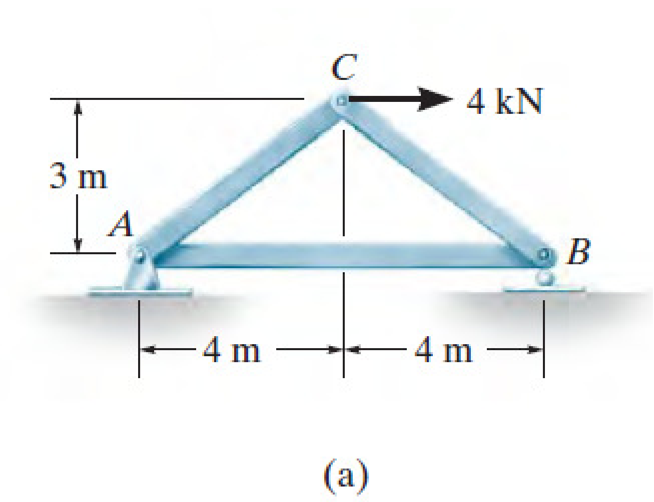


Figure 19.2 a

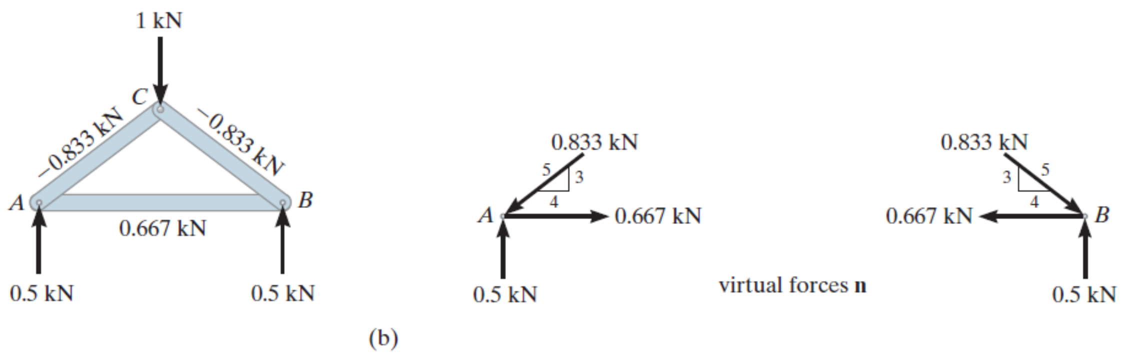


Figure 19.2 b

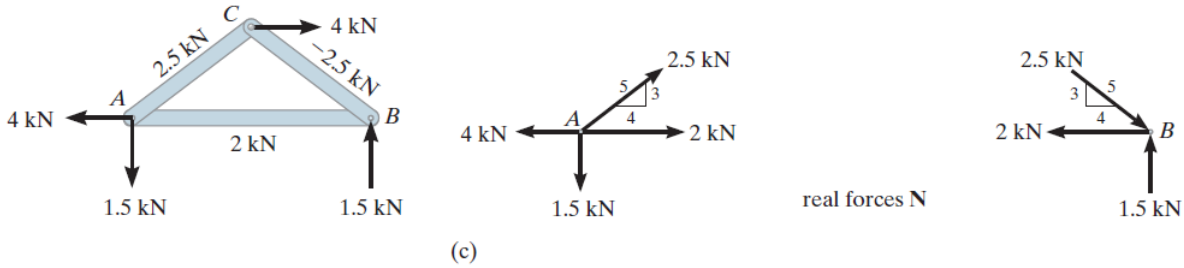


Figure 19.2 c

Solution:

Virtual Forces n. Since the vertical displacement of joint C is to be determined, a virtual force of 1 kN is applied at C in the vertical direction. The units of this force are the same as those of the real loading. The support reactions at A and B are calculated and the n force in each member is determined by the method of joints as shown on the free-body diagrams of joints A and B, Fig. 19–2b.

Real Forces N. The joint analysis of A and B when the real load of 4 kN is applied to the truss is given in Fig. 19–2c.

Virtual-Work Equation. Since AE is constant, each of the terms nNL can be arranged in tabular form and computed. Here positive numbers indicate tensile forces and negative numbers indicate compressive forces.

Member	n(KN)	N (KN)	L (m)	n N L (KN^2 m)
AB	0.667	2	8	10.67
AC	-0.833	2.5	5	-10.41
CB	-0.833	-2.5	5	10.41

$\sum 10.67$

Thus

$$1KN.\Delta c_v = \sum \frac{nNL}{AE} = \frac{10.67KN^2m}{AE}$$

Substituting the values A=400 mm^2 = 400(10^{-6}) m^2, E=200 GPa = 200 (10^6) KN/m^2 we have

$$1KN.\Delta c_v = \sum \frac{nNL}{AE} = \frac{10.67KN^2m}{400(10^6)m^2(200(10^6)KN/m^2)}$$

$$\Delta c_v = 0.000133m = 0.133m$$

Case (b)

Here we must apply $1.\Delta = \sum n\Delta L$. Since the vertical displacement of *C* is to be determined, we can use the results of Fig. 19.1 *b*. Only member *AB* undergoes a change in length, namely, of $\Delta L = -0.005m$

Thus

$$1.\Delta = \sum n\Delta L$$
$$1KN.\Delta_{C_v} = (0.667KN)(-0.005m)$$
$$.\Delta_{C_v} = -0.00333m = -3.33mm$$

The negative sign indicates joint *C* is displaced *upward,* opposite to the 1-kN vertical load. Note that if the 4-kN load and fabrication error are both accounted for, the resultant displacement is then $.\Delta_{C_v}$ 0.133 - 3.33 = -3.20 mm (upward).

19.2 Analysis of Statically Determinate Trusses by Method of Joints

The determination of the member forces and support reactions in the truss shown in Figure 19.3 requires that two force equilibrium equations be written for each of the 8 joints of the truss. This yields a total of 16 equations that can be solved to yield the forces in the 13 members of the truss, and the 3 reactions at the supports at A and E.

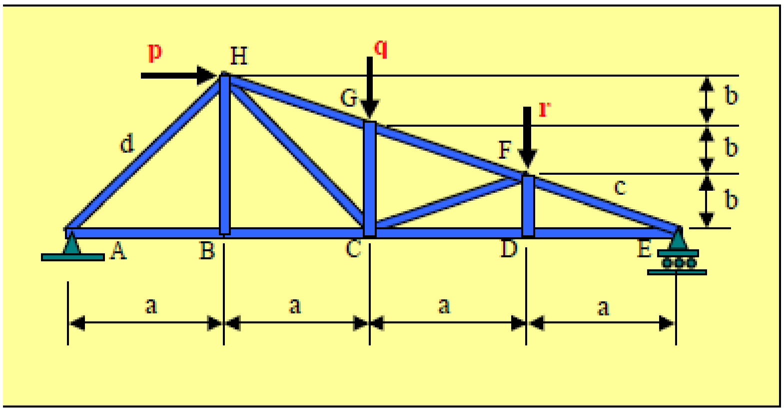


Figure 19.3 A Statically Determinate Truss

To obtain the MATLAB solution to this problem, the equilibrium equations for the truss are first formulated in a general format as shown in Eq. (19.1). Using this equation, the unknown column vector X can be solved for using the MATLAB left division operation as shown in Eq. 19.2. For the above truss, the matrices A and B, and the column vector X containing the unknowns in the problem are provided in Eq. 19.3. Note that in this equation $c = \sqrt{a^2 + b^2}$ and $9 = \sqrt{a^2 + 9b^2}$. Note that c and d are the length of the members EF and AH respectively as shown in Figure 19.3. When formulating the MATLAB program for this type of problem, the “zeros” function can be utilized to generate a series of zeros in the matrix in an easy and convenient way. The script file for this truss and the generated output for the case when a = 10 ft, b = 4 ft, p = 2 kip, q = 3 kip, and r = 3 kip are shown in Figure 19.4. Note that the script file for the problem is developed in a fashion to allow the user to enter any specific values for the dimensions a and b, and for the applied loads p, q, and r.

AX=B (19.1)

X=A\B (19.2)

$$
\begin{bmatrix}
1 & a/d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 3b/d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 1 & a/c & 0 & -a/d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & b/c & 1 & 3b/d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -a/c & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b/c & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -a/c & 0 & 0 & 0 & 0 & a/c & -a/c & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -b/c & 0 & 0 & -1 & 0 & -b/c & b/c & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a/c & -a/c & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -b/c & b/c & 0 & 0 & 0 \\
0 & -a/d & 0 & 0 & 0 & 0 & 0 & a/d & 0 & 0 & 0 & 0 & a/c & 0 & 0 & 0 \\
0 & -3b/d & 0 & -1 & 0 & 0 & 0 & -3b/d & 0 & 0 & 0 & 0 & -b/c & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} F_{AB} \\ F_{AH} \\ F_{BC} \\ F_{BH} \\ F_{CD} \\ F_{CF} \\ F_{CG} \\ F_{CH} \\ F_{DF} \\ F_{DE} \\ F_{EF} \\ F_{FG} \\ F_{GH} \\ A_y \\ E_x \\ E_y \end{Bmatrix}
=
\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ r \\ 0 \\ q \\ -p \\ 0 \end{Bmatrix}
\quad (19.3)
$$

MATLAB Script for method of joints

```
% Analysis of a Truss Utilizing the Method of Joints
%

% Program objective:
% To compute the member forces and support reactions of the given truss
utilizing
% the method of joints.
%

% Data acquisition:
p=2;
q=3;
r=3;
a=10;
b=4;
%

% Computation of the truss member forces and support reactions:
c=sqrt(a^2+b^2);
d=sqrt(a^2+9*b^2);
A(1,:)=[1,a/d,zeros(1,14)];
A(2,:)=[0,3*b/d,zeros(1,11),1,zeros(1,2)];
A(3,:)=[-1,0,1,zeros(1,13)];
A(4,:)=[zeros(1,3),1,zeros(1,12)];
A(5,:)=[zeros(1,2),-1,0,1,a/c,0,-a/d,zeros(1,8)];
A(6,:)=[zeros(1,5),b/c,1,3*b/d,zeros(1,8)];
A(7,:)=[zeros(1,4),-1,zeros(1,4),1,zeros(1,6)];
A(8,:)=[zeros(1,8),1,zeros(1,7)];
A(9,:)=[zeros(1,9),-1,-a/c,zeros(1,3),-1,0];
A(10,:)=[zeros(1,10),b/c,zeros(1,4),1];
A(11,:)=[zeros(1,5),-a/c,zeros(1,4),a/c,-a/c,zeros(1,4)];
A(12,:)=[zeros(1,5),-b/c,zeros(1,2),-1,0,-b/c,b/c,zeros(1,4)];
A(13,:)=[zeros(1,11),a/c,-a/c,zeros(1,3)];
```

```
A(14,:)=[zeros(1,6),-1,zeros(1,4),-b/c,b/c,zeros(1,3)];
A(15,:)=[0,-a/d,zeros(1,5),a/d,zeros(1,4),a/c,zeros(1,3)];
A(16,:)=[0,-3*b/d,0,-1,zeros(1,3),-3*b/d,zeros(1,4),-b/c,zeros(1,3)];
B=[zeros(11,1);r;0;q;-p;0];
X=A\B;
%
```

```
% Outputing the member forces and support reactions:
fprintf('Member Forces(kip):\n\n')
fprintf('AB = %5.3f AH = %5.3f BC = %5.3f BH = %5.3f CD = %5.3f CF = %5.3f
CG = %5.3f\n\n',...
X(1),X(2),X(3),X(4),X(5),X(6),X(7))
fprintf('CH = %5.3f DF = %5.3f DE = %5.3f EF= %5.3f FG = %5.3f GH =
%5.3f\n\n',...
X(8),X(9),X(10),X(11),X(12),X(13))
fprintf('Support Reactions(kip):\n\n')
fprintf('Ay = %5.3f Ex = %5.3f Ey = %5.3f\n',X(14),X(15),X(16))
```

Matlab output

```
Member Forces(kip):

AB = 1.375 AH = -2.148 BC = 1.375 BH = 0.000 CD = 8.875 CF = -4.039 CG = -3.000

CH = 5.858 DF = 0.000 DE = 8.875 EF= -11.713 FG = -7.674 GH = -7.674

Support Reactions(kip):

Ay = 1.650 Ex = 2.000 Ey = 4.350
```

Figure 19.4 MATLAB Script for method of joints

19.3 Analysis of Statically Determinate Trusses by Method of Sections

The method of sections for analyzing a truss is normally utilized in situations where only the forces in specific members are to be computed. In the given truss, to determine the forces in members BC, CH, and GH using this method, these members are cut and two force equilibrium equations and one moment equilibrium equation are written for the section of the truss as shown in Figure 19.5. The three equations can be arranged in the matrix form as shown in Eq. (19.4), and the MATLAB solution for the unknowns in the problem can be obtained using the left-division operation as done in the previous method. Note that prior to determining the unknown member forces FBC, FCH, and FGH in Eq. (19.4), it is necessary to compute the reaction Ay at support A using the equilibrium of the entire truss. The expression for Ay is provided in Eq. 19.5. The script file for this problem and the corresponding results for the data already presented are provided in Figure 19.6.

$$\begin{bmatrix} 1 & a/d & a/c \\ 0 & -3b/d & -b/c \\ 0 & -3ab/d & -3ab/c \end{bmatrix} \begin{Bmatrix} F_{BC} \\ F_{CH} \\ F_{GH} \end{Bmatrix} = \begin{Bmatrix} -p \\ -Ay \\ A_y\, a + 3pb \end{Bmatrix} \tag{19.4}$$

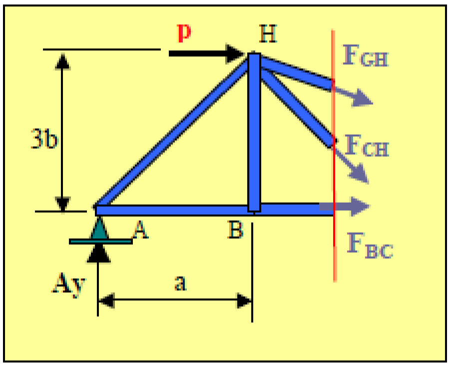


Figure 19.5 Free Body Diagram of the Left Section of the Truss

$$A_y = \frac{ra + 2qa - 3pb}{4a}. \quad (19.5)$$

MATLAB SCRIPT FOR TRUSSES WITH METHOD OF Sections

```
% Analysis of a Truss Utilizing the Method of Sections.
%
________________________________________________________________________
____________________
% Program objective:
% To compute the forces in members BC, CH, and GH of the given truss
utilizing the method
% of sections.
%
________________________________________________________________________
____________________
% Data acquisition:
p=2;
q=3;
r=3;
a=10;
b=4;
%
________________________________________________________________________
____________________
% Computation of the support reaction at A:
Ay=(r*a+2*q*a-3*p*b)/(4*a)
%
________________________________________________________________________
____________________
% Computation of the truss member forces:
c=sqrt(a^2+b^2);
d=sqrt(a^2+9*b^2);
A(1,:)=[1,a/d,a/c];
A(2,:)=[0,-3*b/d,-b/c];
A(3,:)=[0,-3*a*b/d,-3*a*b/c];
B=[-p;-Ay;Ay*a+3*p*b];
F=A\B;
% ______________________________________________________________________
% ______________________
% Outputting the member forces:
fprintf('Member Forces(kip):\n\n')
```

```
fprintf('BC = %5.3f CH = %5.3f GH = %5.3f\n',F(1),F(2),F(3))
```

Matlab output

```
Ay =

    1.6500

Member Forces(kip):

BC = 1.375 CH = 5.858 GH = -7.674
```

Figure 19.6 MATLAB SCRIPT FOR TRUSSES WITH METHOD OF Sections

Lecture 20 Deflection of statically determinate beams by energy methods- strain energy method, castiglianos theorems, reciprocal theorem, unit load method. Deflection of pin-jointed trusses, **Williot-Mohr diagram**

All the discussions so far on joint displacements of trusses were centred on algebraic methods using only geometric relations. It can be easily inferred that the horizontal and vertical deflections of all joints can be found by graphical solution. Theoretically, it is possible to draw the shape of the transformed truss by using the lengths of members which have been changed due to applied loading. These altered lengths can be used as the sides of the component triangles. However, the lengths of the members which have undergone alterations differ only slightly in terms of stretching or shortening over the original lengths. If plotted to the same scale, there can not be any substantial difference between the shape of deformed truss and its original. This difficulty can be overcome by using two difference scales in plotting the original length L and the changes in length.

In a truss, a member undergoes translation or rotation under loading. It may be stretched or shortened or rotated. We consider a member IJ as shown in Figure 20.1.

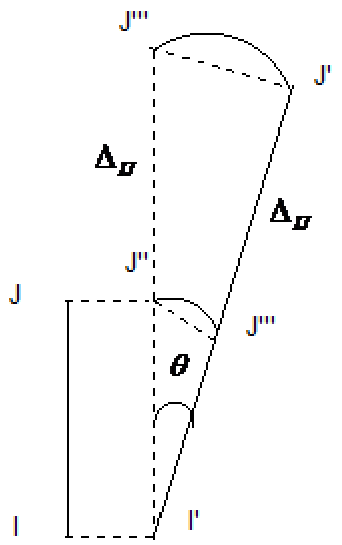


Figure 20.1 Deformations of a member ij

After the occurrence of all deformations the member eventually takes a position I'J'. The total deformation of the bar consists of (i). A translation of a bar IJ, (ii). A rotation of the bar IJ and (iii). A stretching or shortening of the bar IJ. We can analyse the deformation of the bar considering three steps. First, we consider the bar to translate by δ parallel to itself so that I takes the position I' and J takes the position J'' as shown in Figure 20.1. Next we consider rotation θ of the bar i.e I'J'' rotates about I' and assumes new position as I'J''' as shown in Figure 20.1. Now J'' rotates over to J'''. Finally we consider stretching Δ_{IJ} of the bar due to its own axial force. In Figure 20.1 the member IJ has been shown as elongated and hence it is under tension. In case the force in the member is compressive, it would have been shortened.

In frames, the rotation of a member is quite small. Therefore, we take the linear length J''J''' perpendicular to I'J'' as rotation in stead of the arc length J''J'''. The order of the displacements the member undergone is immaterial. It can be any combination of translation, rotation and stretching or shortening.

In contrast to a single member, we now consider two members PQ and QR as parts of a frame. Both members are jointed at Q. We assume that P is displaced to P' and R to R'. Further let PQ be shortened by Δ_{PQ} and QR be stretched by Δ_{QR}. Let Q' be the final position of Q' which is determined as described below.

The member PQ and QR are separated out at Q. Member PQ moves parallel to itself and occupies position P'Q_1 as shown in Figure 20.2. Similarly member QR moves parallel to itself and occupies the new position R'Q_2 as shown in Figure 20.2. Member P'Q_1 in its new position is made short by a quantity Δ_{PQ}. Therefore Q_1 moves to Q_3. Similarly member R'Q_2 after its transformation is made elongate by a quantity Δ_{QR}. So Q_2 moves over to Q_4 . We draw a perpendicular to P'Q_1 at Q_3 . Likewise, we draw a perpendicular to R'Q_4 at Q_4. Both the perpendiculars intersect at Q'. This intersection point will be the position of Q. The distance QQ' indicates the movement of joint Q as shown in Figure 20.2.

Normally, the displacements of members in a truss are very small when compared to their lengths. Therefore, the displacement diagram is drawn on a larger scale showing only the actual displacements undergone by members without drawing their specified lengths. In Figure 20.2, $QQ_1Q_3Q'Q_4Q_2Q$ is the displacement diagram. Such a diagram is called Williot diagram of displacement.

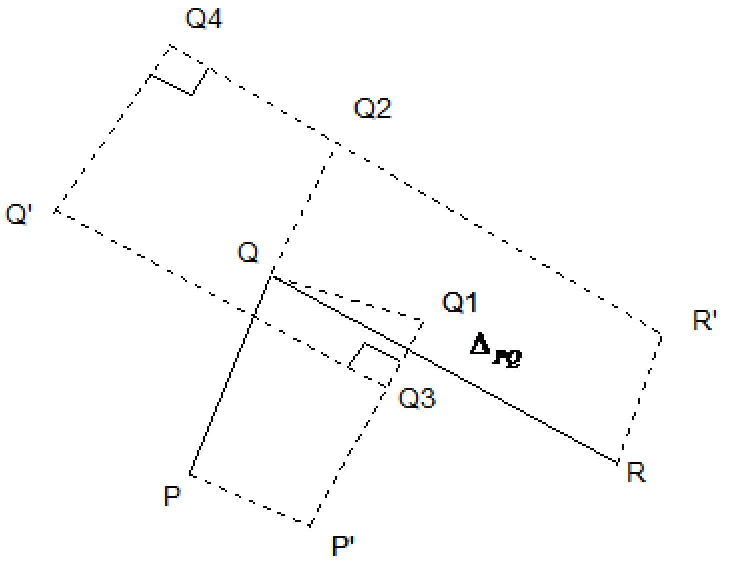


Figure 20.2 Deformations of truss members

Now we can draw the displacement diagram separately as depicted in Figure 20.3. We take a pole o; we draw op corresponding to the displacement of P and or corresponding to the displacement of R. Now, we draw deformation pq_1 equal to Δ_{PQ} and parallel to PQ. Because there is shortening of member PQ, we draw Δ_{PQ} towards P as shown in Figure 20.3. Draw rq_2 deformation of RQ equal to Δ_{RQ} and parallel to RQ. As the member RQ is elongated, we draw Δ_{RQ} away from R. Draw perpendiculars at q_1 and q_2 . The intersection of these perpendiculars i.e q gives the position of the point. The displacement of Q is given by oq. The diagram drawn to determine the deflection is known as Williot diagram.

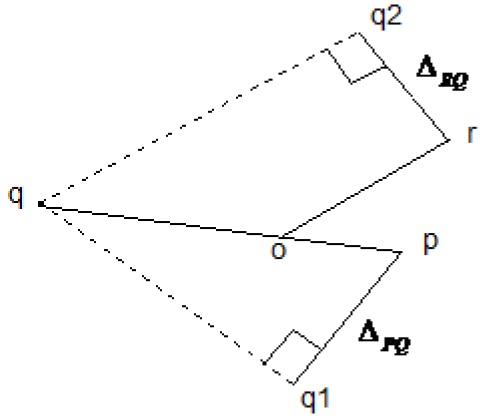
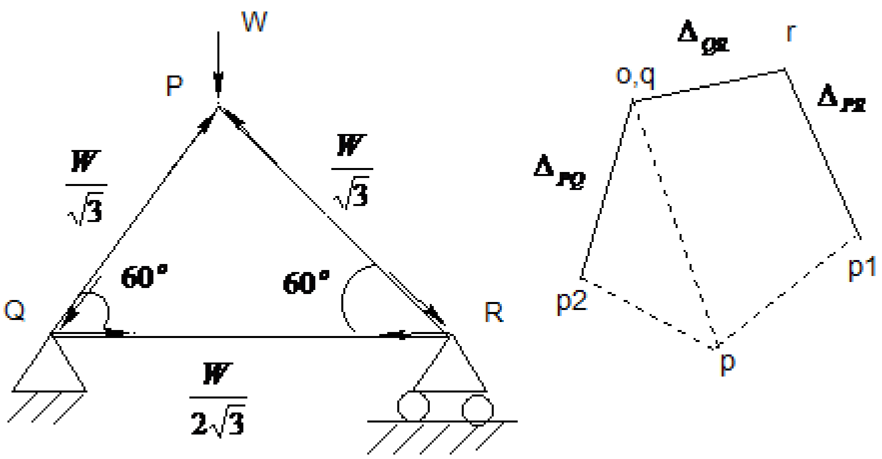


Figure 20.3 Williot diagram – typical

Now, we consider a truss with three members PQ, QR and RP as shown in Fig 20.4 (a). Support Q is hinged. So it can not move and hence there can not be any displacement of Q. Because of load W acting at P, members PQ and PR are under compression. The magnitude of force in each member is (W/2) cosec 60°= $W/\sqrt{3}$. The force in the member QR is tensile and its magnitude is W_{PQ} cos 60° = $W/2\sqrt{3}$. The point is put on roller supports. Therefore, it can move only to the extent of the elongation of the member QR.



(a) Truss (b) Williot diagram

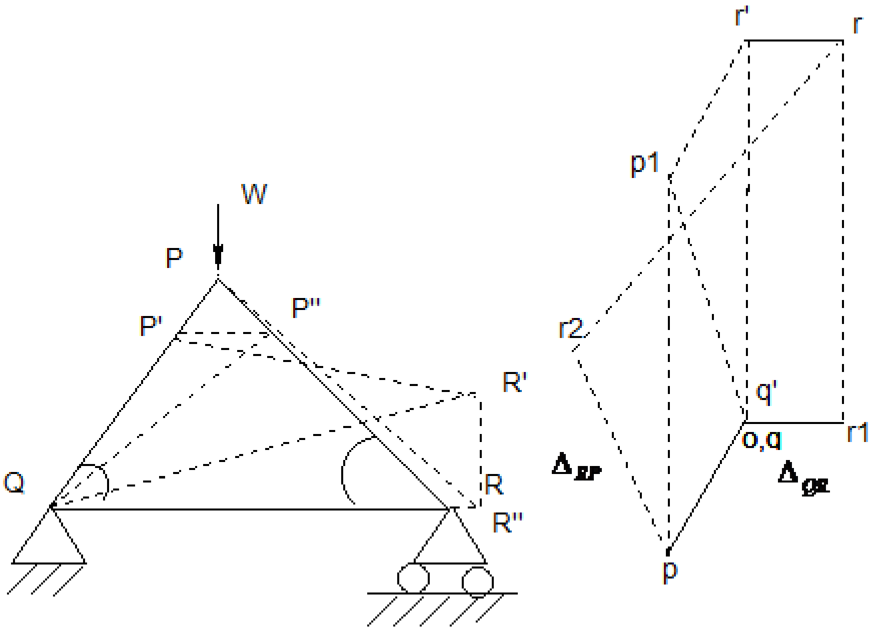
Figure 20.4 Typical truss deformation

Now, we will discuss the construction of displacement diagram of the truss. We take a pole o as shown in Figure 20.4 b. Point Q will coincide with this pole o because its position is fixed. In order to get the position of P on Williot diagram, we proceed as follows.

The shortening Δ_{PQ} of member PQ is drawn towards Q in the direction PQ and parallel to it by qp_2 as shown in Figure 20.4 (b). Likewise, the shortening Δ_{PR} of member PR is drawn towards R in the direction of PR and parallel to it by rp_1 as shown in Figure 20.4 b. We draw perpendiculars at p_1 and p_2 which intersect at point p indicating its position. The displacement of point P is equal to op in direction and magnitude.

20.2 MOHR'S CORRECTION

Sometimes it may so happen that the displacement of two consecutive joints remains unknown. In such a case, the direction of displacement of one of the joints is assumed and finally a correction is applied. Let the displacement of joints P and R in the truss as shown in Figure 20.5 is unknown. We assume that P moves along PQ. With this assumption, we draw the deflected form of the truss. We take a pole o. As the position of the point Q is fixed because of hinge, point q coincides with the pole o as shown in Figure 20.5. The joint P moves towards Q by Δ_{PQ} because the force in PQ is compressive. Therefore, op is drawn parallel to PQ to some scale in the direction of PQ in the Williot diagram. At q, we draw qr_1 by an amount Δ_{QR} in the direction of QR because QR is in tension. At p, we draw r_2p by an amount Δ_{RP} in the direction of RP as the member is in compression. We erect perpendiculars at r_1 and r_2 . The position of r is fixed by the intersection of these two perpendiculars. The point r is shown in Figure 20.5. The displacement of r is given by or in Williot diagram. The deflected truss will be P'QR'. In fact R can have only horizontal movement along QR. However, in Figure 20.5 (a) it is seen as rotated. So it necessitates a correction for the rotation of the truss to bring the movement of R along horizontal line. Because of this reason, the deflected truss is rotated by θ so that R' will occupy the position R'' in line with QR and P' will move to P'' so that the deflected truss can be denoted as P''QR''. The displacements of R' and P' due to rotation θ will be proportional to the lengths RQ and PR respectively.



(a). Deflected truss (b). Polar diagram

Figure 20.5 Williot-Mohr diagram

The application of correction to Williot diagram is based on the consideration of the fact that the movement of R must be only horizontal and that the vertical movement is zero. The displacement or in Figure 20.5 (b) is equal to or' plus r'r. By applying r'o correction, the remaining displacement will be the required displacement r'r. The correction for other joints can be obtained by drawing the shape of the truss on or' as the base. Therefore p'qr' is the correction applied. For PQ. The rotation correction is p'o. This is at right angle to PQ. Similarly r'o is the rotation correction for QR and it is at right angles to QR. The deflection of P will be p'p and the deflection of r is r'r. The correction applied is known as Mohr's correction. The final deflection diagram is known as Williot-Mohr diagram.

Example 1: Now consider the Howe truss as shown in Figure 20.6. Determine the downward deflections at joint B by graphical method. Area of cross section of each member is 400 mm^2 and E=200 KN/mm^2

Solution:

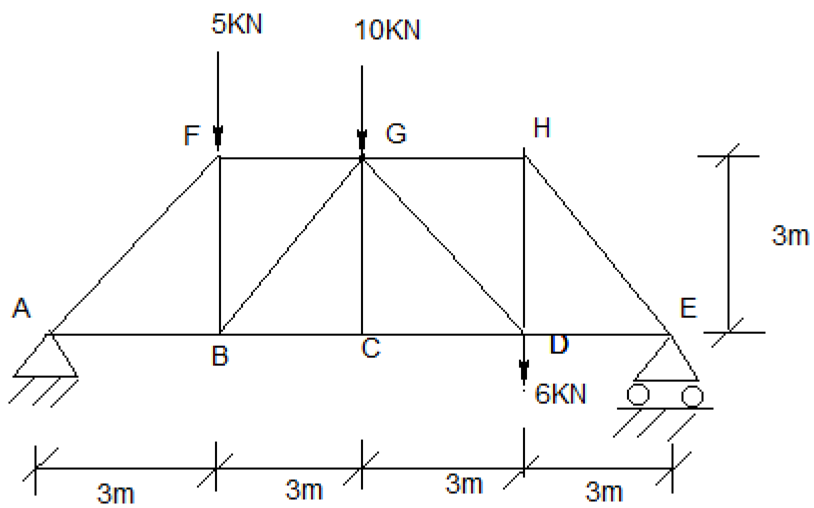


Figure 20.6 Howe truss with loading

The forces and displacements in various members due to applied loading are calculated and summarized in Table 20.1

Table 20.1 Forces and displacements

Member	F_P (KN)	L(mm)	AE(KN)	$\Delta = F_p L / AE$ (mm)
AB	+10.25	3000	80000	+0.384
BC	+15.50	3000	80000	+0.581
CD	+15.50	3000	80000	+0.581
DE	+10.75	3000	80000	+0.403
AF	-14.50	4242.64	80000	-0.769
BF	+5.25.	3000	80000	+0.197
FG	-10.25	3000	80000	-0.384
BG	-7.42	4242.64	80000	-0.394
GC	0	3000	80000	+0.000
GH	-10.75	3000	80000	-0.403
GD	-6.72	4242.64	80000	-0.356
HD	+10.75	3000	80000	+0.403
HE	-15.2	4242.64	80000	-0.806

We take the pole as o in Figure 20.7. As point A is fixed, it will coincide with o. Displacement of B is assumed along AB. Therefore ab will be parallel to AB. The Williot-Mohr diagram is completed as shown in Figure 20.7. The Willot diagram is plotted to some scale.

Vector bb' represents the displacement of joint B in both direction and magnitude. The vertical component of the vector bb' gives the downward displacement of joint B. Therefore $\Delta_{VB} = 3.16mm$.

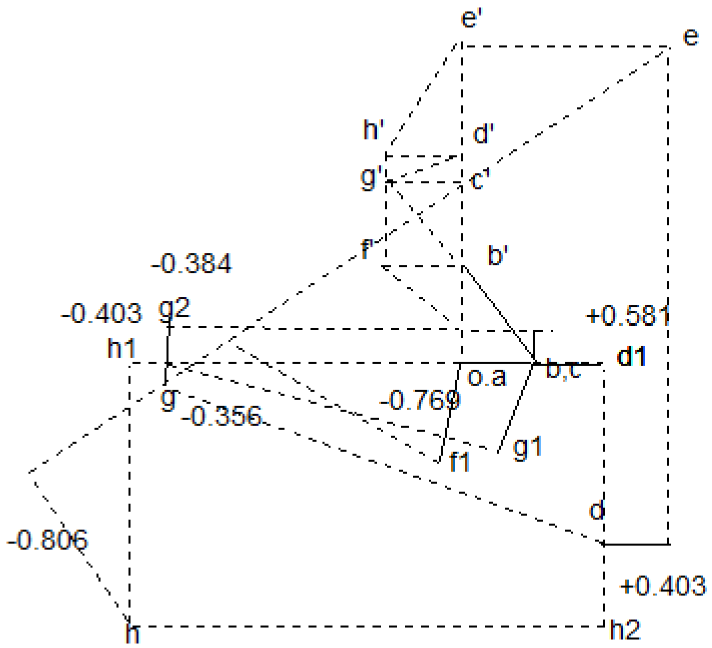


Figure 20.7 Williot-Mohr diagram for a given Howe truss

Lecture 21 B M. and S.F. diagrams for statically indeterminate beams – **propped cantilever** and fixed beams.

21.1 Introduction

We know that a cantilever is a beam with one of its ends fixed and the other free. It is a determinate structure. Therefore, we can easily calculate the bending moment and shear force in a cantilever. However, when the free end or for that matter at any point in the span of the cantilever is supported additionally, the support is called a prop. Hence, the name propped cantilever. Because of this additional support, a reaction is set up at the propping point. Already there exists a vertical reaction and moment at the fixed end. Along with this two reactions, one more reaction at the prop is added. So a propped cantilever will have three unknowns. But under vertical loading, we have only two equations of static equilibrium namely $\sum F_V = 0$ and $\sum M = 0$. Hence one of the reactions can not be determined with the application of static equilibrium conditions alone. Because of this, a propped cantilever becomes an indeterminate structure. The degree of indeterminancy here is one. Therefore, we require one more equation to solve the problem. The third equation can be obtained from the consideration of deflections or slopes.

21.2 Shear force and Bending moment diagrams

Once we determine the redundant in the propped cantilever, we can calculate the other support reactions. With known reactions, it is now possible to determine the shear force and the bending moment at any section in the beam. This facilitates the drawing of the shear force and the bending moment diagrams as usual.

Example 1: Determine the prop reaction in a propped cantilever as shown in Figure 21.1. Also draw the SFD and BMD for the propped cantilever.

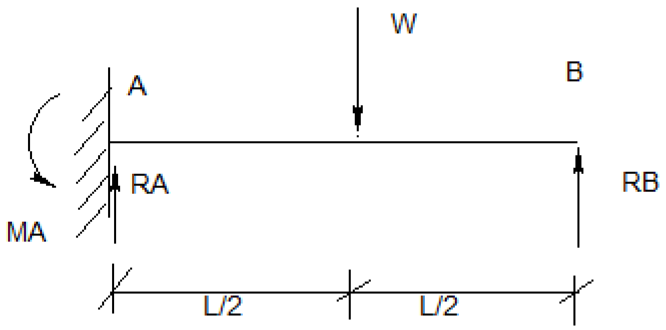


Figure 21.1 Beam with central load

Solution:

First we remove the prop and calculate the deflection at B under the central point load as

$$y_{PL} = \frac{W(L/2)^3}{3EI} + \frac{W(L/2)^2}{2EI} \times \frac{L}{2} = \frac{5WL^3}{48EI}$$

Now we remove the central load W and introduce the prop. With this prop reaction R_B, we calculate the upward deflection as

$$y_{PR} = \frac{R_B L^3}{3EI}$$

As the net deflection at B=0, we have $\frac{5WL^3}{48EI} = \frac{R_B L^3}{3EI} \Rightarrow R_B = \frac{5}{16}W$

Now we can apply static equilibrium equations and determine the other reaction. Therefore $+\downarrow \sum F_V = 0 \quad R_A = W - 5W/16 = 11W/16$ Taking moment of forces about A

+ Sum of anticlockwise moment is zero. $M_A + R_B \times L - W \times L/2 = 0$ $M_A = (WL/2) - 5W \times L/16 = (3/16)WL$

Following the usual procedure, the SFD and BMD can be shown in Figure 21.2.

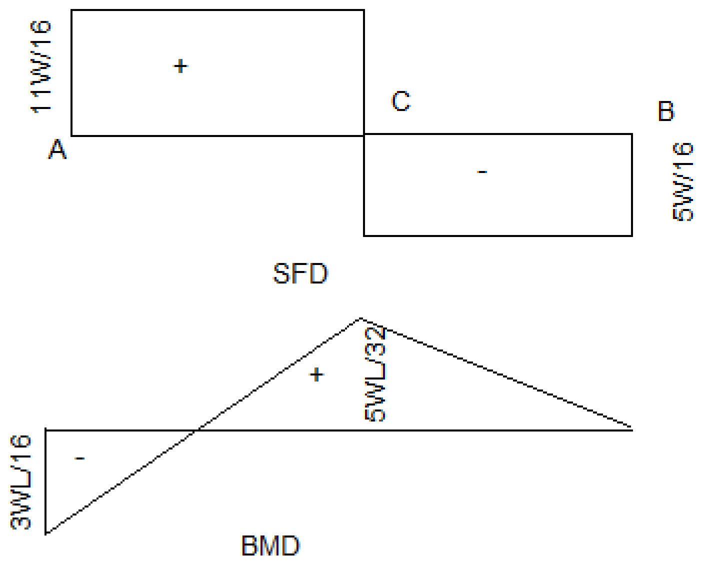


Figure 21.2 SFD and BMD diagrams of a propped cantilever beam

Sample Example Problems

Example 2: Determine the prop reaction in the beam shown in the Figure below. EI is constant.

Solution:

The propped cantilever with loading is shown in Fig. 21.3. We remove the prop at A and calculate the deflection at free end due to the given loads (Fig. 21.3(a)).

$$y_{PL} = \frac{12\times3^3}{3EI} + \frac{12\times3^2}{2EI}\times6 + \frac{12\times6^3}{3EI} + \frac{12\times6^2}{2EI}\times3$$

$$y_{PL} = \frac{108}{EI} + \frac{324}{EI} + \frac{864}{EI} + \frac{648}{EI} = \frac{1944}{EI}$$

We now introduce the prop at A and calculate the deflection at A (Fig. 21.3(a)).

$$y_{PR} = \frac{R_A \times 9^3}{3EI} = \frac{243R_A}{EI}$$

From the relation

$$y_{PL} - y_{PR} = 0$$

$$\frac{1944}{EI} - \frac{243R_A}{EI} = 0$$

$$R_A = 8KN$$

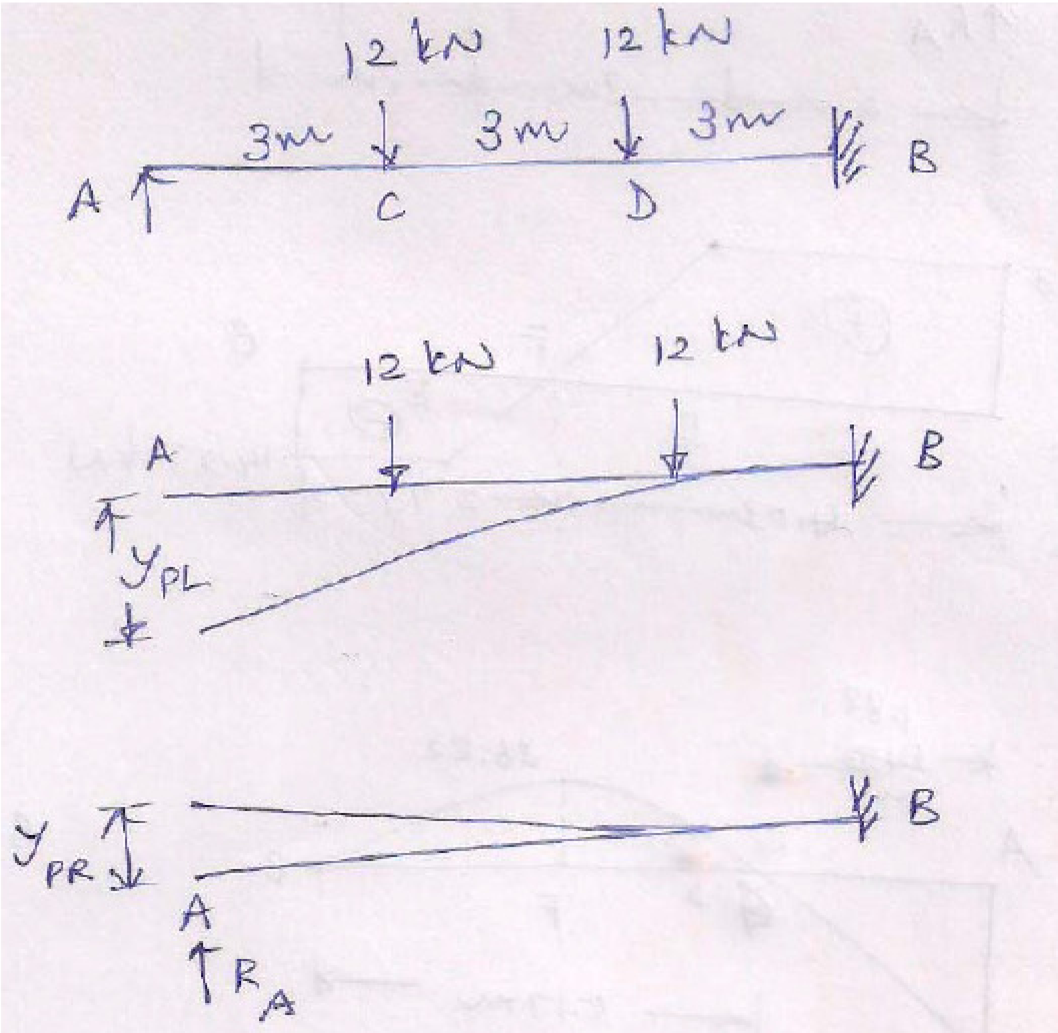


Figure 21.3 Deflections of a propped cantilever

Example 2

Using the consistent deformation method, evaluate the prop reaction in the beam as shown in the Figure 21.4 below. EI Constant.

Solution:

The propped cantilever is shown in Fig. 21.4. We remove the prop at B and allow the cantilever to deflect under the given loading as shown in Fig. 21.4(a).

$$y_{PL} = \frac{8\times 2^4}{8EI} + \frac{8\times 2^3}{6EI}\times 2 + \frac{12\times 3^3}{3EI} + \frac{12\times 3^2}{2EI}\times 1 = \frac{16}{EI} + \frac{21.33}{EI} + \frac{108}{EI} + \frac{54}{EI} = \frac{199.33}{EI}$$

Now we apply a unit load at prop support (Fig. 21.4a)).

The deflection at prop due to this unit load is

$$y_{1R} = \frac{1\times 4^3}{3EI} = \frac{21.33}{EI}$$

The geometrical condition is that the net deflection at prop due to applied loading and the prop R_B is zero.

$$y_{PL} = R_B\, y_{1R}$$

$$R_B = \frac{y_{PL}}{y_{1R}} = \frac{199.33}{EI}\times\frac{EI}{21.33} = 9.35KN$$

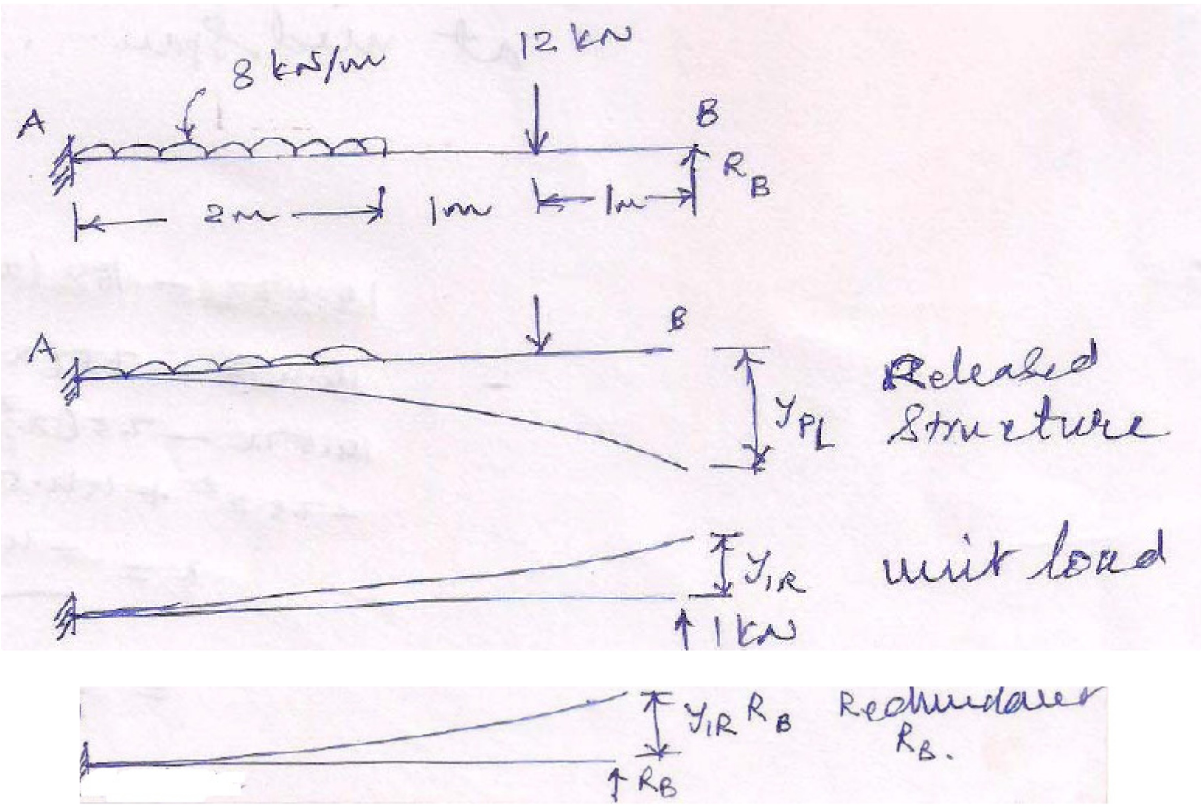


Figure 21.4 Beam with UDL and point load

Lecture 22 B M. and S.F. diagrams for statically indeterminate beams – **propped cantilever** and fixed beams.

Example 1: In the beam shown in Figure 22.1, the prop has sunk by 15 mm. Calculate the prop reaction Take E=200 x 10^6 KN/m^2 I=5 x 10^{-6} m^4

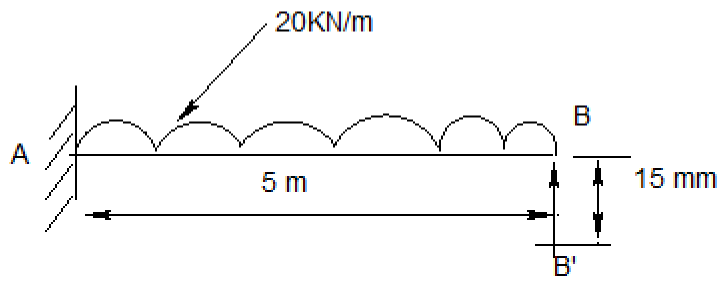


Figure 22.1 Prop with UDL and settlement

Solution:

The propped beam with UDL is shown in Fig. 22.1. We remove the prop and calculate the deflection at prop.

$$y_{PL} = \frac{20\times5^4}{8\times200\times10^6\times5\times10^{-6}} = 1.56m$$

The deflection due to R_B is

$$y_{PR} = \frac{R_B\times5^3}{3\times200\times10^6\times5\times10^{-6}} = 0.042R_B$$

$$1.56 - 0.042R_B = 0.015$$
$$R_B = 36.79KN$$

Lecture 23 B M. and S.F. diagrams for statically indeterminate beams – propped cantilever and **fixed beams**.

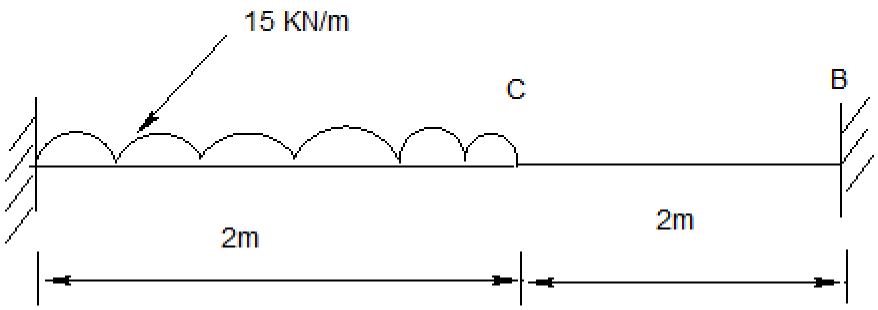


Figure 23.1 Fixed beam with partial varying load

The fixed beam with partial UDL is shown in Fig. 23.1.We adopt here consistent deformation method to determine the fixed-end moments and reactions. We release the fixity at both ends and introduce in their places hinge at A and roller at B as shown in Fig. 23.2(a).

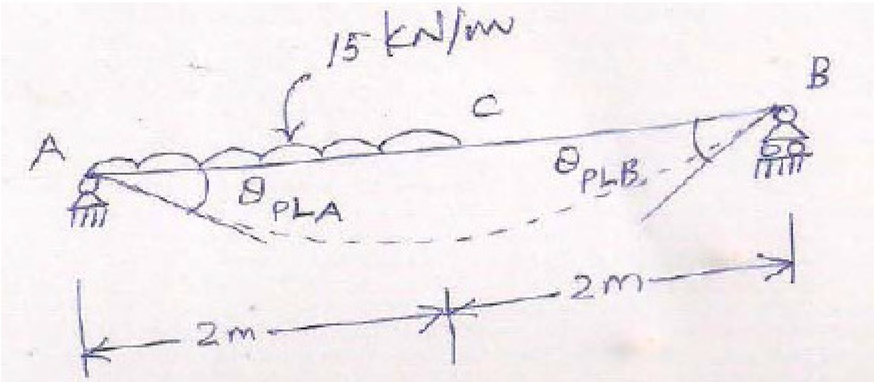


Figure 23.2 Determinate beam

Thus the fixed-end moments MA and MB are treated as redundants. We denote the rotations at A and B under the applied loading as shown in Fig. 23.2(b).

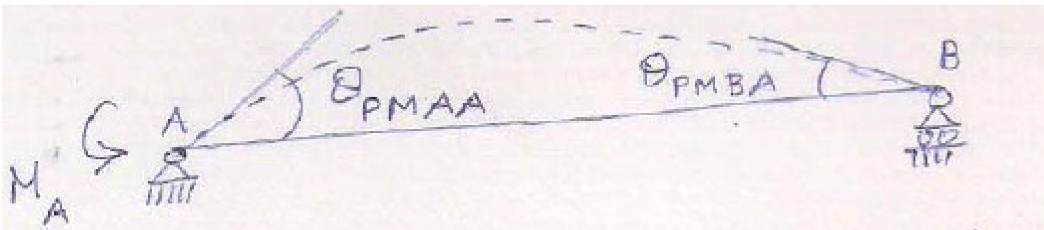


Figure 23.2 b) Rotations under redundant MA

Next we apply the redundant MA at A and denote the rotations at both ends as shown in Fig. 23.2 (b). Finally, we apply the redundant MB at B and denote the rotations at both ends as shown in Fig. 23.2 (c).

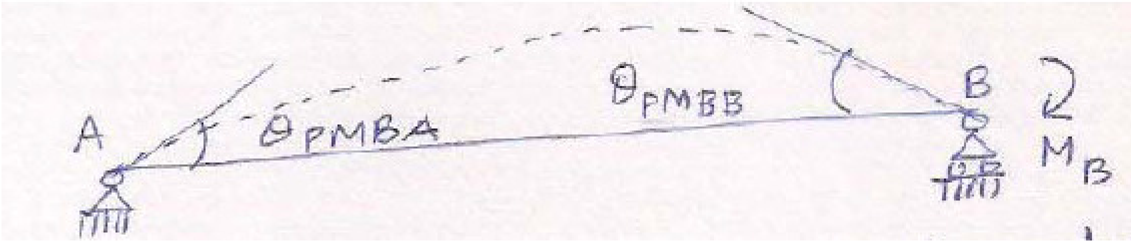


Figure 23.2 c) Rotations under redundant MB

As the slopes at both ends are zeros, the geometrical conditions require that,

$\theta_{PLA} = \theta_{PMAA} + \theta_{PMBA}$ (a)

$\theta_{PLB} = \theta_{PMBA} + \theta_{PMBB}$ (b)

$$\theta_{PLA} = \frac{3\times15\times4^3}{128EI} = \frac{22.5}{EI} \quad \theta_{PMAA} = \frac{M_A\times4^3}{3EI} = \frac{1.33M_A}{EI} \quad \theta_{PMBA} = \frac{M_A\times4}{6EI} = \frac{0.67M_A}{EI}$$

$$\theta_{PLB} = \frac{7\times15\times4^3}{384EI} = \frac{17.5}{EI} \quad \theta_{PMBA} = \frac{M_B\times4}{6EI} = \frac{0.67M_B}{EI} \quad \theta_{PMBB} = \frac{M_B\times4}{3EI} = \frac{1.33M_B}{EI}$$

Substituting these values in (a) and (b),

$$\frac{22.5}{EI} = \frac{1.33M_A}{EI} + \frac{0.67M_B}{EI} \quad \text{(c)}$$

$$\frac{17.5}{EI} = \frac{0.67M_A}{EI} + \frac{1.33M_B}{EI} \quad \text{(d)}$$

Solving (c) and (d), we get

$$M_A = 13.79KNm \quad M_B = 6.21KNm$$

The calculation of the reactions is shown in Fig. 23.2(d). The SFD is shown in Fig. 23.2(e). Figure 23.2(f) shows the BMD.

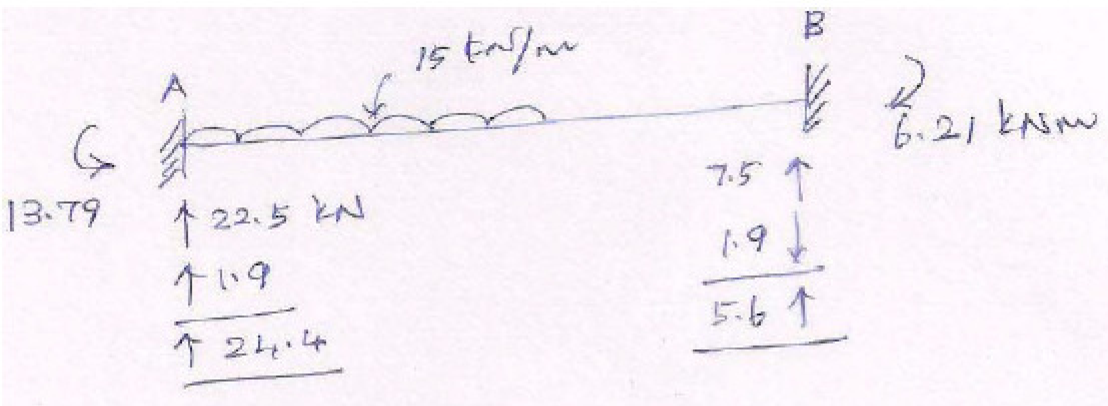


Figure 23.2 d: Reactions

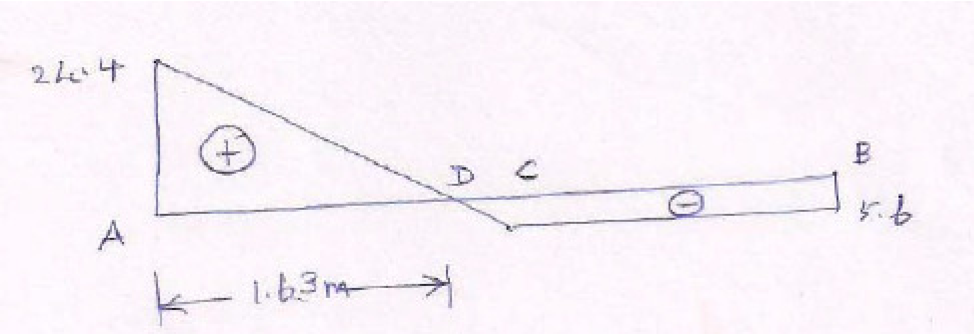


Figure 23.2 e SFD

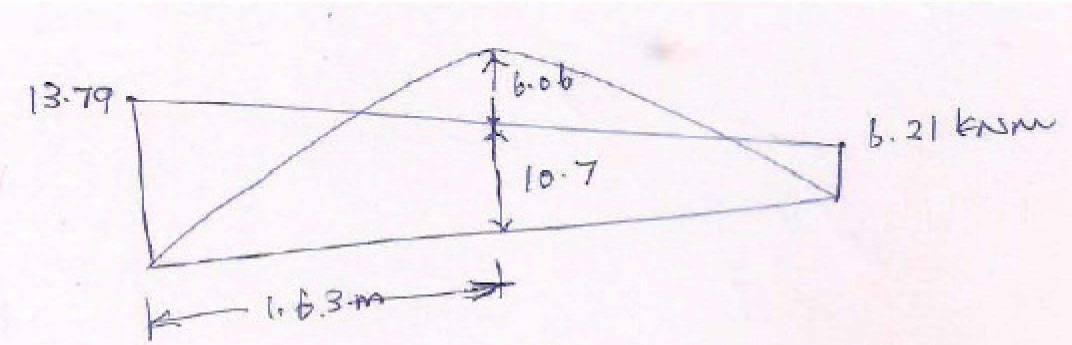


Figure 23..2 f BMD

Lecture 24 Application of three moment theorem to **continuous** and other indeterminate beams.

24.1 Introduction

A beam having many supports and covering more than one span is called continuous beam. It is nothing but a simple beam extended over many supports on either side and consisted of many spans. A continuous beam must have a minimum of two spans and three supports. The ends of the continuous beam may be either fixed or propped. The length of spans of a continuous beam may be either fixed or propped. The length of spans of a continuous beam may be either equal or unequal. Similarly, the moment of inertia in various spans may be same or different. A continuous beam is statically indeterminate. Owing to the continuity of the beam over the supports, bending moments will exist at these supports. Until these bending moments are known, the bending moment diagram (BMD) and the shear force diagram (SFD) can not be drawn.

When a continuous beam is loaded, it deflects with convexity upward at intermediate supports and concavity upward over the region of mid-span like that as shown in Figure 24.1.

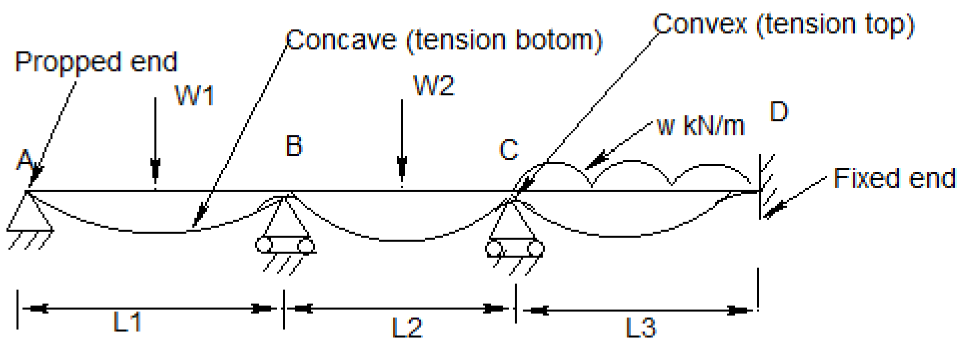


Figure 24.1 Typical continuous beam

This means hogging moments with tension at the top occur at the supports and sagging moments with tension at the bottom at the mid-span. If the ends were simply supported, then there would be no bending moment at these ends as shown in Figure 24.1 and the slope at the ends would not be zero. On the other hand if the ends were fixed, the slopes at these ends would be zero and fixed end moments would occur at these ends.

A continuous beam can be analysed by various methods. A more general method was proposed by Clapeyron. This is called Clapeyron's theorem of three moments.

In this method we develop an expression for the slope of the beam at a support, say, at B in Figure 24.1 in terms of both known and unknown bending moments, considering first span AB and then span BC as shown in Figure 24.1. Both the expressions thus developed represent one and the same aspect, namely, the slope of the beam at B. Hence they are equal and result in an equation connecting the values of the bending moments at A, B and C. If we apply in turn this methodology to each support over which the beam is continuous, we can get a sufficient number of equations to evaluate all the unknown bending moments, provided that the bending moment is zero at two extreme supports if they are pinned or the bending moments at two supports are known otherwise or can be determined from the conditions of the problem.

Once we determine the bending moment at all supports, the BMD and the SFD for the beam can be drawn.

24.2 Clapeyron's theorem of three moments

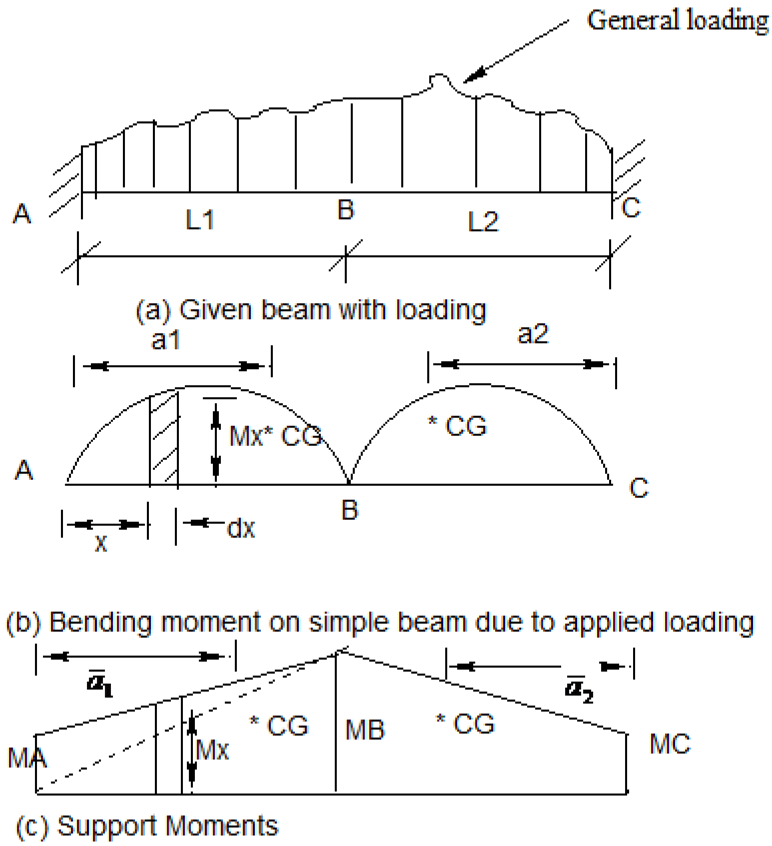


Figure 24.2 Derivation of the theorem

Let us consider a two span continuous beam as shown in Figure 24.2. on which a general loading acts. We assume EI to be constant throughout.

The span of AB is L_1 and that of BC is L_2. The BMD on a corresponding simple beam is shown in Figure 24.2 (b). The fixed end moments M_A, M_B and M_C are the support moments at A, B and C respectively (Fig 24.2 (c)). In Fig 24.2 (b), a_1 is the distance of the center of gravity (CG) of the area of the BMD for the span AB from the end A and a_2 is the corresponding quantity for the span BC from the end C. In Fig 24.2 (c), $\overline{a}_1$ is the distance of the CG of the area of the support moments for the span AB from the end A and $\overline{a}_2$ is the distance of the corresponding area for the span BC from the end C.

Let us first consider AB. The bending moment at any section distance x from A due to applied loading as shown in Figure 24.2 (b) is given by M_x. Therefore, the net moment at that section is given by M_x-M_x'. From the fundamental bending equation,

$$EI\frac{d^2y}{dx^2} = M_x - M_x^{'} \qquad \text{(a)}$$

Multiplying both sides of eq(a) by x

$$EIx\frac{d^2y}{dx^2} = xM_x - xM_x^{'} \qquad \text{(b)}$$

Integrating Eq. (b) from 0 to L_1, we obtain

$$\int_0^{L_1} EIx\frac{d^2y}{dx^2}dx = \int_0^{L_1} xM_x d_x - \int_0^{L_1} xM'_x d_x \qquad (c)$$

$$EIx\left[x\frac{dy}{dx} - y\right]_0^{L_1} = A_1a_1 - \overline{A}_1\overline{a}_1 \qquad (d)$$

Where A_1 is the area of the BMD for the span AB and is given by $\int_0^{L_1} M_x dx$ and A_1a_1 is the moment of area of BMD for the span AB about B and is given by $\int_0^{L_1} xM_x dx$. Similarly $\overline{A}_1\overline{a}_1$ holds good for support moments for the span AB and is equal to $\int_0^{L_1} xM_x dx$.

Substituting the limits on the left hand side of eq (d),

$$EI\left\{\left[L_1\left(\frac{dy}{dx}\right)_{atB} - y_B\right] - \left[0\times\left(\frac{dy}{dx}\right)_{atA} - y_A\right]\right\} = A_1a_1 - \overline{A}_1\overline{a}_1 \qquad (e)$$

$$EI\left[(L_1\theta_B - y_B) - (0 - y_A)\right] = A_1a_1 - \overline{A}_1\overline{a}_1 \qquad (f)$$

However deflections at supports A and B are zero. So $y_A = y_B = 0$. With these quantities, eq (f) becomes

$$EIL_1\theta_B = A_1a_1 - \overline{A}_1\overline{a}_1 \qquad (g)$$

Here $\overline{A}_1$ is the area of support moment diagram shown in Figure 24.2 (b) for span AB. It is also equal to the area of trapezium ABDE as shown in Figure 24.2 (c) i.e

$$\frac{1}{2}(M_A + M_B)\times L_1 \qquad (h)$$

And $\overline{a}_1$ is the distance of the CG of area ABDE from A and equal to

$$\frac{\frac{1}{2}M_BL_1\times\frac{2}{3}L_1 + \frac{1}{2}M_AL_1\times\frac{1}{3}L_1}{\frac{1}{2}M_BL_1 + \frac{1}{2}M_AL_1}$$

Obtained by dividing trapezium ABDE into two triangles shown in Figure 24.2 (b) which is equal to

$$\left(\frac{M_A + 2M_B}{M_A + M_B}\right)\frac{L_1}{3} \qquad (i)$$

From Eqs (h) and (i),

$$\overline{A}_1\overline{a}_1 = \frac{1}{2}(M_A + M_B)L_1\times\frac{(M_A + 2M_B)L_1}{(M_A + M_B)3} = \frac{1}{6}(M_A + 2M_B)L_1^2 \qquad (j)$$

Substituting Eq (i) into eq (g),

$$EIL_1\theta_B = A_1a_1 - \frac{L_1^2}{6}(M_A + 2M_B)$$

$$6EI\theta_B = 6A_1a_1/L_1 - L_1(M_A + 2M_B) \qquad (k)$$

Similarly, we can consider the span BC and take C as the origin, with x positive to the left and it can be derived that

$$6EI(-\theta_B) = \frac{6A_2a_2}{L_2} - L_2(M_C + 2M_B) \tag{l}$$

As the directions of x from A in the span AB and from C in the span BC are of the opposite nature, the value of the slope at B, θ_B will have the opposite sign for the span BC. Therefore Eq(1) becomes

$$-6EI\theta_B = \frac{6A_2a_2}{L_2} - L_2(M_C + 2M_B) \tag{m}$$

Addition of Eqs (k) and (m) yields

$$0 = \frac{6A_1a_1}{L_2} - L_1(M_A + 2M_B) + \frac{6A_2a_2}{L_2} - L_2(M_C + 2M_B)$$

$$= \frac{6A_1a_1}{L_2} + \frac{6A_2a_2}{L_2} - L_1M_A - 2L_1M_B - L_2M_C - 2L_2M_B$$

$$L_1M_A + 2L_1M_B + L_2M_C + 2L_2M_B = \frac{6A_1a_1}{L_2} + \frac{6A_2a_2}{L_2}$$

$$M_AL_1 + 2M_B(L_1 + L_2) + M_CL_2 = \frac{6A_1a_1}{L_2} + \frac{6A_2a_2}{L_2} \tag{24.1}$$

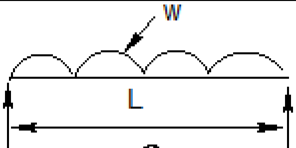
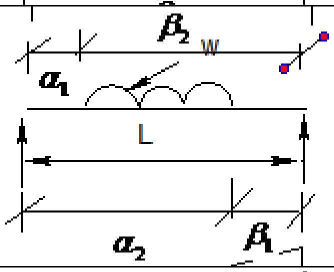
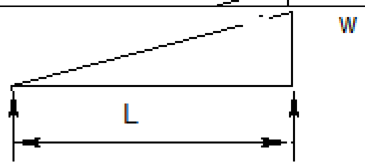
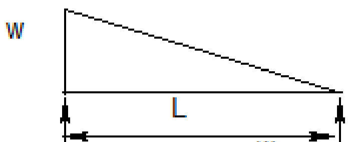
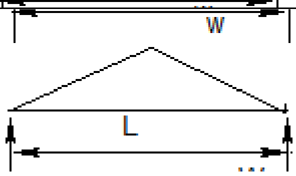
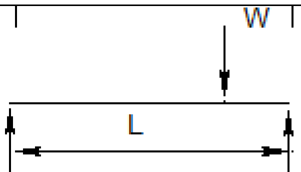
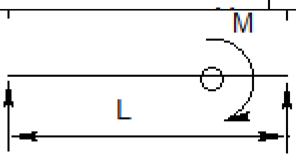
Equation (24.1) is the Clapeyron's theorem of three moments which can be used to solve problems on continuous beams of varying span lengths, different moments of inertia and subjected to different types of loading. The three moment equation expresses the relation between the bending moments at three successive supports of a continuous beam subjected to a certain applied loading on different spans.

Lecture 25 Application of three moment theorem to **continuous** and other indeterminate beams

25.1 Introduction

In this lecture, some example problems will be discussed. The Table 25.1 shows the values of and for standard loading

Table 25.1 Values of $\left(\frac{6A_1a_1}{L}\right)$ and $\left(\frac{6A_2a_2}{L}\right)$ for standard loading

Sl No	Type of loading	$6A_1a_1/L$	$6A_2a_2/L$
1		$wL^3/4$	$wL^3/4$
2		$w/4L\left[\begin{matrix}\alpha_2^2(2L^2-\alpha_2^2)\\-\alpha_1^2(2L^2-\alpha_1^2)\end{matrix}\right]$	$w/4L\left[\begin{matrix}\beta_2^2(2L^2-\beta_2^2)\\-\beta_1^2(2L^2-\beta_1^2)\end{matrix}\right]$
3		$(8/60)wL^3$	$(7/60)wL^3$
4		$(7/60)wL^3$	$(8/60)wL^3$
5		$(5/32)wL^3$	$(5/32)wL^3$
6		$(w(L-\alpha)/L)\left[L^2-(L-\alpha)^2\right]$	$(W\alpha/L)(L^2-\alpha^2)$
7		$-(M/L)\left[3(L-\alpha)^2-L^2\right]$	$+(M/L)\left[3\alpha^2-L^2\right]$

Example 1:

Analyse the beam as shown in Figure 25.1 below with constant EI using Clapeyron's three moment equation.

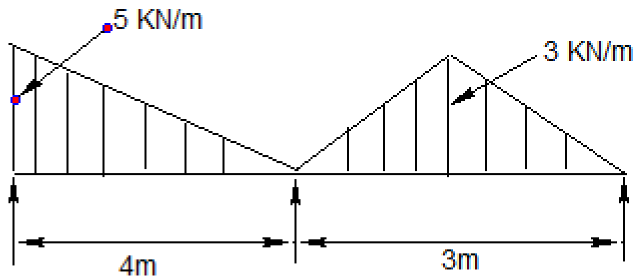


Figure 25.1 Two span beam

Solution:

The continuous beam is shown in Fig. 25.1. We denote the left support as A, middle support as B, and right support as C. MA = MC = 0 because the supports are simple. From Table 25.1,

For the left span $\frac{6A\bar{a}}{L} = \frac{7}{60} \times 5 \times 4^3 = 37.33 KNm^2$

For the right span, $\frac{6A\bar{a}}{L} = \frac{5}{32} \times 3 \times 3^3 = 12.66 KNm^2$

From the expression $M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6A_1 a_1}{L_2} + \frac{6A_2 a_2}{L_2}$, we have

$$2M_B(4+3) = 37.33 + 12.66$$
$$M_B = 3.57 KNm$$

Example 2:

The moment of inertia of s continuous beam is different for different spans as shown in Figure 25.2 . Find the reactions.

Solution:

The beam with M.I. different for different spans is shown in Fig. 25.2. From left to right supports are denoted as A, B, and C, respectively.

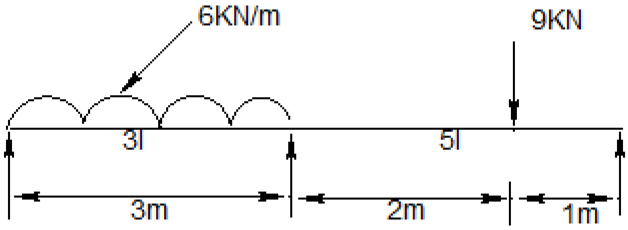


Figure 25.2 Varying MI of beam

$$\frac{6A\bar{a}}{I_1L_1} = \frac{6\times 3^3}{3I\times 3} = \frac{18}{I}$$
$$\frac{6A\bar{a}}{I_2L_2} = \frac{9\times 1}{5I\times 3}\left(3^2 - 1^2\right) = \frac{4.8}{I}$$
$$2M_B\left(\frac{3}{3I}+\frac{3}{5I}\right) = \frac{18}{I}+\frac{4.8}{I}$$

$M_B = 14.25KNm$. The computed reactions are shown in Figure 25.3.

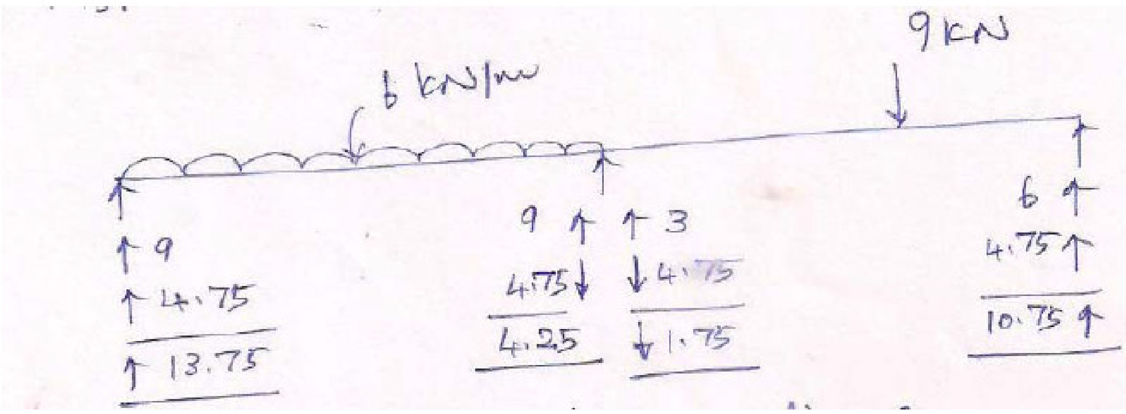


Figure 25.3 Computed Reactions

Example 3: Draw the SFD and BMD for the continuous beam shown in Figure 25.4.

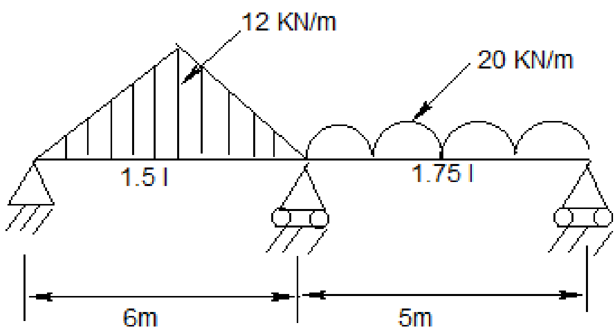


Figure 25.4 Varying load on span

Solution:

The two span beam is shown in Fig. 25.4. We denote the supports as A, B, and C.

From Table 25.1, we have for both spans

$$\frac{6A\overline{a}}{I_1L_1} = \frac{5\times 12\times 6^3}{1.5I\times 32} = \frac{270}{I}$$

$$\frac{6A\overline{a}}{I_2L_2} = \frac{20\times 5^3}{1.75I\times 5} = \frac{285.71}{I}$$

From the expression

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6A_1a_1}{L_2} + \frac{6A_2a_2}{L_2}$$

$$2M_B\left(\frac{6}{1.5I} + \frac{5}{1.75I}\right) = \frac{270}{I} + \frac{285.71}{I}$$

$M_B = 40.53KNm$

The reactions have been calculated in Figure 25.5.

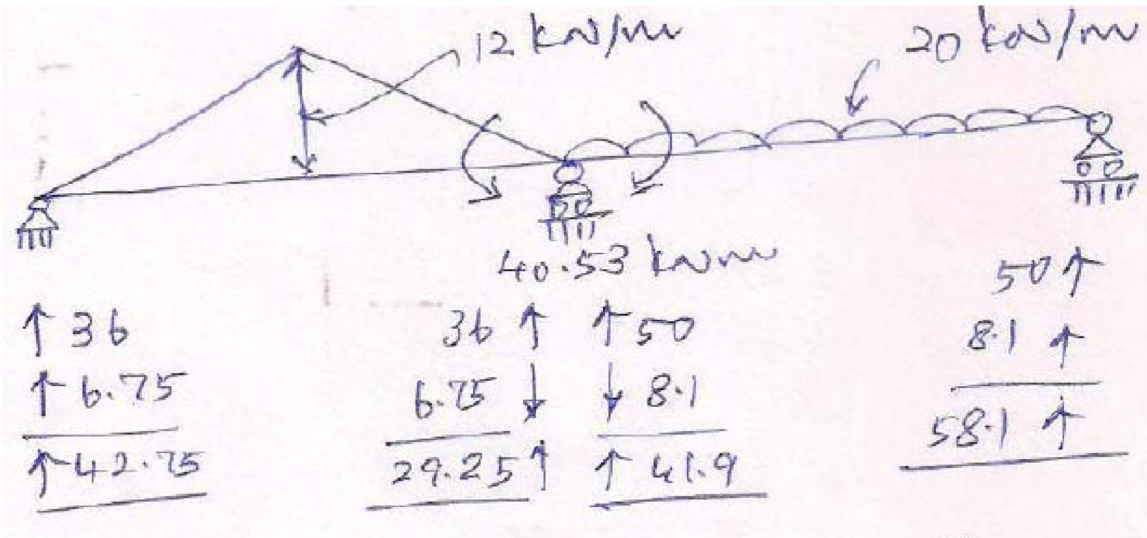


Figure 25.5 (a) Computed reactions

The SFD is shown in Figure 25.5 (b) and the BMD is shown in Figure 25.5 (c).

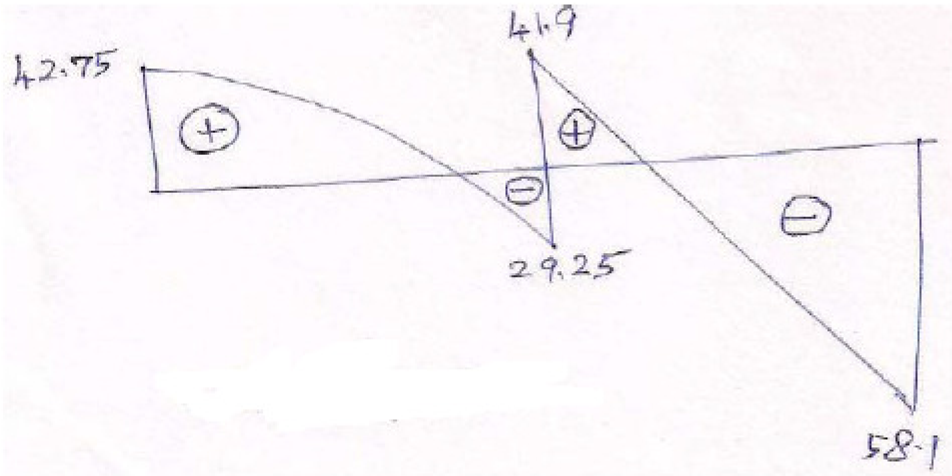


Figure 25.5 (b) SFD

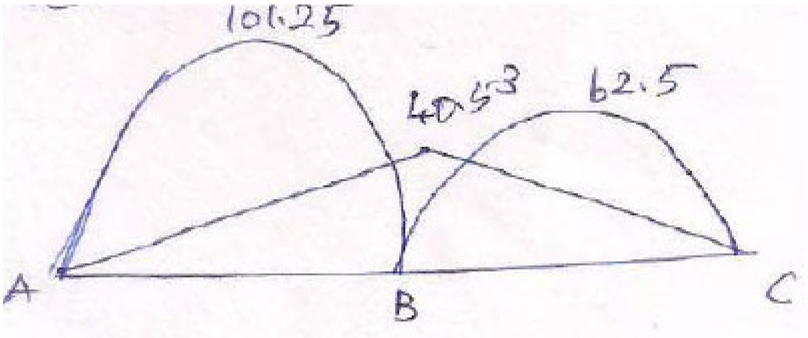


Figure 25.5 c BMD

Lecture 26 Application of three moment theorem to continuous and **other indeterminate beams**

In this Lecture, some example problems on other indeterminate beams will be discussed.

Example 1: Using Clapeyron's theorem, solve the problem of the continuous beam as shown in Figure 26.1. EI is constant throughout.

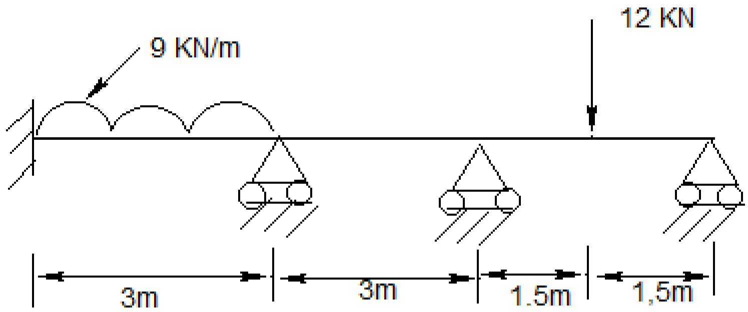


Figure 26.1 One end fixed beam

Solution:

The three span continuous beam is shown in Fig. 26.1. The left end of the beam is fixed. Now we insert an imaginary span to the left of A as shown in Fig. 26.1(a).

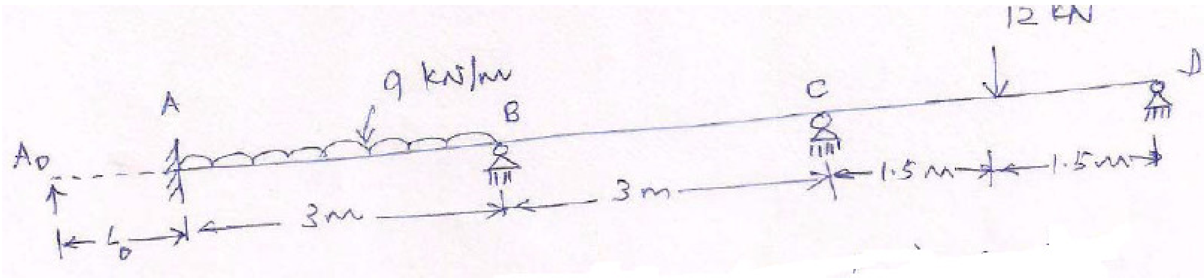


Figure 26.1 (a) Imaginary span L_o inserted

The imaginary span is A_0A, $M_{Ao} = 0$

From Table 25.1 for span AB,

$$\frac{6A\bar{a}}{L} = \frac{9\times 3^3}{4} = 60.75 KNm^2$$

From the expression

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6A_1 a_1}{L_2} + \frac{6A_2 a_2}{L_2}$$

$$2M_A(0+3) + M_B \times 3 = 60.75$$

$$6M_A + 3M_B = 60.75$$

$2M_A + M_B = 20.25$ (a)

Again from expression $M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6A_1 a_1}{L_2} + \frac{6A_2 a_2}{L_2}$

$M_A \times 3 + 2M_B(3+3) + M_C \times 3 = 60.75$

$M_A + 4M_B + M_C = 20.25$ (b)

From Table 25.1 for span CD, $\frac{6A\bar{a}}{L} = \frac{12 \times 15}{3}(3^2 - 1.5^2) = 40.5KNm^2$

$M_D = 0$

From expression $M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 = \frac{6A_1 a_1}{L_2} + \frac{6A_2 a_2}{L_2}$

$M_B \times 3 + 2M_C(3+3) = 40.5$

$M_B + 4M_C = 13.5KNm^2$ (c)

Solving (a), (b) and (c), we get M_A = 9.09 kNm, M_B = 2.08 kNm, M_C = 2.86 kNm.

Problem 2: A continuous beam has overhangs on both sides as shown in Figure 26.2 below. Apply three moment equation to determine the support moments. EI is constant throughout.

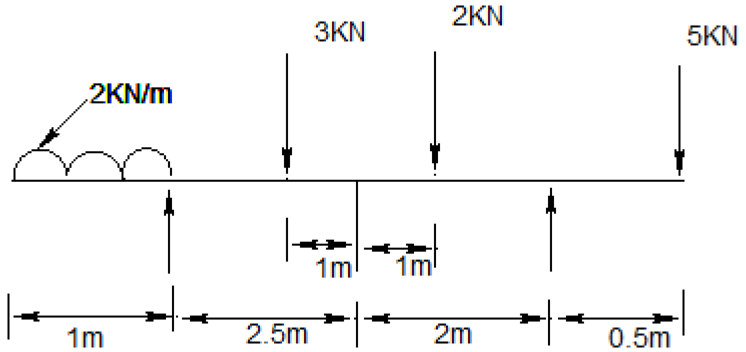


Figure 26.2 Beam with double overhangs

Solution:

The continuous beam with double overhangs is shown in Fig. 26.2. We denote the left support as A, middle support B and right support C. The UDL on left overhang will transfer a moment = 2 × 1 × 0.5 = 1 kNm to the left support. Similarly, the concentrated load on right overhang will transfer a moment = 5 × 0.5 = 2.5 kNm to the right support. For the loaded span from Table 25.1,

$$\frac{6A\bar{a}}{L} = \frac{3 \times 1.5 \times (2.5^2 - 1^2)}{2.5} = 9.45KNm^2$$

$$\frac{6A\overline{a}}{L}=\frac{2\times1\times\left(2^{1}-1^{2}\right)}{2}=3KNm^{2}$$

From expression $M_{A}L_{1}+2M_{B}\left(L_{1}+L_{2}\right)+M_{C}L_{2}=\frac{6A_{1}a_{1}}{L_{2}}+\frac{6A_{2}a_{2}}{L_{2}}$

$$1\times2.5+M_{B}\left(2.5+2\right)+2.5\times2=9.45+3$$

$$M_{B}=1.1KNm$$

Lecture 27 ILD for determinate structures for **reactions at supports**, S.F. at given section, B.M. at a given section, Maximum shear and maximum bending moment at given section, Problems relating to series of wheel loads, UDL less than or greater than the span of the beam, Absolute Maximum bending moment.

Moving loads caused by trains must be considered when designing the members of this bridge. The influence lines for the members become an important part of the structural analysis.

27.1 Introduction

In the previous lectures we developed techniques for analyzing the forces in structural members due to dead or fixed loads. It was shown that the shear and moment diagrams represent the most descriptive methods for displaying the variation of these loads in a member. If a structure is subjected to a live or moving load, however, the variation of the shear and bending moment in the member is best described using the influence line. An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a concentrated force moves over the member. Once this line is constructed, one can tell at a glance where the moving load should be placed on the structure so that it creates the greatest influence at the specified point. Furthermore, the magnitude of the associated reaction, shear, moment, or deflection at the point can then be calculated from the ordinates of the influence-line diagram. For these reasons, **influence lines** play an important part in the design of bridges, industrial crane rails, conveyors, and other structures where loads move across their span.

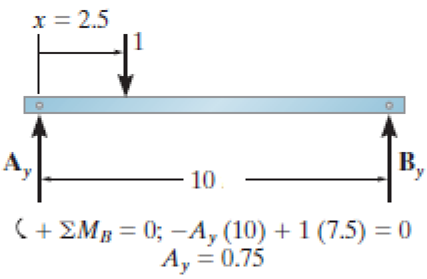
Although the procedure for constructing an influence line is rather basic, one should clearly be aware of the difference between constructing an influence line and constructing a shear or moment diagram. Influence lines represent the effect of a moving load only at a specified point on a member, whereas shear and moment diagrams represent the effect of fixed loads at all points along the axis of the member.

Problem 1: Construct the influence line for the vertical reaction at A of the beam in Fig. 27.1 (a)

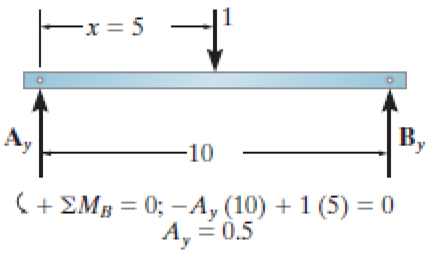
(a)

Solution:

Tabulate Values. A unit load is placed on the beam at each selected point x and the value of A_y is calculated by summing moments about B. For example, when x=2.5 m and x=5m and see Figs. 27–1b and 27–1c, respectively. The results for A_y are entered in the table, Fig. 27–1d.A plot of these values yields the influence line for the reaction at A, Fig. 27–1e.



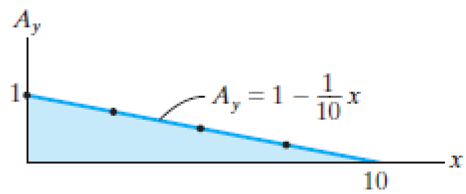
(b)



(c)

x	A_y
0	1
2.5	0.75
5	0.5
7.5	0.25
10	0

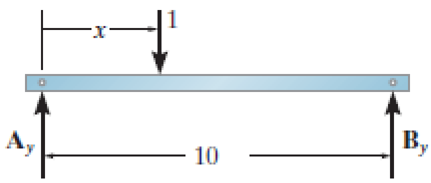
(d)



influence line for A_y

(e)

Influence-Line Equation. When the unit load is placed a variable distance *x* from *A*, Fig. 27–1*f*, the reaction A_y as a function of *x* can be determined from



(f)

$\circlearrowright + \; M_B = 0; \quad -A_y(10) + (10 - x)(1) = 0$

$A_y = 1 - \frac{1}{10}x$

This line is plotted in Fig. 27–1e.

Problem 1: Draw the influence line for support reaction at B of an overhanging beam as shown in Figure 27.2. and determine the maximum reaction at B when a single load of 30 KN rolls across the span.

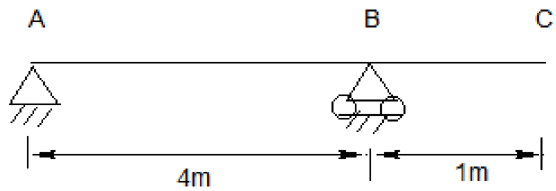


Figure 27.2 Overhang beam

Solution:

The overhang beam is shown in Fig. 27.2. The influence line for reaction R_B is shown in Fig. 27.2 (a). The reaction R_B is maximum when the load is at C. $R_B = 30 \times 1.25 = 37.5$ kN.

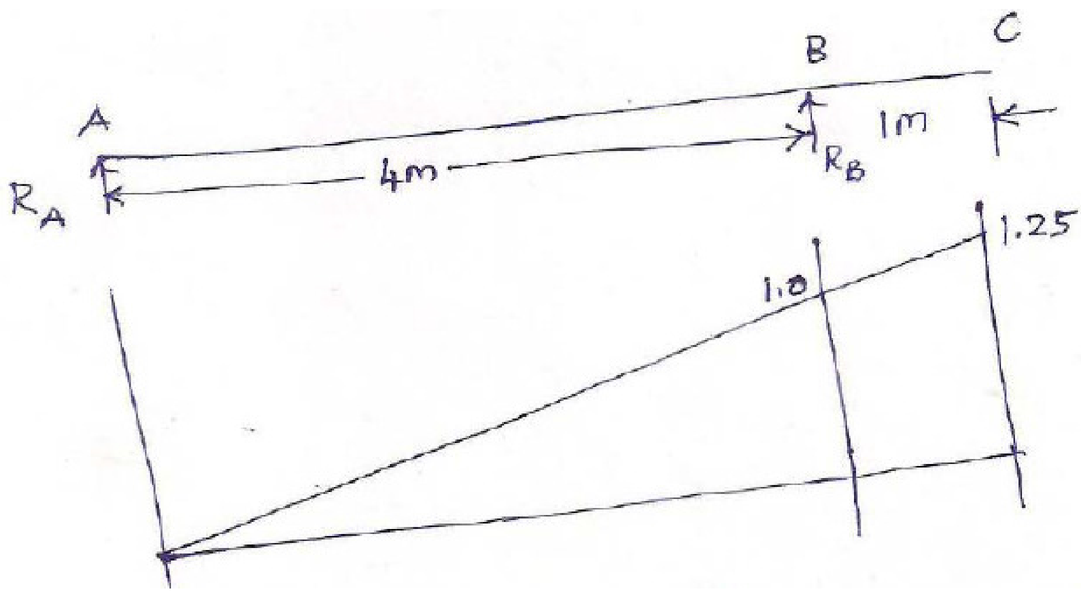


Figure 27.2 Influence line for reaction R_B

Lecture 28 ILD for determinate structures for reactions at supports, **S.F. at given section**, B.M. at a given section, Maximum shear and maximum bending moment at given section, Problems relating to series of wheel loads, UDL less than or greater than the span of the beam, Absolute Maximum bending moment.

Problem 1:

Construct the influence line for the shear at point *C* of the beam in Fig. 28–1*a*.

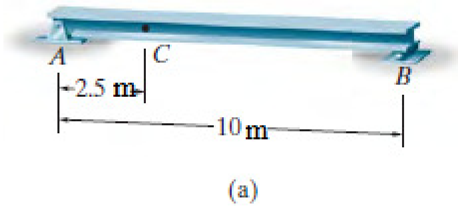


Figure 28.1 a

Solution:

Tabulate Values. At each selected position x of the unit load, the method of sections is used to calculate the value of Note in particular that the unit load must be placed just to the left (x=2.5 m) and just to the right (x=2.5m) of point C since the shear is discontinuous at C, Figs. 28.1 b and 28.1 c. A plot of the values in Fig. 28.1 d yields the influence line for the shear at C, Fig. 28.1 e.

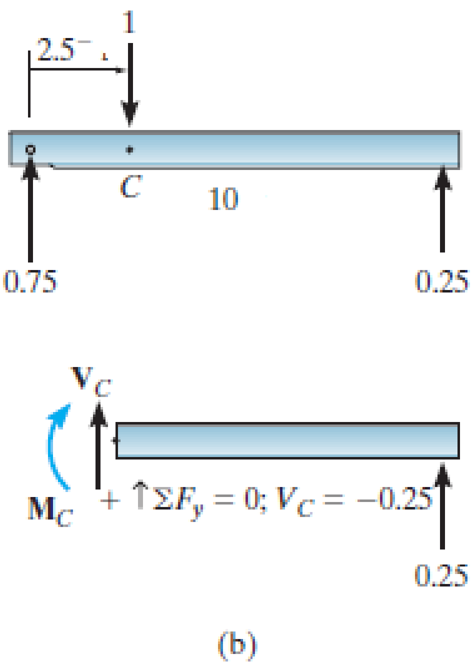
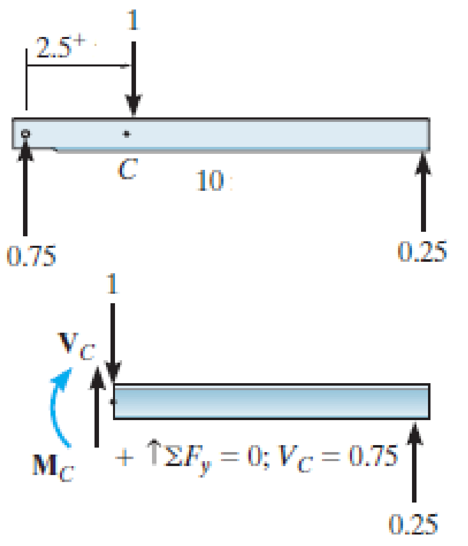


Figure 28.1 b

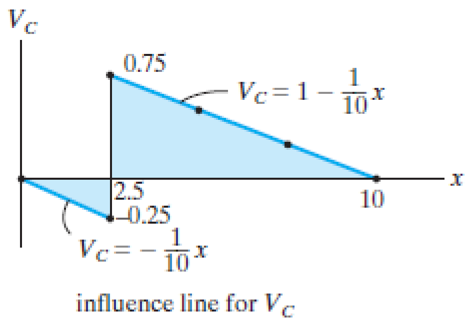


(c)

Figure 28.1 c

x	V_C
0	0
2.5^-	-0.25
2.5^+	0.75
5	0.5
7.5	0.25
10	0

(d)



(e)

Figure 28.1 d and e

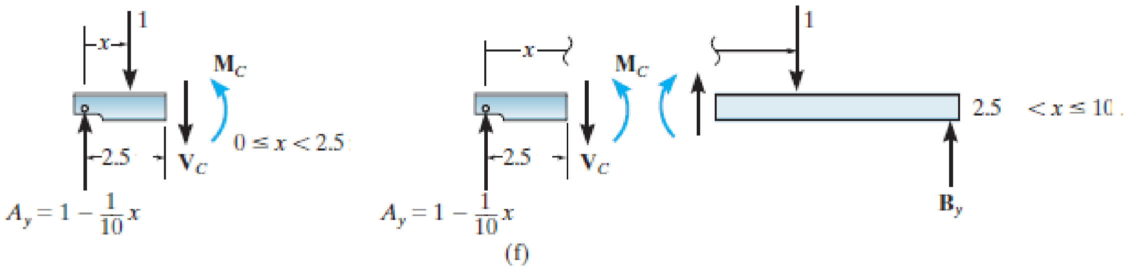


Figure 28.1 f

Influence-Line Equations. Here two equations have to be determined since there are two segments for the influence line due to the discontinuity of shear at C, Fig. 28.1 f. These equations are plotted in Fig. 28.1 e.

Lecture 29 ILD for determinate structures for reactions at supports, S.F. at given section, **B.M. at a given section**, Maximum shear and maximum bending moment at given section, Problems relating to series of wheel loads, UDL less than or greater than the span of the beam, Absolute Maximum bending moment.

Example 1:

Construct the influence line for the moment at point C of the beam in Fig. 29.1 a.

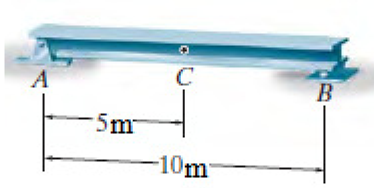


Figure 29.1 a)

Solution:

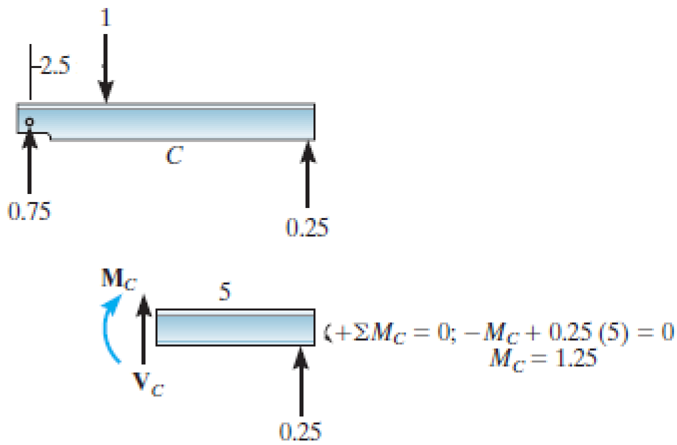


Figure 29.1 (b)

x	M_C
0	0
2.5	1.25
5	2.5
7.5	1.25
10	0

Figure 29.1 (c)

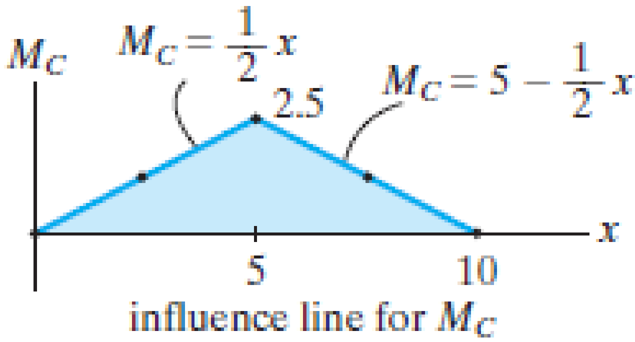


Figure 29.1 (d)

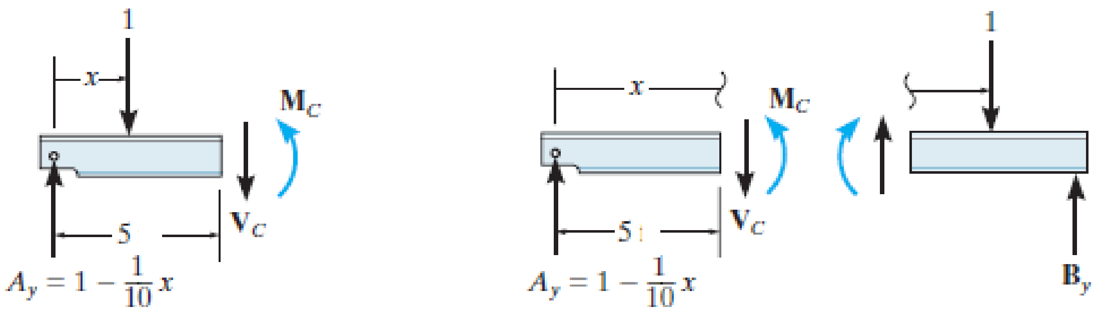


Figure 29.1 (e)

Influence-Line Equations. The two line segments for the influence line can be determined using $\sum M_C = 0$ along with the method of sections shown in Fig. 29.1 e. These equations when plotted yield the influence line shown in Fig. 29.1 d.

$$\circlearrowleft + \sum M_C = 0 \quad M_C + 1(5 - x) - \left(1 - \frac{1}{10}x\right)5 = 0$$

$$M_C = \frac{1}{2}x \quad 0 \leq x \prec 5m$$

$$\circlearrowleft + \sum M_C = 0 \quad M_C - \left(1 - \frac{1}{10}x\right)5 = 0$$

$$M_C = 5 - \frac{1}{2}x \qquad 5m \prec x \leq 10m$$

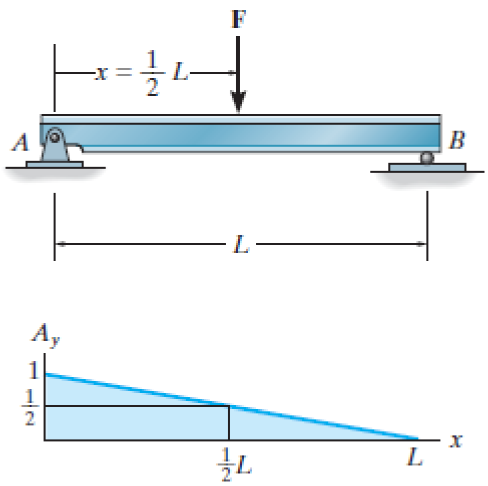
Lecture 30 ILD for determinate structures for reactions at supports, S.F. at given section, B.M. at a given section, **Maximum shear and maximum bending moment at given section**, Problems relating to series of wheel loads, UDL less than or greater than the span of the beam, Absolute Maximum bending moment.

30.1 Influence lines for beams

Since beams (or girders) often form the main load-carrying elements of a floor system or bridge deck, it is important to be able to construct the influence lines for the reactions, shear, or moment at any specified point in a beam.

Loadings. Once the influence line for a function (reaction, shear, or moment) has been constructed, it will then be possible to position the live loads on the beam which will produce the maximum value of the function. Two types of loadings will now be considered.

Concentrated Force. Since the numerical values of a function for an influence line are determined using a dimensionless unit load, then for any concentrated force F acting on the beam at any position x, the value of the function can be found by multiplying the ordinate of the influence line at the position x by the magnitude of F. For example, consider the influence line for the reaction at A for the beam AB, Fig. 30.1. If the unit load is at x=L/2. the reaction at A is A_e = ½ as indicated from the influence line. Hence, if the force F KN is at this same point, the reaction is A_y = 1F/2 KN. Of course, this same value can also be determined by statics. Obviously, the maximum influence caused by F occurs when it is placed on the beam at the same location as the peak of the influence line— in this case at x=0 where the reaction would be A_y = (1)F KN.



influence line for A_y

Figure 30.1

Uniform Load. Consider a portion of a beam subjected to a uniform load w_o . Figure 30.2 . As shown, each dx segment of this load creates a concentrated force of dF=w_o dx on the beam. If dF is located at x, where the beam's influence-line ordinate for some function (reaction, shear, moment) is y, then the value of the function is dF y = w_o dx y The effect of all the concentrated forces dF is determined by integrating over the entire length of the beam, that is, $\int w_o y dx = w_o \int y dx$. Also, since $\int y dx$ is equivalent to the area under the influence line, then, in general, the value of a function caused by a uniform distributed load is simply the area under the influence line for the function multiplied by the intensity of the uniform load. For example, in the case of a uniformly loaded beam shown in Fig. 30.3, the reaction A_y can be determined from the influence line as $A_y = area \times w_o = \frac{1}{2} \times 1 \times L \times w_o = \frac{1}{2} w_o L$ This value can of course also be determined from statics.

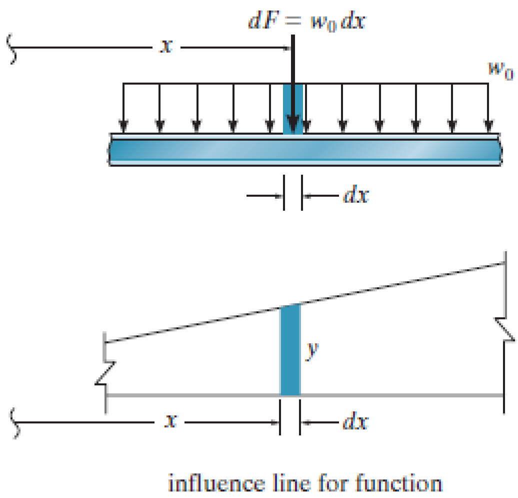


Figure 30.2

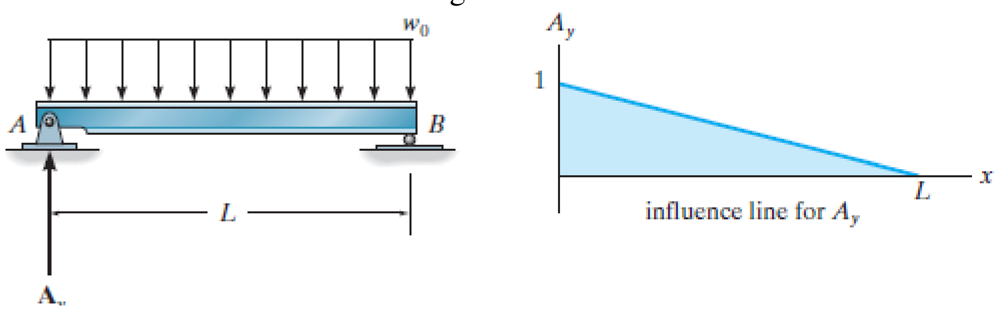


Figure 30.3

Example 1:

Calculate the maximum negative and positive SF and maximum BM at a section 3 m from left support in a simple beam of 8 m when a UDL of 13 KN/m for a length of 2 m rolls across the beam.

Solution:

The beam is shown in Fig. 30.1 (a). Let the section C be situated at a distance of 3 m from left support, i.e., a = 3 m. To get the maximum negative SF we position the head of the load at C as in Fig. 30.1 (b). In this position of the load, the ordinate of the negative SF at C is (3/8) = 0.375. From similar triangles, the ordinate at tail end of the load is 0.125. The maximum negative SF is given by the product of the area of the trapezoidal influence diagram and the intensity of load.

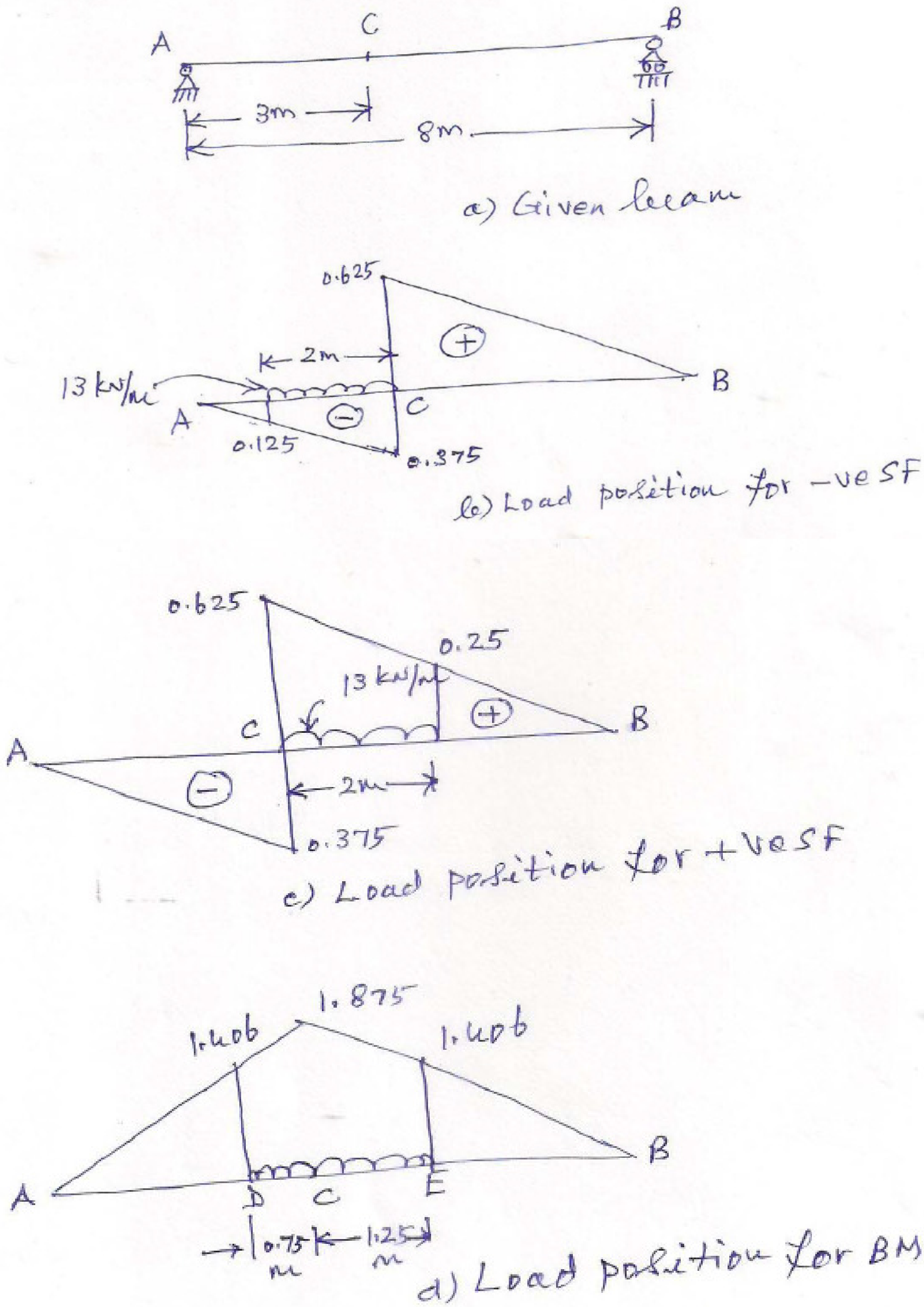


Figure 30.1

$$V_{-\max} = -13 \times \left(\frac{0.125 + 0.375}{2} \right) \times 2 = -6.5KN$$

We can get the maximum positive SF by placing the tail of the load at C as in Fig. 30.1 (c). The ordinate of the positive SF is (5/8) = 0.625. The ordinate at the head of the load is obtained from similar triangle as 0.25. The maximum positive SF is given by

$$V_{+\max} = 13 \times \left(\frac{0.625 + 0.25}{2} \right) \times 2 = 11.375 KN$$

We can obtain the maximum BM by placing the load about C and as per Fig. 30.1(d). From the relationship

$$\frac{AC}{CB} = \frac{DC}{CE}$$

$$\frac{3}{5} = \frac{DC}{(2 - DC)}$$

Solving DC = 0.75 and CE = 1.25 m

The peak ordinate at C is (3×5)/8 = 1.875. From similar triangles we can get the ordinates at the ends of load as 1.406 and 1.406, respectively as in Fig. 30.1 (d).

The maximum BM is

$$M_{\max} = 13 \left[\frac{(1.406 + 1.875)}{2} \times 0.75 + \frac{(1.406 + 1.875)}{2} \times 1.25 \right] = 42.653 KNm$$

Lecture 31 ILD for determinate structures for reactions at supports, S.F. at given section, B.M. at a given section, Maximum shear and maximum bending moment at given section, **Problems relating to series of wheel loads**, UDL less than or greater than the span of the beam, Absolute Maximum bending moment.

As the train passes over this girder bridge the engine and its cars will exert vertical reactions on the girder. These along with the dead load of the bridge must be considered for design.

31.1 Maximum Influence at a Point due to a Series of Concentrated Loads

Once the influence line of a function has been established for a point in a structure, the maximum effect caused by a live concentrated force is determined by multiplying the peak ordinate of the influence line by the magnitude of the force. In some cases, however, several concentrated forces must be placed on the structure. An example would be the wheel loadings of a truck or train. In order to determine the maximum effect in this case, either a trial-and-error procedure can be used or a method that is based on the change in the function that takes place as the load is moved. Each of these methods will now be explained specifically as it applies to shear and moment.

Example 1:

Determine the maximum shear at a point 6 m from left support of a simple beam of 15 m span when the loading in Figure 31.1 rolls across the beam

Solution:

We use influence line diagram for SF to determine its maximum value. The heavier load on the
section would yield the maximum SF. When the load moves from right to left we place the load ① of 75 kN on C. The Position 1 of loading is shown in Fig. 31.1 (a)(i). The Influence Line diagram (ILD) is shown in Fig. 31.1 (a)(ii). For this Position 1 the SF is

$$V_{\max} = 75\times 0.6 + 50\times 0.43 + 100\times 0.3 + 140\times 0.03 = 59.9KN$$

We now place the load ③ of 100 kN on C. This is shown in Fig. 31.1 (a)(iii). The corresponding ILD is shown in Fig. 31.1 (a)(iv). For this Position 3 of loading SF is

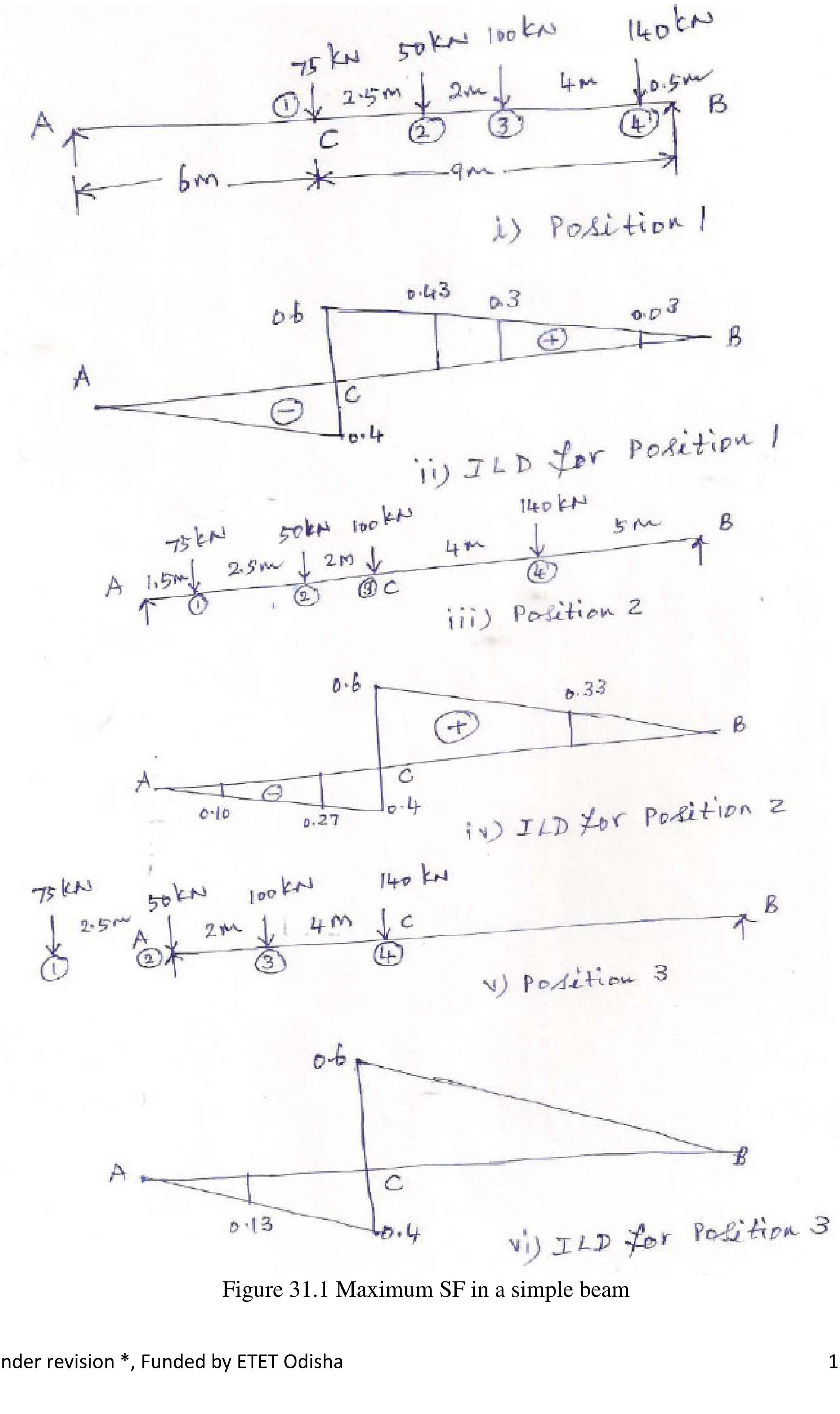


Figure 31.1 Maximum SF in a simple beam

$$V_{max} = -75\times0.6 - 50\times0.27 + 100\times0.6 + 140\times0.33 = 85.2KN$$

If we move the loads to the left by an infinitesimal amount we get

$$V_{max} = -75\times0.1 - 50\times0.27 - 100\times0.4 + 140\times0.33 = -14.8KN$$

A slight change in position of loads causes the SF to change its sign. This means the maximum SF is 85.2 kN.

When the load ④ of 140 kN on C as in Fig. 31.1 (a)(v). This is called Position 3. The corresponding ILD is shown in Fig. 31.1 (a)(vi). For this Position 3 the SF is

$$V_{max} = -100\times0.13 - 140\times0.4 = -69KN$$

Hence, Position 2 gives the absolute maximum SF.

Lecture 32 ILD for determinate structures for reactions at supports, S.F. at given section, B.M. at a given section, Maximum shear and maximum bending moment at given section, Problems relating to series of wheel loads, **UDL less than or greater than the span of the beam**, Absolute Maximum bending moment.

Example 1:

Construct the influence line for BM at a section 2.5 m from left support of a simple beam of span of 6m. Determine the maximum BM when a UDL of 10 KN/m longer than the span moves across the beam.

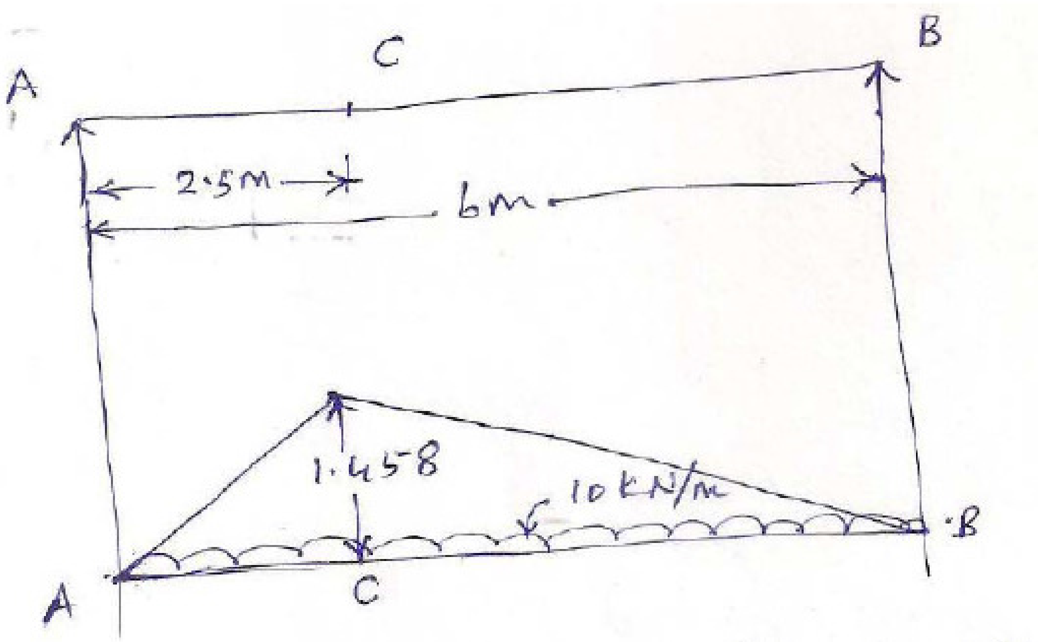


Figure 32.1 Influence line for BM at section 2.5 m

Solution:

The influence line for BM at a section 2.5 m from left support is shown in Fig. 32.1. The value of BM under the section is

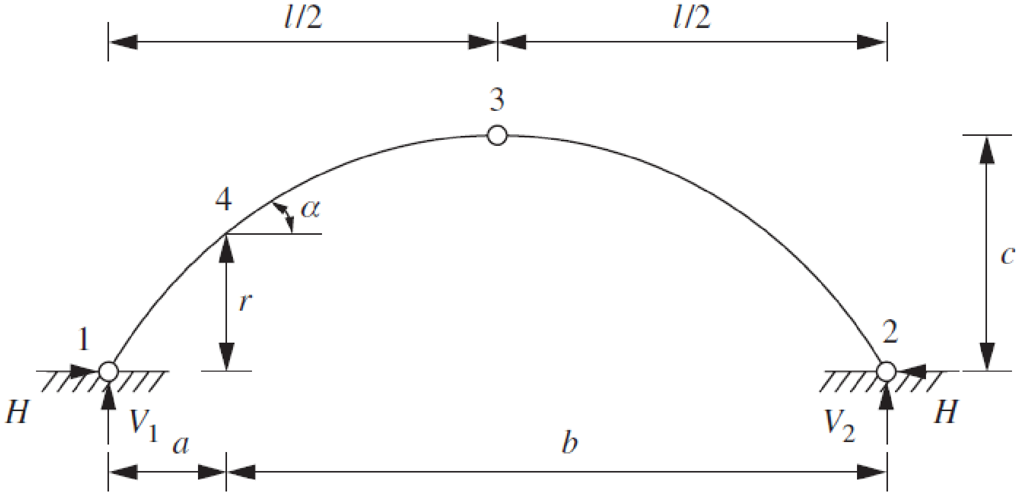
$$M_x = \frac{2.5 \times 3.5}{6} = 1.458$$

The travelling load consists of UDL longer than the span. The maximum BM is obtained when the UDL occupies the entire span.

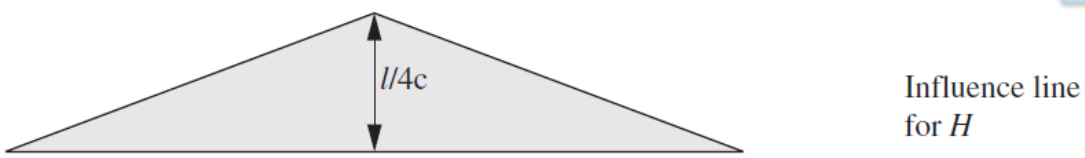
$$M_{\max} = \frac{1}{2} \times 1.458 \times 6 \times 10 = 46.44 KNm$$

Lecture 33 ILD for **B.M., S.F.**, normal thrust and radial shear of a three hinged arch.

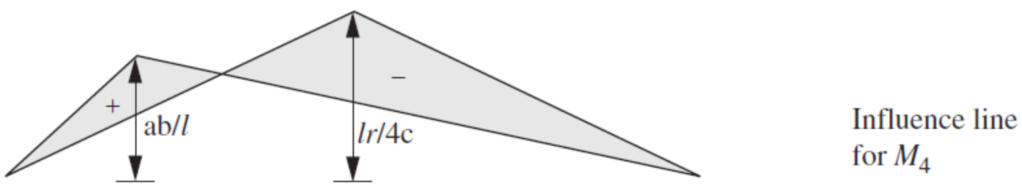
33.1 Three Hinged Arch



(a)



(b)



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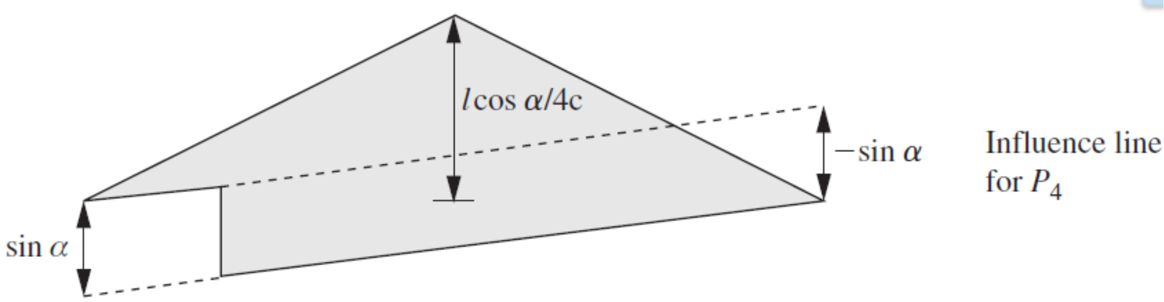
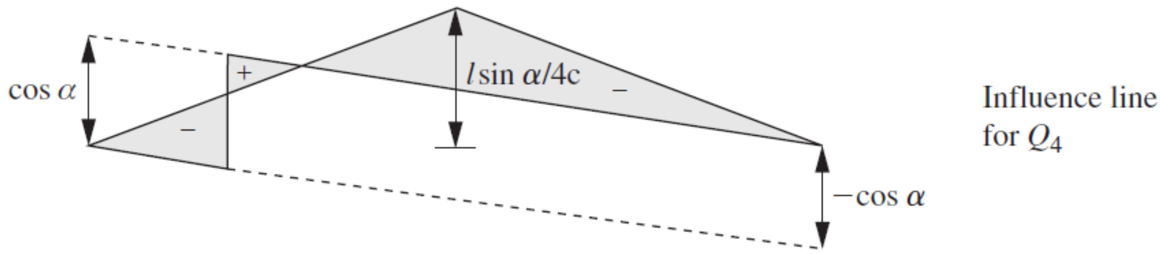


(d)



(e)
Figure 33.1 Three hinged arches

The horizontal thrust at the springings of a three-hinged arch is equal to the bending moment at the center of a simply supported beam of the same Influence lines 121 length multiplied by 1/c. Hence, the influence line for horizontal thrust is the influence line for the free bending moment multiplied by 1/ c and is shown in Figure 33.1 (b)

The influence line for bending moment at point 4 is the influence line for free bending moment at 4 minus the horizontal thrust multiplied by r and is given by:

$$M_4 = (M_s)_4 - Hr$$

The influence line is shown at (iii).
The influence line for thrust at point 4 is given by:

$$P_4 = H\cos\alpha - V_2\sin\alpha \ldots \text{ unit load from 1 to 4}$$
$$= H\cos\alpha + V_2\sin\alpha \ldots \text{ unit load from 4 to 2}$$

The influence line is shown at (iv).
The influence line for shear at point 4 is given by:

$$Q_4 = -H\sin\alpha - V_2\cos\alpha \ldots \text{ unit load from 1 to 4}$$
$$= -H\sin\alpha + V_1\cos\alpha \ldots \text{ unit load from 4 to 2}$$

The influence line is shown at (v).

Lecture 34 ILD for B.M., S.F., **normal thrust and radial shear** of a three hinged arch.

Example 34.1:

Find the maximum horizontal thrust at a section 10 m from left support in a three hinged arch of span 50 m and rise 8 m when a concentrated load of 70 KN rolls across the arch.

Solution:

The three-hinged arch is similar to that shown in Fig. 33.1 (a). Here rise h = 8 m and span L = 50 m. The magnitude of the rolling load is 70 kN.

From Fig. 33.1 and the expression H=PL/4h, the maximum horizontal thrust is

$$H = \frac{70 \times 50}{4 \times 8} = 109.375 KN$$

The given section is at 10 m from the left support. We assume that the arch is parabolic. With the origin at left support we can write the equation of the parabola using generalized parameters for span and rise

$$y = \frac{4h}{L^2} \times (L - x)$$

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{L^2}(L - 2x)$$

Substituting values,

$$\tan\theta = \frac{4 \times 8}{50^2}(50 - 10) = 0.512$$

$$\theta = 29.34^o$$

The reaction at right support is

$$V_B = \frac{10 \times 70}{50} = 14 KN$$

When the load is on the left half of the arch, the normal thrust from Expression $N = H\cos\theta - V_B \sin\theta$ is

$$N = 109.375\cos 29.34^o - 14\sin 29.34^o = 88.485 KN$$

When the load is on the right half of the arch, the normal thrust from Expression $N = H\cos\theta + V_A \sin\theta$ is

$$N = 109.375\cos 29.34^o + 14\sin 29.34^o = 102.21 KN$$

Lecture 35 **Suspension cables**, three hinged stiffening girders

The deck of a cable-stayed bridge is supported by a series of cables attached at various points along the deck and pylons

35.1 Introduction

Cables are often used in engineering structures for support and to transmit loads from one member to another. When used to support suspension roofs, bridges, and trolley wheels, cables form the main load-carrying element in the structure. In the force analysis of such systems, the weight of the cable itself may be neglected; however, when cables are used as guys for radio antennas, electrical transmission lines, and derricks, the cable weight may become important and must be included in the structural analysis. Two cases will be considered in the sections that follow: a cable subjected to concentrated loads and a cable subjected to a distributed load. Provided these loadings are coplanar with the cable, the requirements for equilibrium are formulated in an identical manner.

When deriving the necessary relations between the force in the cable and its slope, we will make the assumption that the cable is perfectly flexible and inextensible. Due to its flexibility, the cable offers no resistance to shear or bending and, therefore, the force acting in the cable is always tangent to the cable at points along its length. Being inextensible, the cable has a constant length both before and after the load is applied. As a result, once the load is applied, the geometry of the cable remains fixed, and the cable or a segment of it can be treated as a rigid body.

35.2 Cable Subjected to Concentrated Loads

When a cable of negligible weight supports several concentrated loads, the cable takes the form of several straight-line segments, each of which is subjected to a constant tensile force. Consider, for example, the cable shown in Fig. 35–1. Here θ specifies the angle of the cable's cord AB, and L is the cable's span. If the distances L_1, L_2 and L_3 and the loads P_1 and P_2 are known, then the problem is to determine the nine unknowns consisting of the tension in each of the three segments, the four components of reaction at A and B, and the sags y_C and y_D. At the two points C and D. For the solution we can write two equations of force equilibrium at each of points A, B, C, and D. This results in a total of eight equations. To complete the solution, it will be necessary to know something about the geometry of the cable in order to obtain the necessary ninth equation. For example, if the cable's total length $\mathscr{L}$ is specified, then the Pythagorean theorem can be used to relate $\mathscr{L}$ to each of the three segmental lengths, written in terms of θ, y_C, y_D, L_1 and L_2 and L_3. Unfortunately, this type of problem cannot be solved easily by hand. Another possibility, however, is to specify one of the sags, either y_C and y_D. instead of the cable length. By doing this, the equilibrium equations are then sufficient for obtaining the unknown forces and the remaining sag. Once the sag at each point of loading is obtained, $\mathscr{L}$ can then be determined by trigonometry.

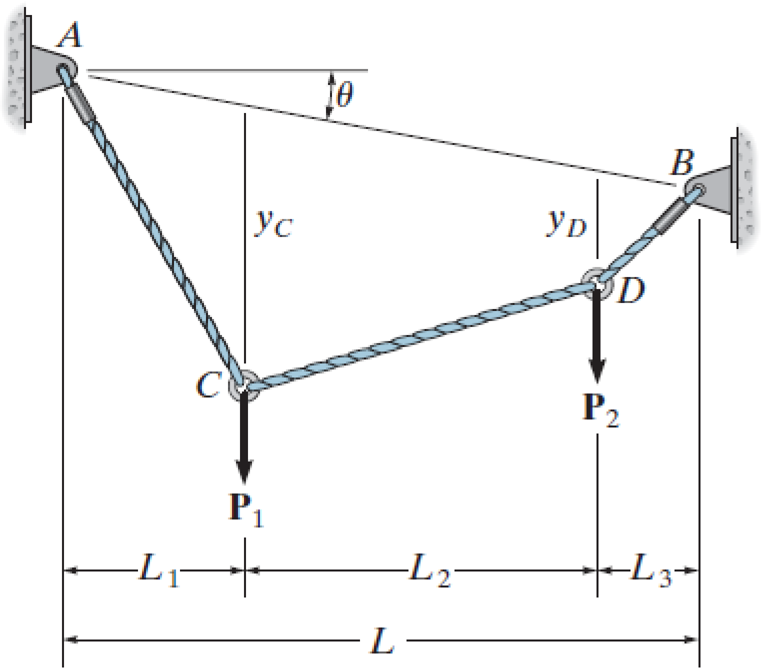


Figure 35.1

When performing an equilibrium analysis for a problem of this type, the forces in the cable can also be obtained by writing the equations of equilibrium for the entire cable or any portion thereof.

35.3 Cable Subjected to Uniform distributed Loads

Cables provide a very effective means of supporting the dead weight of girders or bridge decks having very long spans. A suspension bridge is a typical example, in which the deck is suspended from the cable using a series of close and equally spaced hangers.

In order to analyze this problem, we will first determine the shape of a cable subjected to a uniform horizontally distributed vertical load w_o, Fig. 35–2a. Here the x,y axes have their origin located at the lowest point on the cable, such that the slope is zero at this point. The free-body diagram of a small segment of the cable having a length Δs is shown in Fig. 35.2 b. Since the tensile force in the cable changes continuously in both magnitude and direction along the cable's length, this change is denoted on the free-body diagram by ΔT. The distributed load is represented by its resultant force $w_o \Delta x$ which acts at $\Delta x/2$ from point O. Applying the equations of equilibrium yields

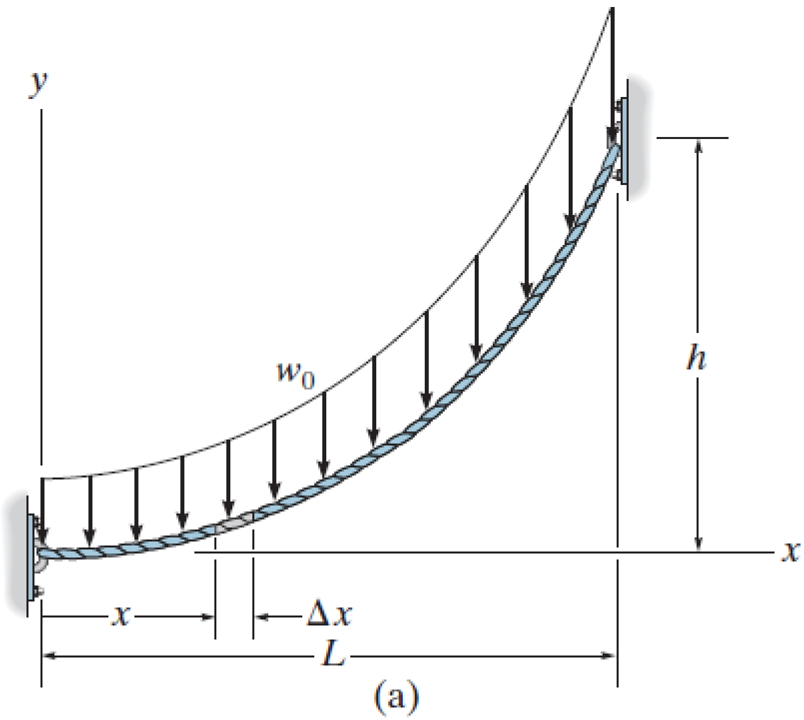


Figure 35.2 (a)

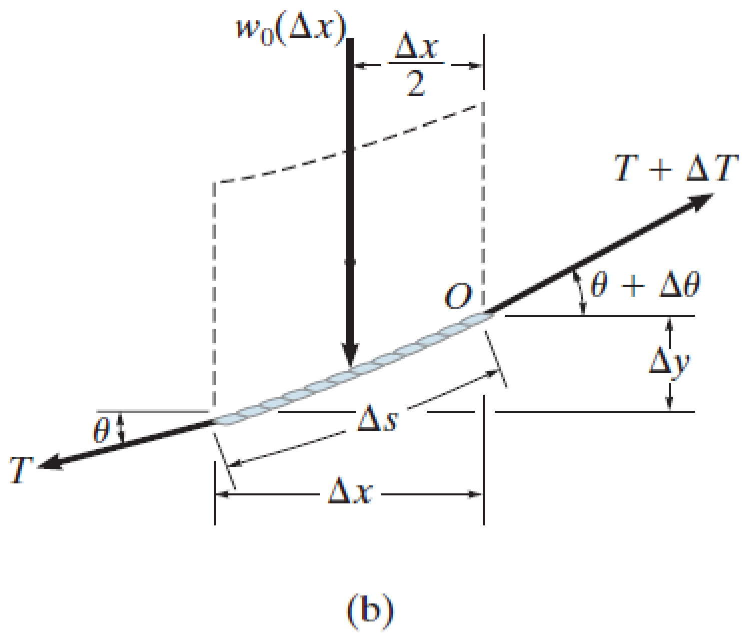


Figure 35.2 (b)

$$\sum F_x = 0 \quad -T\cos\theta + (T + \Delta T)\cos(\theta + \Delta\theta) = 0$$

$$\sum F_y = 0 \quad -T\sin\theta - w_o(\Delta x) + (T + \Delta T)\sin(\theta + \Delta\theta) = 0$$

$$\sum M_o = 0 \quad w_o(\Delta x)(\Delta x / 2) - T\cos\theta\Delta y + T\sin\theta\Delta x = 0$$

Dividing each of these equations by Δx and taking the limit as $\Delta x \to 0$ and hence $\Delta y \to 0$, $\Delta\theta \to 0$ and $\Delta T \to 0$, we obtain

$$\frac{d(T\cos\theta)}{dx} = 0 \qquad (35.1)$$

$$\frac{d(T\sin\theta)}{dx} = w_o \qquad (35.2)$$

$$\frac{dy}{dx} = \tan\theta \qquad (35.3)$$

Integrating Eq. 35–1, where T=F_H at x=0 we have:

$$T\cos\theta = F_H \qquad (35.4)$$

which indicates the horizontal component of force at any point along the cable remains constant.

Integrating Eq. 35–2, realizing that $T\sin\theta = 0$ at x=0 gives

$$T\sin\theta = w_o x \qquad (35.5)$$

Dividing Eq. 35–5 by Eq. 35–4 eliminates T. Then using Eq. 35–3, we can obtain the slope at any point,

$$\tan\theta = \frac{dy}{dx} = \frac{w_o x}{F_H} \qquad (35.6)$$

Performing a second integration with y=0 at x=0 yields

$$y = \frac{w_o}{2F_H}x^2 \qquad (35.7)$$

This is the equation of a parabola. The constant F_H may be obtained by using the boundary condition y=h at x=L. Thus

$$F_H = \frac{w_o L^2}{2h} \qquad (35.8)$$

Finally, substituting into Eq. 35–7 yields

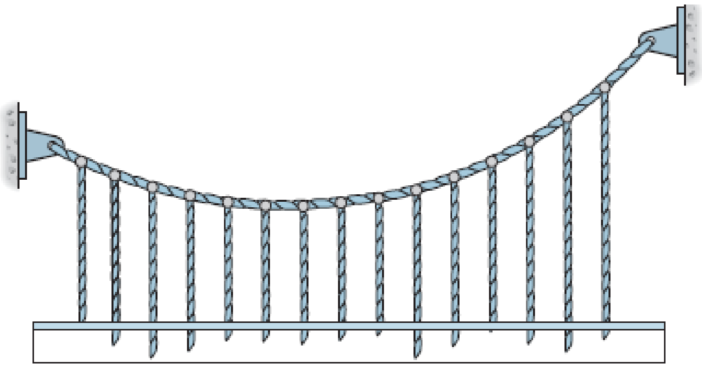
$$y = \frac{h}{L^2}x^2 \qquad (35.9)$$

From Eq. 35–4, the maximum tension in the cable occurs when θ is maximum; i.e., at x=L. Hence, from Eqs. 35–4 and 35–5,

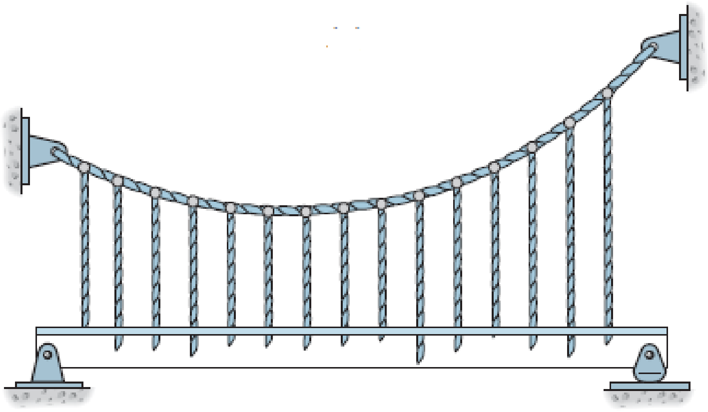
$$T_{\max} = \sqrt{F_H^2 + (w_o L)^2} \qquad (35.10)$$

Or, using Eq. 35–8, we can express T_{max} in terms of w_o i.e

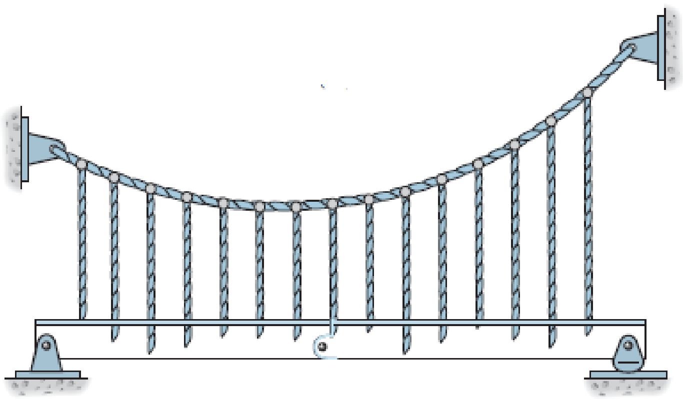
$$T_{\max} = w_o L\sqrt{1+(L/2h)^2} \tag{35.11}$$



(a)



(b)



(c)

Figure 35.3

Realize that we have neglected the weight of the cable, which is uniform along the length of the cable, and not along its horizontal projection. Actually, a cable subjected to its own weight and free of any other loads will take the form of a catenary curve. However, if the sag-to span ratio is small, which is the case for most structural applications, this curve closely approximates a parabolic shape, as determined here. From the results of this analysis, it follows that a cable will maintain a parabolic shape, provided the dead load of the deck for a suspension bridge or a suspended girder will be uniformly distributed over the horizontal projected length of the cable. Hence, if the girder in Fig. 35–3a is supported by a series of hangers, which are close and uniformly spaced, the load in each hanger must be the same so as to ensure that the cable has a parabolic shape.

Using this assumption, we can perform the structural analysis of the girder or any other framework which is freely suspended from the cable. In particular, if the girder is simply supported as well as supported by the cable, the analysis will be statically indeterminate to the first degree, Fig. 35–3b. However, if the girder has an internal pin at some intermediate point along its length, Fig.35–4c, then this would provide a condition of zero moment, and so a determinate structural analysis of the girder can be performed.

Lecture 36 **Suspension cables**, three hinged stiffening girder

Example 1:

A light cable of length 20 m is supported at two ends at the same level. The supports are at 16 m apart. The cable supports three loads of 10, 12 and 16 KN dividing the 16 m distance in four equal parts. Find the shape of the string and tension in various portions.

Solution:

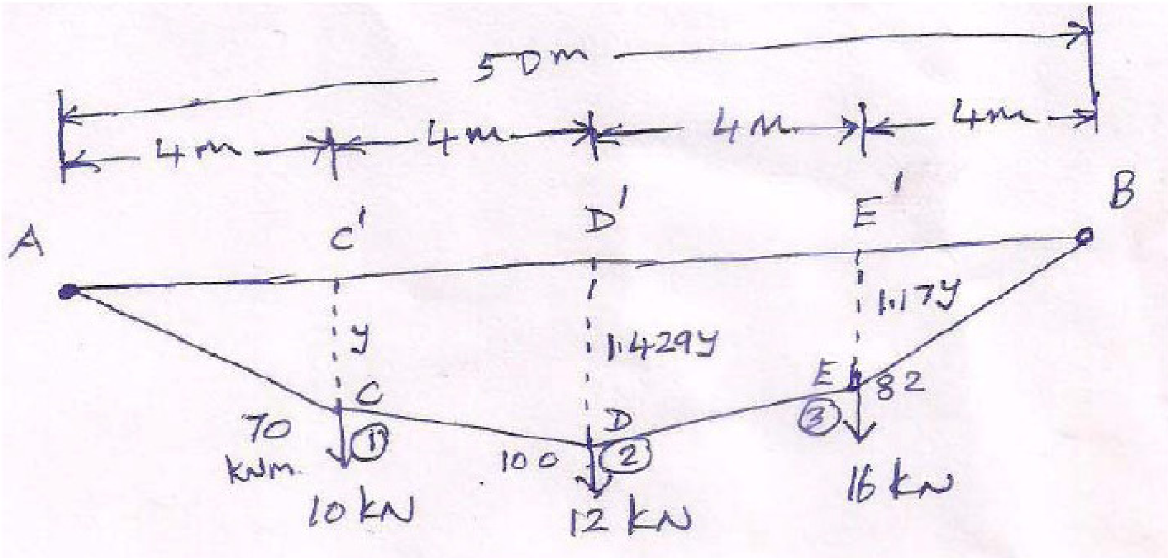


Figure 36.1 Cable with three loads

Naturally the shape of the cable is the BM diagram for these loads. Under the three concentrated loads the cable takes a profile shown in Fig. 36.1.

The vertical reaction at A

$$V_A = 10\times\frac{12}{16}+\frac{12}{2}+16\times\frac{4}{16}=7.5+6+4=17.5KN$$

We now calculate the BM at each joint.

$$M_C = 17.5\times 4 = 70KNm$$

$$M_D = 17.5\times 8 - 10\times 4 = 140-40 = 100KNm$$

$$M_E = 17.5\times 12 - 10\times 8 - 12\times 4 = 210-80-48 = 82KNm$$

The BMD is shown in Fig. 36.1. Let us assume that the ordinate CC' be y m. Then,

$$DD' = 100\times\frac{y}{70} = 1.429y$$

$EE^{'} = 82 \times \frac{y}{70} = 1.17y$

Now

Length AC = $\sqrt{4^2 + y^2}$

Length CD = $\sqrt{4^2 + (1.429y - y)^2} = \sqrt{4^2 + 0.184y^2}$

Length DE = $\sqrt{4^2 + (1.429y - 1.17y)^2} = \sqrt{4^2 + 0.067y^2}$

Length EB = $\sqrt{4^2 + (1.17y)^2} = \sqrt{4^2 + 1.3689y^2}$

Total length of the cable = AC+CD+DE+EB

$$l = 4\left[\left(1+\frac{y^2}{16}\right)^{1/2} + \left(1+\frac{0.184y^2}{16}\right)^{1/2} + \left(1+\frac{0.067y^2}{16}\right)^{1/2} + \left(1+\frac{1.3689y^2}{16}\right)^{1/2}\right]$$

As y is less than 4, we can approximate the length of the cable as

$$= 4\left[1+\frac{y^2}{2\times16}+1+\frac{0.184y^2}{2\times16}+1+\frac{0.067y^2}{2\times16}+1+\frac{1.3689y^2}{2\times16}\right]$$
$$= 4\left[4 + y^2\{0.03125 + 5.75\times10^{-3} + 2.09375\times10^{-3} + 0.0428\}\right]$$
$$= 4\left[4 + 0.0819y^2\right] = 16 + 0.3276y^2$$

When l=20

$\therefore 16 + 0.3276y^2 = 20$

Solving y=3.49 m

Now CC'=3.49 m

$DD' = 1.429 \times 3.49 = 4.99m$

$EE' = 1.17 \times 3.49 = 4.08m$

For joints (1), (2) and (3) we get

$T_o(\tan\theta_2 - \tan\theta_1) = 10$

$T_o(\tan\theta_3 - \tan\theta_2) = 12$

Adding all the ab $T_o(\tan\theta_4 - \tan\theta_3) = 16$ ove three equations we get

$T_o(\tan\theta_4 - \tan\theta_1) = 38$

Now $\tan\theta_1 = -\frac{3.49}{4} = -0.8725$ because it is clockwise

$$\tan\theta_4 = \frac{4.08}{4} = 1.02$$

Substituting in the above expression $T_o\{1.02-(-0.8725)\} = 38;\ T_o = 20.08KN$
Now

$$AC = \sqrt{4^2 + 3.49^2} = 5.31m$$
$$CD = \sqrt{4^2 + 0.184\times 3.49^2} = 4.27m$$
$$DE = \sqrt{4^2 + 0.067\times 3.49^2} = 4.1m$$
$$EB = \sqrt{4^2 + 1.3689\times 3.49^2} = 5.72m$$

$$\cos\theta_1 = \frac{AC'}{AC} = \frac{4}{5.31} = 0.753$$
$$\cos\theta_2 = \frac{C'D''}{CD} = \frac{4}{4.27} = 0.937$$
$$\cos\theta_3 = \frac{D'E''}{DE} = \frac{4}{4.1} = 0.976$$
$$\cos\theta_4 = \frac{E'B}{EB} = \frac{4}{5.72} = 0.7$$

$$T_1\cos\theta_1; T_1 = \frac{T_o}{\cos\theta_1} = \frac{20.08}{0.753} = 26.67KN$$
$$T_2\cos\theta_2; T_2 = \frac{T_o}{\cos\theta_2} = \frac{20.08}{0.937} = 21.43KN$$

$$T_3\cos\theta_3; T_3 = \frac{T_o}{\cos\theta_3} = \frac{20.08}{0.976} = 20.57KN$$
$$T_4\cos\theta_4; T_4 = \frac{T_o}{\cos\theta_4} = \frac{20.08}{0.7} = 28.69KN$$

Tension in segment AC, T_1 = 26.67 KN
Tension in segment CD, T_2=21.43KN
Tension in segment DE, T_3 = 20.57 KN
Tension in segment EB, T_4 = 26.67 KN

The shape of the cable is shown in Fig. 36.1.

Lecture 37 **Suspension cables**, three hinged stiffening girders

Example 1: A cable is supported at the same level between two points spanning a distance of 300 m. It carries UDL of 100 KN/m horizontally. If the central dip is 30 m , compute the maximum tension in the Cable.

Solution:

The thrust H is given by $H = \frac{wl^2}{8h}$

$$H = \frac{100 \times 300^2}{8 \times 30} = 37500 KN$$

The maximum tension T occurs at the supports of a cable. Therefore, the maximum tension in a uniformerly distributed cable is given by

$$T_{max} = H\sqrt{1 + \left(\frac{wL/2}{H}\right)^2}$$

$$T_{max} = 37500 \times \sqrt{1 + \left(\frac{100 \times 300}{2 \times 37500}\right)^2} = 40388.74 KN$$

Example 2: A suspension cable has a span of 250 m and its central dip is 25m. It is subjected to UDL of 3 KN per horizontal metre. Determine the maximum and minimum tension in the cable. Calculate the horizontal and vertical forces in each tower under the following two conditions a) when the cable passes over frictionless rollers on the top of the tower and b) when the cable is firmly clamped to saddles carried on frictionless rollers on the top of the tower. In each case, the anchor cable also called back stay is inclined at 30^o to the vertical.

Solution:

The tension in the cable is maximum at its ends and minimum at its lowest point. The minimum tension is equal to H.

$$H = \frac{3 \times 250^2}{8 \times 25} = 937.5 KN$$

Sag ratio r=25/250=0.1

$$T_{max} = 937.5\sqrt{1 + 16 \times 0.1^2} = 1009.72 KN$$

Case (a) When the cable passes over frictionless rollers on top of the tower

Under this circumstances the tension in the back stay = tension in the cable

From Figure 37.1 we have Vertical load on the tower

$$V_p = T(\cos\alpha_1 + \cos\alpha_2)$$

Here $T\cos\alpha_1 = V$, the vertical reaction at the left end.

i.e $V = (wL/2) = (3\times 250)/2 = 375KN$

α_2 is the angle of the back stay with vertical and = 30°

$V_P = 375 + 1009.72\cos 30 = 1249.44KN$

The horizontal shear is

$H_p = T(\sin\alpha_1 - \sin\alpha_2)$

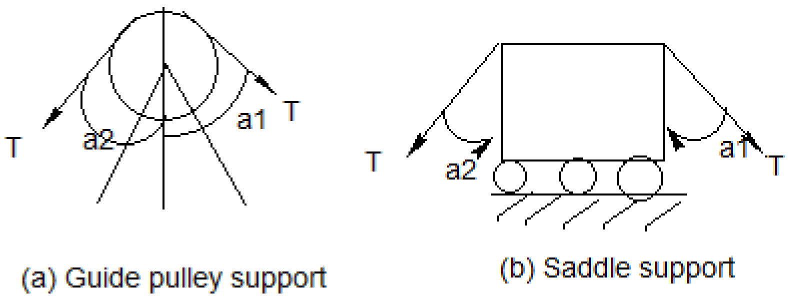


Figure 37.1

However, $T\sin\alpha_1 = H = 937.5KN$

$T = T_{max}\sin\alpha_2 \quad T = 1009.72\sin 30^o = 504.86KN$

$H_P = 937.5 - 504.86 = 432.64KN$

(b) When the cable is connected to the saddle

There is no horizontal shear on towers because rollers do not allow it. Hence, horizontal components are balanced.

$T_C\sin\alpha_1 = T_A\sin\alpha_2 = H = 937.5KN$

T_A = (937.5/sin30)=1875KN

We obtain the vertical force as

$V_P = T_C\cos\alpha_1 - T_A\cos\alpha_2$

However $T_C\cos\alpha_1 = 375KN$
$V_P = 375 + 1875\cos 30 = 1998.8KN$

Lecture 38 Suspension cables, **three hinged stiffening girders**.

38.1 Introduction

Since the cable of the suspension bridge is the main load bearing member, the curvature of the cable of an unstiffened bridge changes as the load moves on the decking. To avoid this, the decking is stiffened by provision of either a three hinged stiffening girder or a two hinged stiffening girder. The stiffening girder transfers a uniform or equal load to each suspender, irrespective of the position of the load on the decking.

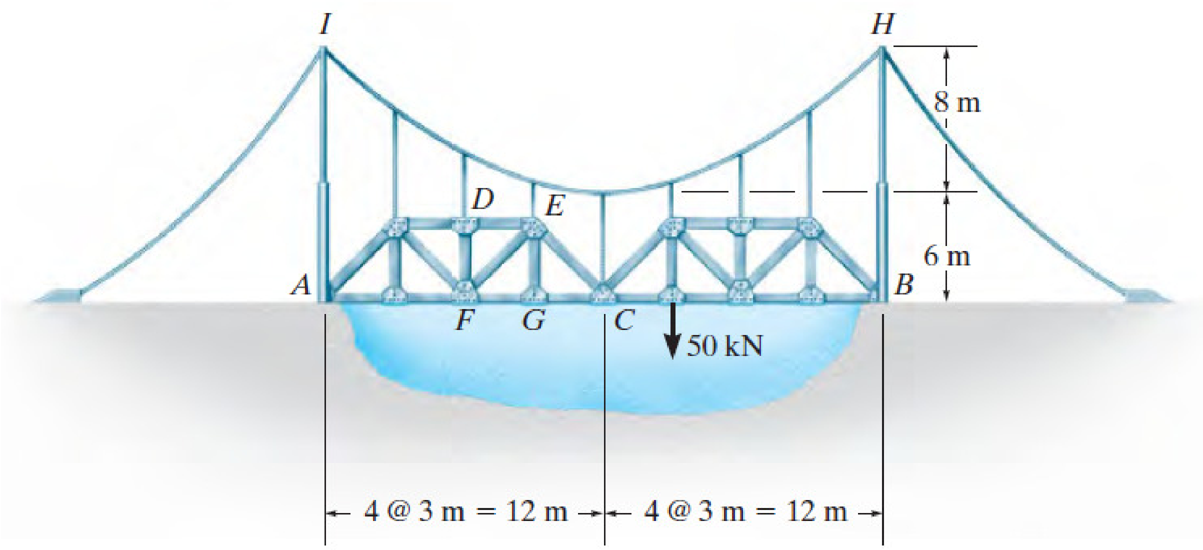
Example 1:

The suspension bridge in Fig. 38.1 a is constructed using the two stiffening trusses that are pin connected at their ends C and supported by a pin at A and a rocker at B. Determine the maximum tension in the cable IH. The cable has a parabolic shape and the bridge is subjected to the single load of 50 kN.

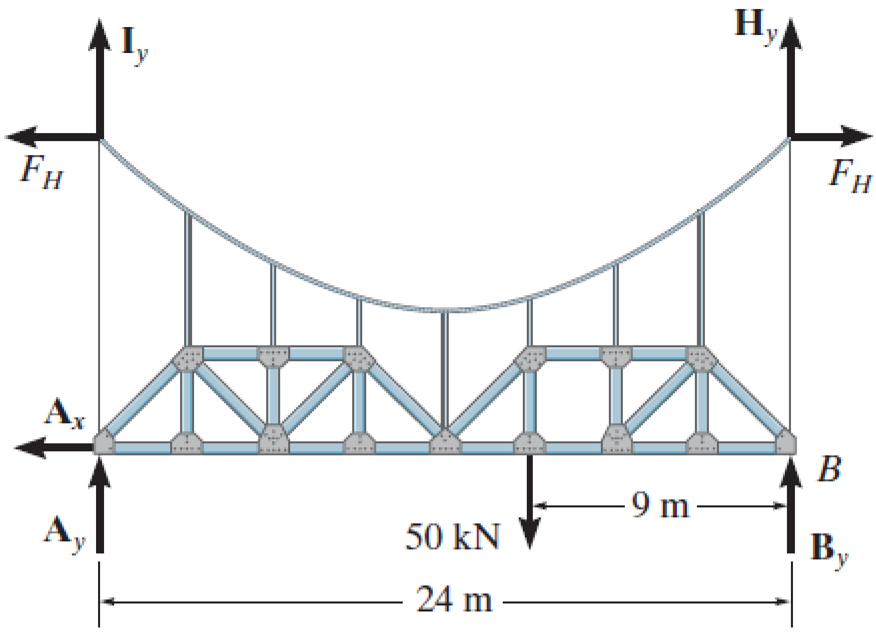
Solution:

The free-body diagram of the cable-truss system is shown in Fig. 38.1 b. As per the expression $T\cos\theta = F_H$, the horizontal component of cable tension at I and H must be constant, F_H, Taking moments about B, we have

$$-I_y(24m) - A_y(24m) + 50KN(9m) = 0$$
$$I_y + A_y = 18.75$$



(a)



(b)

Figure 38.1

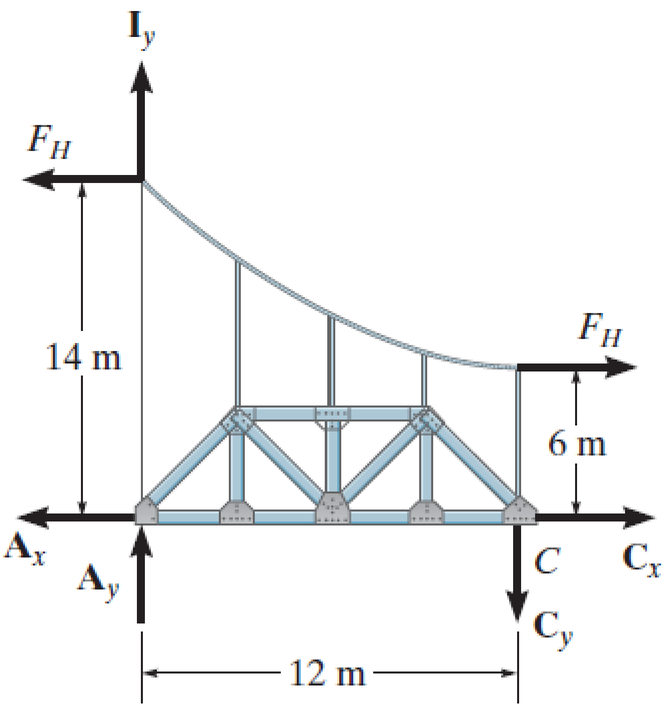


Figure 38.1 (c)

If only half the suspended structure is considered, Fig. 38.1 *c*, then summing moments about the pin at *C*, we have

$$F_H(14m) - F_H(6m) - I_y(12m) - A_y(12m) = 0$$

$$I_y + A_y = 0.667F_H$$

From these two equations,

$$18.75 = 0.667 F_H$$

$$F_H = 28.125 KN$$

the value of an assumed uniform distributed loading w_o is given by

$$w_o = \frac{2F_H h}{L^2} = \frac{2(28.125KN)(8m)}{(12m)^2} = 3.125KN/m$$

the maximum tension in the cable T_{max} is given by

$$T_{max} = w_o L\sqrt{1+(L/2h)^2} = 3.125(12m)\sqrt{1+(12m/2(8m))^2} = 46.9KN$$

Lecture 39 Suspension cables, **three hinged stiffening girders**.

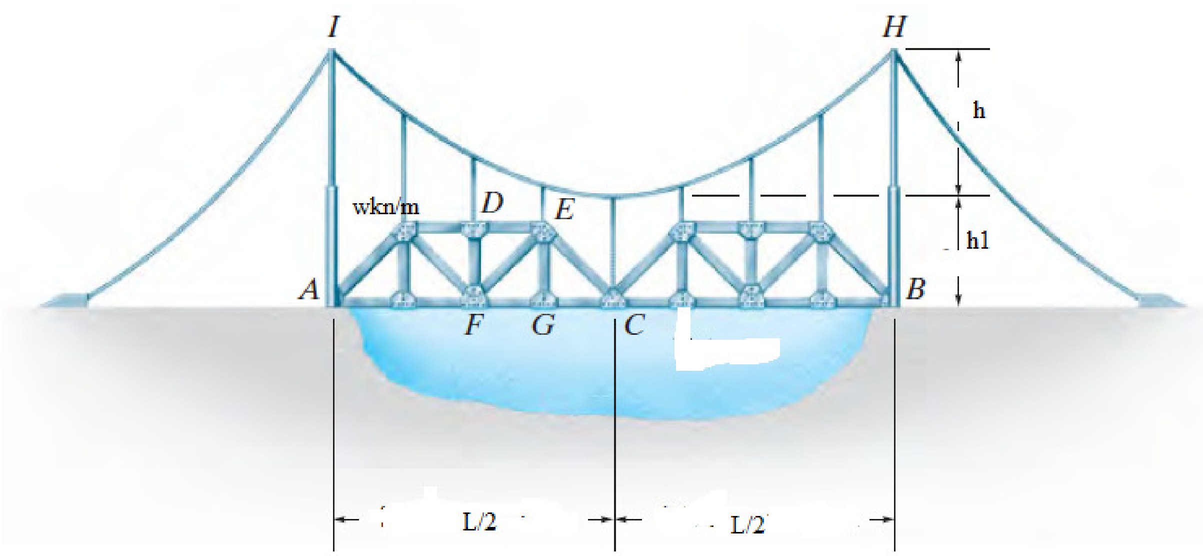
39.1 Influence Line Diagrams of stiffening girder

When the loads move over a suspension bridge, it is quite well known that because of its own characteristics, the shape of the cable gets distorted continuously. However, suspension bridges are meant for heavy traffic. Therefore, it is desirable that the roadway must be maintained at the same grade as far as possible for all conditions of road traffic. This means that the cable must remain in its original designed geometrical configuration. This objective is achieved by stiffening the cable with girders. The girders used to halt the distortion of the cable are called stiffening girders.

Example 1:

The span of a three hinged stiffening girder bridge is 350 m and its central dip is 40 m. A single load of 70 KN rolls along the bridge. Determine the horizontal thrust H at a section 60 m from the left support as well as when the load is at 40 m from the left support. Also find the maximum value of H. Find the maximum load w and the maximum positive moment under the load when it is 50 m from left support as well as the maximum shear force at the same section.

Solution:



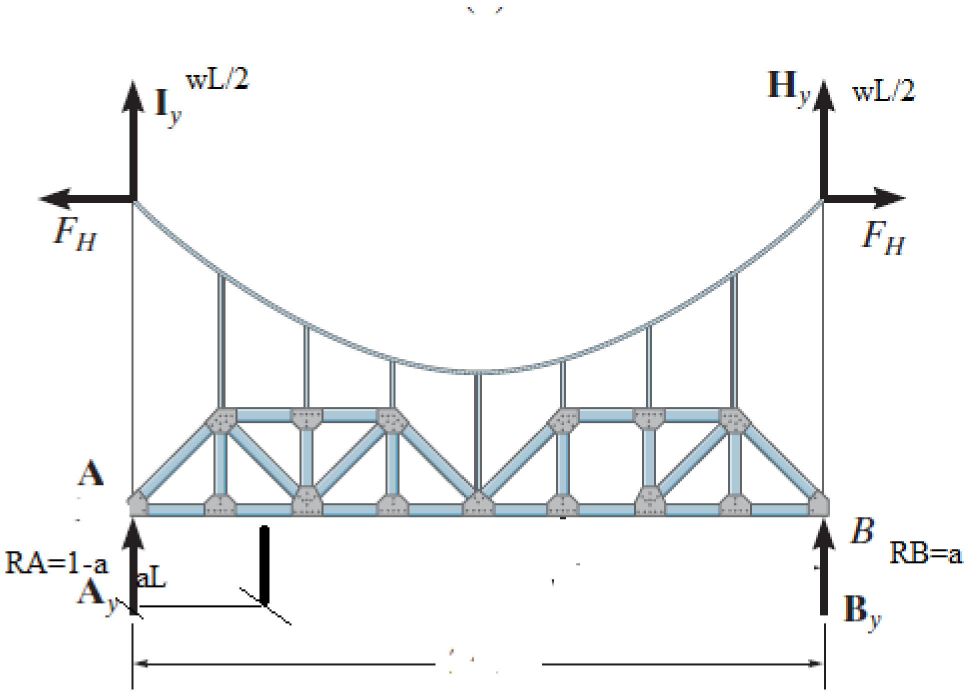


Figure 39.1 a) Three hinged stiffening girder

The ILD for H is shown in Fig. 39.1 (b). The maximum ordinate is at mid span.

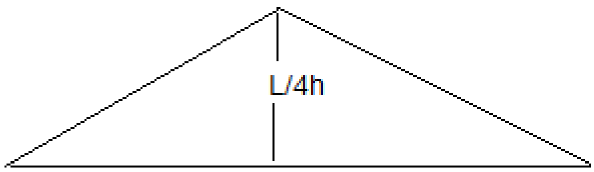


Figure 39.1 (b) Influence line for H

$$H_{max} = \frac{350}{4 \times 40} = 2.1875 KN$$

The horizontal reaction at 60 m from left support is obtained from similar triangles.

$$H_{60} = 70 \times \frac{2.1875}{175} \times 60 = 52.5 KN$$

The horizontal reaction at 40 m from left support where the load is situated is obtained from similar triangles.

$$H_{40} = 70 \times \frac{2.1875}{175} \times 40 = 35 KN$$

Maximum value of H = 2.1875 x 70 = 153.125 KN

From Figure 39.1 (c),

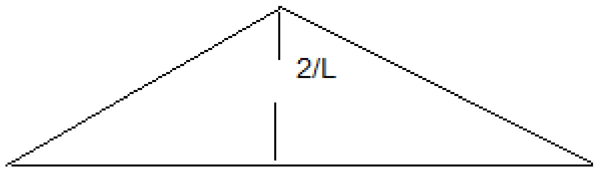


Figure 39.1 c Influence line for w

$$w_{max} = 70 \times \frac{2}{350} = 0.4 KN/m$$

The maximum positive BM under the load, i.e., at 50 m from the left support is

$$M_{max} = 70 \times \frac{50 \times (350 - 50) \times (350 - 2 \times 50)}{350^2} = 2142.86 KNm$$

Here the section is at 50 m from left support. It is < [(350/4) = 87.5 m]. Therefore from Fig. 39.1 (d), the maximum positive shear at the section is

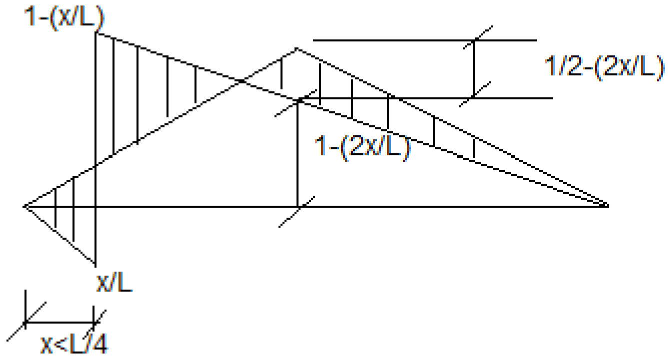


Figure 39.1 d Influence line for shear force x < L/4

$$F_{max} = 70\left(\frac{1}{2} - \frac{2 \times 50}{350}\right) = 15 KN$$

Lecture 40 Suspension cables, **three hinged stiffening girders**.

40.1 Introduction to Three hinged stiffening girder

Problem 1: The two parts of a three hinged stiffening girder of a suspension bridge are 200 m and 150 m long respectively. Find the position and magnitude of the maximum bending moment due to a UDL of w KN/m for 75 m on both sides of the central hinge.

Solution:

The two parts of the three-hinged stiffening girder is shown in Fig. 21.46. The position of the applied UDL w.r.t. the central hinge is also shown in Fig. 21.46.

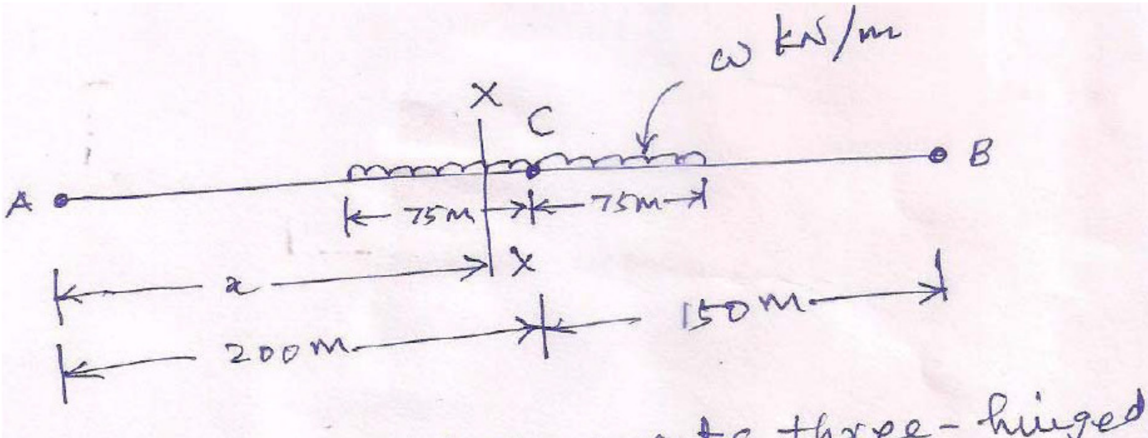


Fig. 21.46 Two parts three-hinged stiffening girder

The left reaction $V_{CA} = \dfrac{75w}{350}\left(150+\dfrac{75}{2}\right)+\dfrac{750w}{350}\left(150-\dfrac{75}{2}\right)=40.18w+24.11w=64.29w$

$V_B = 150w - 64.29w = 85.71w$

We assume that the maximum BM occurs at a section X from left support. The distance of section X is x.

$$M_x = 64.29wx - w\left[\{75-(200-x)\}\times\frac{1}{2}\times\{75-(200-x)\}\right]$$

$$= w\left[-\frac{x^2}{2}+189.29x-7812.5\right] \qquad (40.1)$$

Differentiating Eq. (40.1) w.r.t x and equating to zero,

$$\frac{dM_x}{dx} = w[-x+189.29=0]$$

Solving x=189.29

Substituting this in Equation (41.1), we get

$$M_{\max} = w\left[-\frac{189.29^2}{2} + 189.29x189.29 - 7812.5\right] = 10102.85KNm$$

The maximum BM occurs at 189.29 m from left support.

Problem 2: The span of a three-hinged stiffening girder bridge is 350 m and its central dip is 40 m. A single load of 75 KN rolls along the bridge. Determine the horizontal thrust H at a section 60 m from the left support as well as when the load is at 40 m from the left support. Also, find the maximum value of H. Find the maximum load w and the maximum positive moment under the load when it is 50 m from left support as well as the maximum shear force at the same section.

Solution:

The ILD for H is shown in Fig. 21.42(b). The maximum ordinate is at mid span.

$$H_{\max} = \frac{350}{4 \times 40} = 2.1875KN$$

The horizontal reaction at 60 m from left support is obtained from similar triangles.

$$H_{60} = 70 \times \frac{2.1875}{175} \times 60 = 52.5KN$$

The horizontal reaction at 40 m from left support where the load is situated is obtained from similar triangles.

$$H_{40} = 70 \times \frac{2.1875}{175} \times 40 = 35KN$$

Maximum value of $H = 2.1875 \times 70 = 153.125KN$

From Figure 21.42 (c)

$$w_{\max} = 70 \times \frac{2}{350} = 0.4KN/m$$

The maximum positive BM under the load, i.e., at 50 m from the left support is

$$M_{\max} = 70 \times \frac{50 \times (350 - 50) \times (350 - 2 \times 50)}{350^2} = 2142.86KNm$$

Hence the section is at 50 m from left support. It is < [(350/4)=87.5 m]. Therefore from Fig. 21.43(b), the maximum positive shear at the section is

$$F_{\max} = 70\left(\frac{1}{2} - \frac{2 \times 50}{350}\right) = 15KN$$

Lecture 41 Introduction to **space frames**

41.1 Introduction

Normally the centre lines of bars, forces applied and support reactions in the case of plane trusses lie in a plane. When all these lie in different planes i.e in three dimensional space, such a structure is called a space truss or space frame which is nothing but an assemblage of bars in three dimensional space. Tetrahedron is the simplest space frame consisting of six members. Antenna towers, transmission line towers, guyed masts, derricks, offshore structures etc are some of the common examples of space frames. We can construct a space frame from the basic tetrahedron by adding three new members and a joint. To get a stable space frame, we have to arrange adequate number of bars in a suitable manner starting with a basic tetrahedron. There are six bars and four joints in the basic tetrahedron. For each joint added, we have now three additional members. Therefore, we can have a reaction between the member of bars (b) and the number of joints (j) as given below

$$b-6=3(j-4)$$

$$b=3j-6 \qquad (41.1)$$

Equation (41.1) gives the minimum number of bars required to construct a stable space truss or space frame. If the number of bars in the space truss is less than that required by Equation (41.1), then we consider the space frame as unstable. In Contrast, if the number of bars is more than the minimum number required then the space frame is considered internally indeterminate.

We can now analyze a space frame on the basis of three coordinate axes, namely, X, Y and Z. Here axes X and Y are assumed to lie on a horizontal plane and Z in a vertical plane. We also assume that the joints in a space frame are pinned and that they carry only axial forces. A force in space or in a member of a space frame can be resolved into three components along X, Y and Z axes. So, for maintaining equilibrium at a joint, the algebraic sum of the components of all forces along the reference axes must be zero. Therefore, it can be concluded that a system of concurrent non-planar force is in equilibrium if

$$\sum F_X = 0 \quad \sum F_y = 0 \quad \text{and} \sum F_z = 0 \qquad (41\text{-}2)$$

41.2 Methods of Tension Coefficients

The method of tension coefficients is a tabular technique of carrying out joint resolution in either two or three dimensions. It is ideally suited to the analysis of pin-jointed space-frames.

Consider an individual member from a pin-jointed plane-frame, e.g. member AB shown in Figure 41.1 with reference to a particular X-Y co-ordinate system.

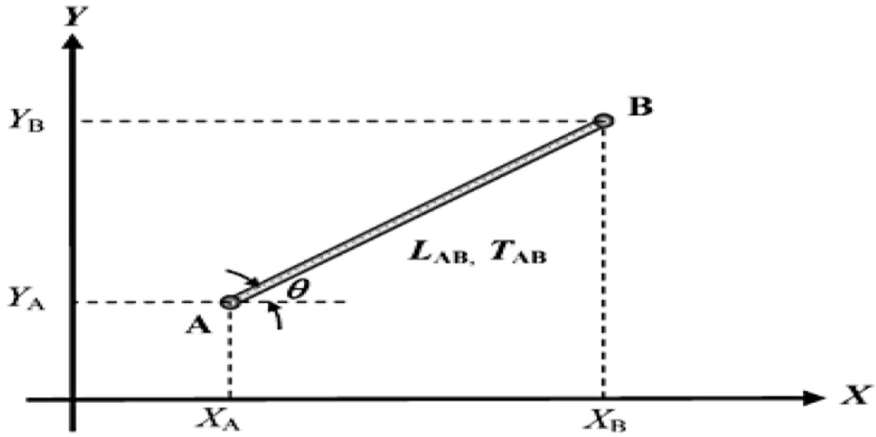


Figure 41.1 Coordinate systems

If AB is a member of length LAB having a tensile force in it of T_{AB}, then the components of this force in the X and Y directions are T_{AB} Cosθ and T_{AB} Sinθ respectively.

If the co-ordinates of A and B are (X_A, Y_A) and (X_B, Y_B), then the component of T_{AB} in the x-direction is given by :

x-component = $T_{AB}\dfrac{(X_B - X_A)}{L_{AB}} = t_{AB}(X_B - X_A)$

where

$$t_{AB} = \frac{T_{AB}}{L_{AB}}$$

and is known as the tension coefficient of the bar. Similarly, the component of TAB in the y-direction is given by:

y-component = $T_{AB}\dfrac{(Y_B - Y_A)}{L_{AB}} = t_{AB}(Y_B - Y_A)$

If at joint A in the frame there are a number of bars, i.e. AB, AC … AN, and external loads X_A and Y_A acting in the X and Y directions, then since the joint is in equilibrium the sum of the components of the external and internal forces must equal zero in each of those directions. Expressing these conditions in terms of the components of each of the forces then gives:

$$t_{AB}(X_B - X_A) + t_{AC}(X_C - X_A) + \ldots\ldots\ldots\ldots t_{AN}(X_N - X_A) + X_A = 0 \qquad (41.3)$$

$$t_{AB}(Y_B - Y_A) + t_{AC}(Y_C - Y_A) + \ldots\ldots\ldots\ldots t_{AN}(Y_N - Y_A) + Y_A = 0 \qquad (41.4)$$

A similar pair of equations can be developed for each joint in the frame giving a total number of equation equal to (2×number of joints) In a statically determinate triangulated plane-frame the number of unknown member forces is equal to [(2×number of joints)–3], hence there are three additional equations which can be used to determine the reactions or check the values of the tension coefficients. Once a tension coefficient (e.g. t_{AB}) has been determined, the unknown member force is given by the product:

$T_{AB} = t_{AB}L_{AB}$ (Note: $T_{AB} = T_{BA}$)

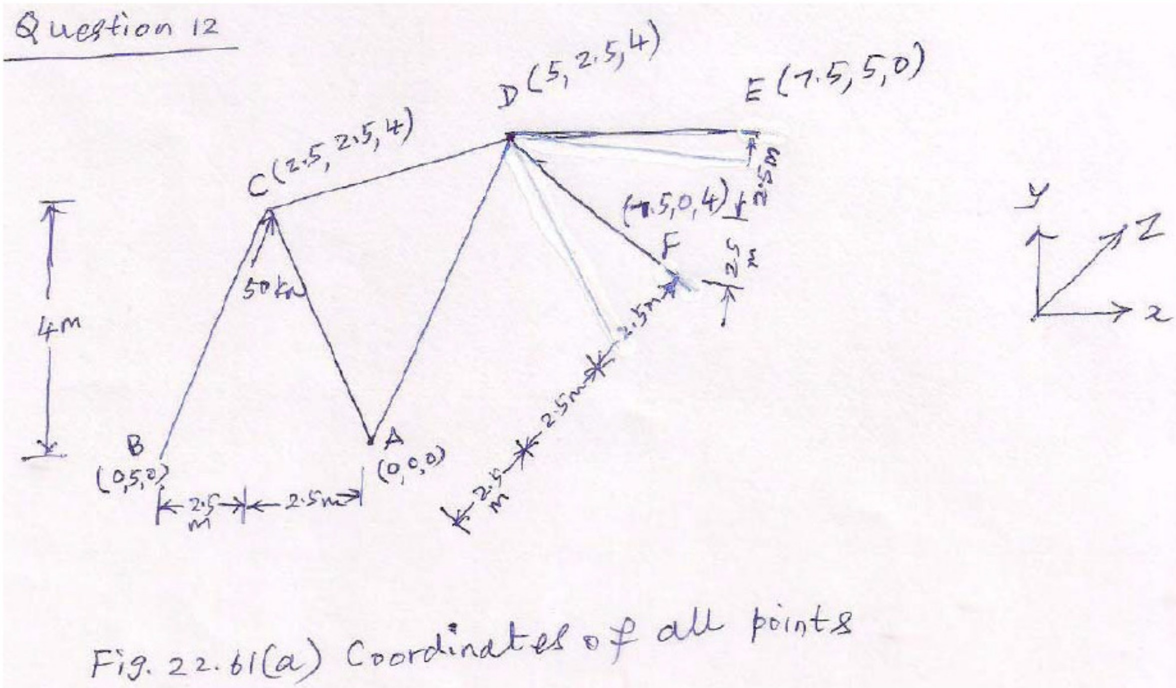
Note: A member which has a –ve tension coefficient is in compression and is a strut.

Lecture 42 Introduction to **space frames**

Problem 1: Analyse the space frame by method of tension coefficients

Solution:

The space frame is shown in Fig. 22.61. Now we determine the coordinates of all points as shown in Fig. 22.61(a) by taking point A as origin with coordinate axes as shown in Fig. 22.61(a).



Using these coordinates we calculate the length of members.

$$L_{AC} = \sqrt{(2.5-0)^2 + (2.5-0)^2 + (4-0)^2} = 5.34m$$

$$L_{BC} = \sqrt{(2.5-0)^2 + (2.5-5)^2 + (4-0)^2} = 5.34m$$

$$L_{CD} = \sqrt{(5-2.5)^2 + (2.5-2.5)^2 + (4-4)^2} = 2.5m$$

$$L_{DE} = \sqrt{(7.5-5)^2 + (5-2.5)^2 + (0-4)^2} = 5.34m$$

$$L_{DF} = \sqrt{(7.5-5)^2 + (0-2.5)^2 + (0-4)^2} = 5.34m$$

$$L_{AD} = \sqrt{(5-0)^2 + (2.5-0)^2 + (4-0)^2} = 6.87m$$

We consider joints one by one. We first consider joint C

$$t_{CA}(x_A - x_C) + t_{CB}(x_B - x_C) + t_{CD}(x_D - x_C) + X_C = 0 \quad \text{(a)}$$

$$t_{CA}(y_A - y_C) + t_{CB}(y_B - y_C) + t_{CD}(y_D - y_C) + Y_C = 0 \quad \text{(b)}$$

$$t_{CA}(z_A - z_C) + t_{CB}(z_B - z_C) + t_{CD}(z_D - z_C) + Z_C = 0 \quad \text{(c)}$$

Substituting values in (a), (b) and (c), we get

$$t_{CA}(0 - 2.5) + t_{CB}(0 - 2.5) + t_{CD}(5 - 2.5) + 50 = 0 \quad \text{(d)}$$

$$t_{CA} + t_{CB} + t_{CD} = 20 \quad \text{(e)}$$

$$t_{CA}(0 - 2.5) + t_{CB}(5 - 2.5) + t_{CD}(2.5 - 2.5) + 0 = 0 \quad \text{(f)}$$

$$t_{CA} + t_{CB} = 0 \quad \text{(g)}$$

$$t_{CA}(0 - 4) + t_{CB}(0 - 4) + t_{CD}(4 - 4) + 0 = 0 \quad \text{(h)}$$

$$t_{CA} + t_{CB} = 0 \quad \text{(i)}$$

Substituting (g) in (e), we get $t_{CD} = 20$ (j)

Next we consider joint D

$$t_{DC}(x_C - x_D) + t_{DE}(x_E - x_D) + t_{DF}(x_F - x_D) + t_{DA}(x_A - x_D) + X_D = 0 \quad \text{(k)}$$

$$t_{DC}(y_C - y_D) + t_{DE}(y_E - y_D) + t_{DF}(y_F - y_D) + t_{DA}(y_A - y_D) + Y_D = 0 \quad \text{(l)}$$

$$t_{DC}(z_C - z_D) + t_{DE}(z_E - z_D) + t_{DF}(z_F - z_D) + t_{DA}(z_A - z_D) + Z_D = 0 \quad \text{(m)}$$

Substituting values

$$t_{DC}(2.5 - 5) + t_{DE}(7.5 - 5) + t_{DF}(7.5 - 5) + t_{DA}(0 - 5) + 0 = 0 \quad \text{(n)}$$

$$-t_{DC} + t_{DE} + t_{DF} - 2t_{DA} = 0 \quad \text{(o)}$$

$$t_{DC}(2.5 - 5) + t_{DE}(5 - 2.5) + t_{DF}(0 - 2.5) + t_{DA}(0 - 2.5) + 0 = 0 \quad \text{(p)}$$

$t_{DE} - t_{DF} - t_{DA} = 0$ (q)

$t_{DC}(4-4) + t_{DE}(0-4) + t_{DF}(0-4) + t_{DA}(0-4) + 0 = 0$ (r)

$t_{DE} + t_{DF} + t_{DA} = 0$ (s)

From (r) $t_{DE} + t_{DF} = -t_{DA}$ (t)

Substituting (j) and (t) in (o), we get $t_{DA} = -6.67 KNm$ (u)

Substituting (u) in (q), we get (v)

$t_{DE} - t_{DF} = -6.67$ (w)

Substituting (u) in (s)

$t_{DE} + t_{DF} = 6.67$ (x)

$(w) + (x) t_{DE} = 0$ (y)

Substituting (y) in (x), $t_{DF} = 6.67$ (z)

Joint A

$t_{AC}(x_C - x_A) + t_{AD}(x_D - x_A) + 0 = 0$

Substituting values

$t_{AC}(x_C - x_A) + t_{AD}(x_D - x_A) + 0 = 0$

Substituting values

$t_{AC}(2.5-0) + t_{AD}(5-0) = 0$

$t_{AC} + 2t_{AD} = 0$ (1)

Substituting (u) in (l), $t_{AC} = 13.34 KNm$ (2)

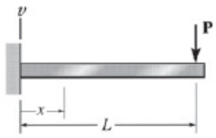
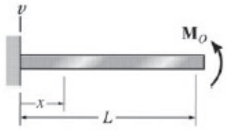
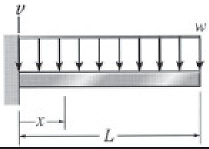
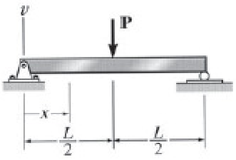
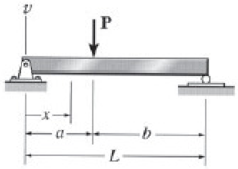
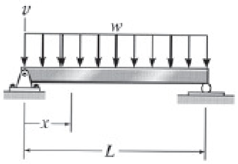
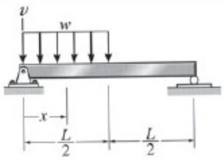
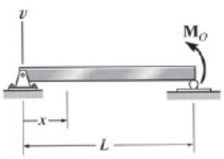
Substituting (2) in (g), $t_{CB} = -13.34 KNm$ (3)

We now calculate the forces in members and present it in the following Table 42.1

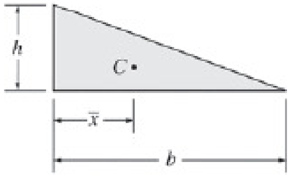
Member	Length in m	Tension Coefficients in KN/m	Force in KN
AC	5.34	13.34	71.24
BC	5.34	-13.34	-71.24
AD	6.87	-6.67	-45.82
CD	2.5	20	50
DE	5.34	0	0
DF	5.34	6.67	35.62

Appendix-A

Slopes and Deflections in Beams

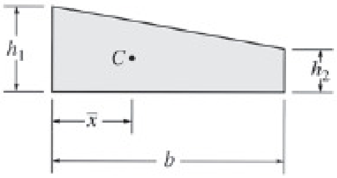
Loading	v + ↑	θ + ↺	Equation + ↑ + ↺
	$v_{max} = -\frac{PL^3}{3EI}$ at $x = L$	$\theta_{max} = -\frac{PL^2}{2EI}$ at $x = L$	$v = \frac{P}{6EI}(x^3 - 3Lx^2)$
	$v_{max} = \frac{M_O L^2}{2EI}$ at $x = L$	$\theta_{max} = \frac{M_O L}{EI}$ at $x = L$	$v = \frac{M_O}{2EI}x^2$
	$v_{max} = -\frac{wL^4}{8EI}$ at $x = L$	$\theta_{max} = -\frac{wL^3}{6EI}$ at $x = L$	$v = -\frac{w}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$
	$v_{max} = -\frac{PL^3}{48EI}$ at $x = L/2$	$\theta_{max} = \pm\frac{PL^2}{16EI}$ at $x = 0$ or $x = L$	$v = \frac{P}{48EI}(4x^3 - 3L^2x)$, $0 \le x \le L/2$
		$\theta_L = -\frac{Pab(L+b)}{6LEI}$ $\theta = \frac{Pab(L+a)}{6LEI}$	$v = -\frac{Pbx}{6LEI}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$v_{max} = -\frac{5wL^4}{384EI}$ at $x = \frac{L}{2}$	$\theta_{max} = \pm\frac{wL^3}{24EI}$	$v = -\frac{wx}{24EI}(x^3 - 2Lx^2 + L^3)$
		$\theta_L = -\frac{3wL^3}{128EI}$ $\theta_R = \frac{7wL^3}{384EI}$	$v = -\frac{wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = -\frac{wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x \le L$
	$v_{max} = -\frac{M_O L^2}{9\sqrt{3}EI}$	$\theta_L = -\frac{M_O L}{6EI}$ $\theta_R = \frac{M_O L}{3EI}$	$v = -\frac{M_O x}{6EIL}(L^2 - x^2)$

Appendix-B
Geometrical Properties of Areas



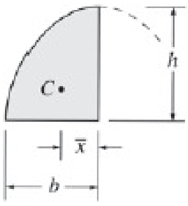
Triangle

$$A = \frac{1}{2}bh$$
$$\bar{x} = \frac{1}{3}b$$



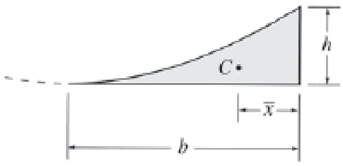
Trapezoid

$$A = \frac{1}{2}b\,(h_1 + h_2)$$
$$\bar{x} = \frac{b\,(2h_2 + h_1)}{3\,(h_1 + h_2)}$$



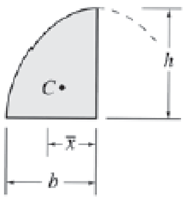
Semi Parabola

$$A = \frac{2}{3}bh$$
$$\bar{x} = \frac{3}{8}b$$



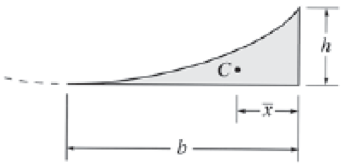
Parabolic spandrel

$$A = \frac{1}{3}bh$$
$$\bar{x} = \frac{1}{4}b$$



Semi-segment of *n*th degree curve

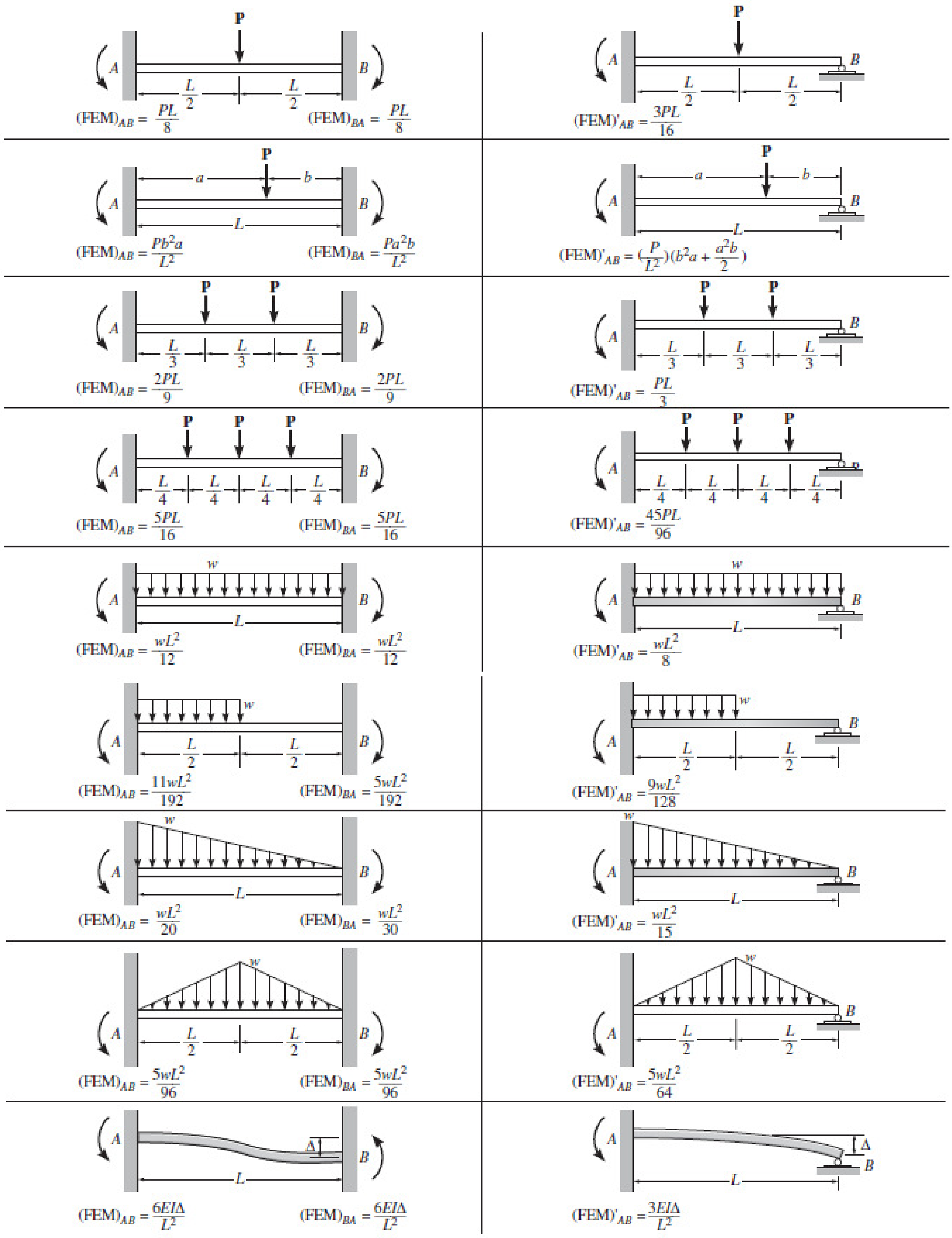
$$A = bh\left(\frac{n}{n+1}\right)$$
$$\bar{x} = \frac{b\,(n+1)}{2\,(n+2)}$$



Spandrel of *n*th degree curve

$$A = bh\left(\frac{1}{n+1}\right)$$
$$\bar{x} = \frac{b}{(n+2)}$$

Appendix-C
Fixed End Moments



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