**NM INSTITUTE OF ENGINEERING AND TECHNOLOGY**

**SIJUA, BHUBANESWAR**

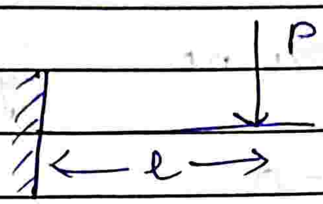
# LEARNING MATERIALS

# STRUCTURAL MECHANICS

Moment :-

- The turning effect of the force on the body on which it is acting is measured by moment.

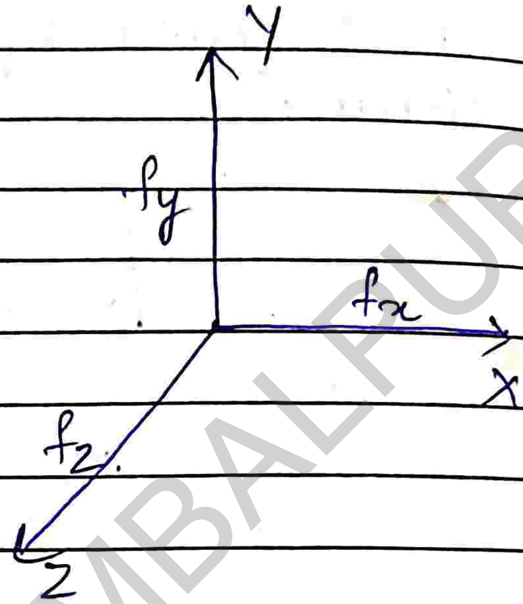
- It is found by multiplying the force by $\perp$r distance




$$M = P \times l$$
$$= N \times m$$
$$= Nm \quad \text{(Newton meter)}$$

## Conditions for Equilibrium :-

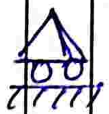
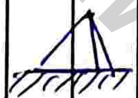
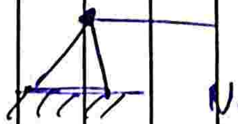
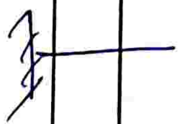
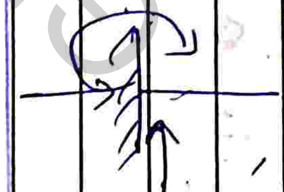
It may be defined as a state of rest or uniform motion of a body whose net force and net moment is acting upon a body is equal t zero.




| Net force | Net moment |
|---|---|
| $\Sigma f_x = 0$ | $\Sigma M_x = 0$ |
| $\Sigma f_y = 0$ | $\Sigma M_y = 0$ |
| $\Sigma f_z = 0$ | $\Sigma M_z = 0$ |

Net force = Net moment = zero.

## Support conditions:-

| SL NO | Types of Supports | Represented By | Reaction force | Resisting load |
|---|---|---|---|---|
| 1. | Roller Support |  |  1 Vertical | Vertical force |
| 2. | Pinned Support/ Hinged support |  |  2 Vertical and Horizontal | Vertical and Horizontal |
| 3. | fixed support |  |  3 Vertical, Horizontal Rotational | ~~Vertical~~ all type of loads & mom (Vertical, Horizontal, Rotational. |
| 4. | Single support. |  |  1 Vertical | Vertical loads |

## Centre of gravity (CG) :-

- Centre of gravity is an imaginary balancing poi[nt] where the body weight can be assumed to [be] concentrated.

- It Symbol is C.G

## Moment of inertia (M.I) :-

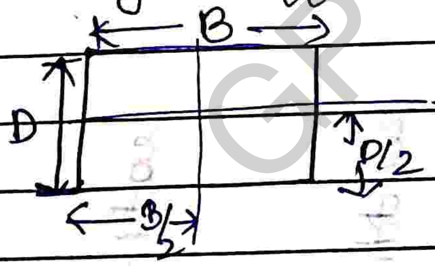
- Moment of inertia measures the resistance to a change in rotation.

- MI = M x Distance

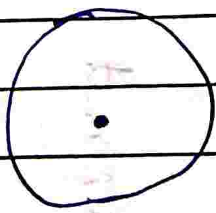
$= Pl \times l$

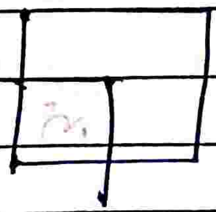
$\boxed{M = Pl^2}$

CG and MI of Different section.

(1) Rectangle 

$\frac{B}{2}, \frac{D}{2}$ | $\frac{BD^3}{12}$

(2) circle 
$\frac{D}{2}$ or $r$ | $\frac{\pi D^4}{64}$

(3) Square 
$\frac{a}{2}, \frac{a}{2}$ | $\frac{a^4}{12}$

Teacher's Signature..........................

## Centre of gravity (CG) :-

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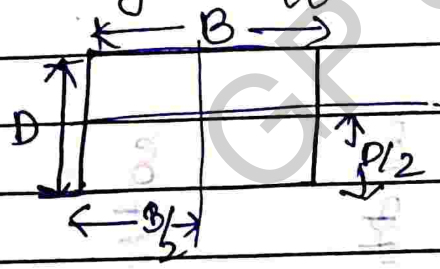
  $= Pl \times l$

  $\boxed{M = Pl^2}$

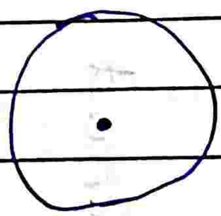
CG and MI of Different section.

① Rectangle



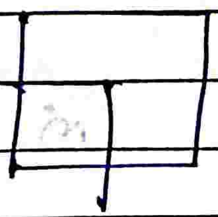

$\frac{B}{2}, \frac{D}{2}$ | $\frac{BD^3}{12}$

② circle



$\frac{D}{2}$ or $r$ | $\frac{\pi D^4}{64}$

③ Square



$\frac{a}{2}, \frac{a}{2}$ | $\frac{a^4}{12}$

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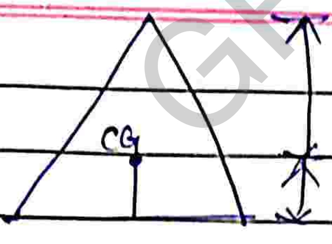
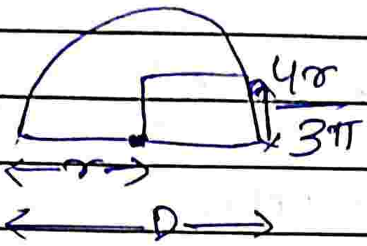
(4) Triangle




$\frac{Bh^3}{36}$

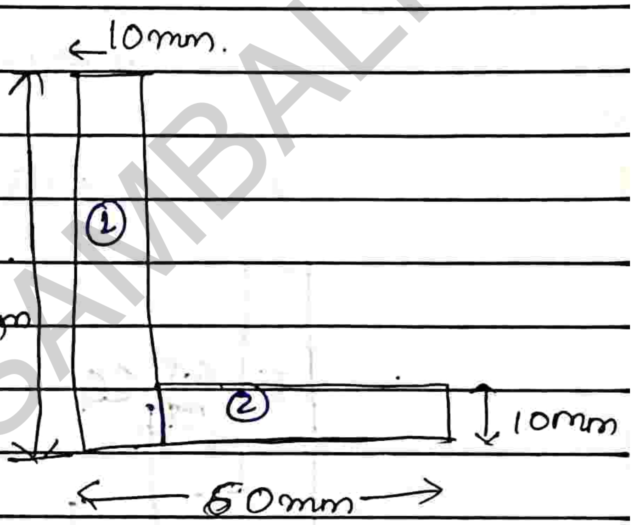

(5) Semicircle




$\frac{\pi D^4}{128}$

CW
22/09/22

find the CG of given section.




Rectangle - (1)

$a_1 = 80 \times 10$
$= 800\ mm^2.$

$x_1 = \frac{10}{2} = 5\ mm.$

$y_1 = \frac{80}{2} = 40\ mm.$

Rectangle - (2).

$a_2 = 50 \times 10$
$= 500\ mm^2.$

$x_2 = \frac{50}{2} = \cancel{25}\ 35\ mm$

$y_2 = \frac{10}{2} = 5\ mm.$

$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}.$

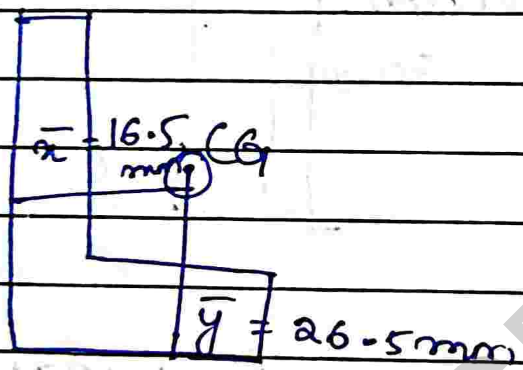
$= \frac{800 \times 5 + 500 \times \cancel{25}\ 35}{800 + 500}.$ ~~16500~~ ~~1300~~ ~~140~~ ~~12~~ 16.5 mm.

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{800 \times 40 + 500 \times 5}{800 + 500}$$

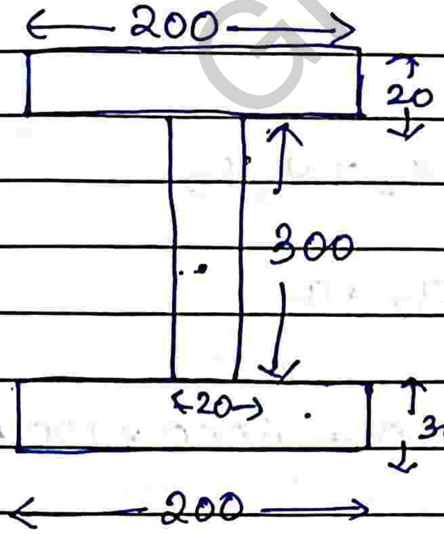
$$= \frac{34500}{1300}$$

$$= 26.5 \text{ mm}$$




Q) Find the CG of a I-section for the given data.

Top flange :- 200x20
Web :- 300x20
Bottom flange :- 200x30




**Bottom flange**

$a_1 = 200 \times 30$

$= 6000 \text{ mm}^2$

$x_1 = \frac{200}{2}$

$= 100 \text{ mm}$

$y_1 = \frac{30}{2}$

$= 15 \text{ mm}$

**web**

$a_2 = 300 \times 20$

$= 6000 \text{ mm}^2$

$\cancel{x_2 = \frac{20}{2} + 30}$

$\cancel{= 10 + 30}$

$\cancel{= 40 \text{ mm}}$

$x_2 = 100 \text{ mm}$

$y_2 = 30 + \frac{300}{2}$

$= 30 + 150$

$= 180 \text{ mm}$

web =

$a_3 = 200 \times 20$

$= 4000 \text{ mm}^2$

$x_3 = \frac{200}{2} = 100 \text{ mm}$

$y_3 = 300 + 30 + \frac{20}{2}$

$= 340 \text{ mm}$

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$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3}$$

$$= \frac{6000 \times 100 + 6000 \times 100 + 4000 \times 100}{6000 + 6000 + 4000}$$

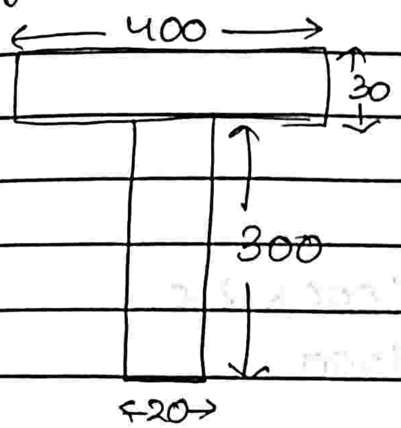
$$= 100\text{mm}$$

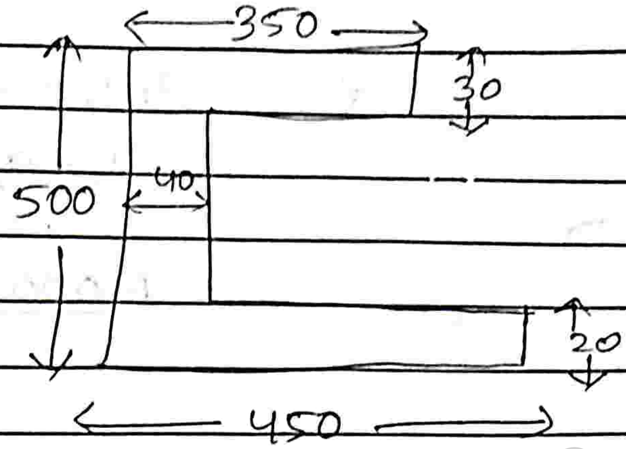
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{a_1 + a_2 + a_3}$$

$$= \frac{6000 \times 15 + 6000 \times 180 + 4000 \times 340}{6000 + 6000 + 4000}$$

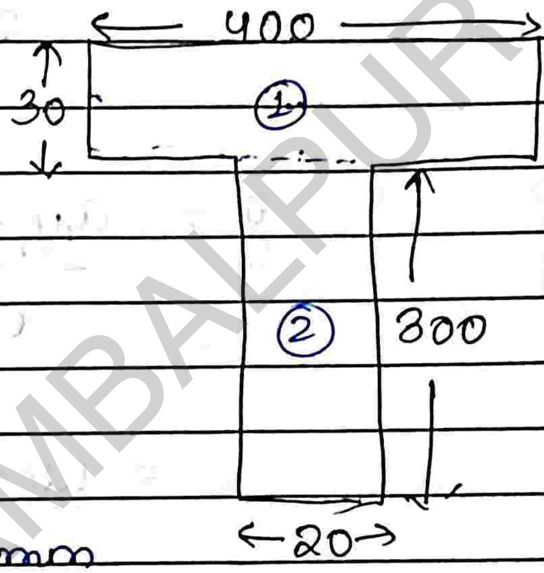
$$= 158.2\text{mm}$$

# Assignment :-




Q1




Rectangle - 1

$$\text{area } (a_1) = 400 \times 30$$
$$= 12000 \text{ mm}^2.$$

$$x_1 = \cancel{200\text{mm}}\ 190 + 10 = 200 \text{ mm}$$

$$y_1 = 300 + \frac{30}{2} = 300 + 15 = 315 \text{ mm}$$

Rectangle - 2

$$\text{area } (a_2) = 300 \times 20.$$
$$= 6000 \text{ mm}^2.$$

$$x_2 = 200 \text{ mm}.$$

$$y_2 = \frac{300}{2} = 150 \text{ mm}$$

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{a_1 + a_2}$$

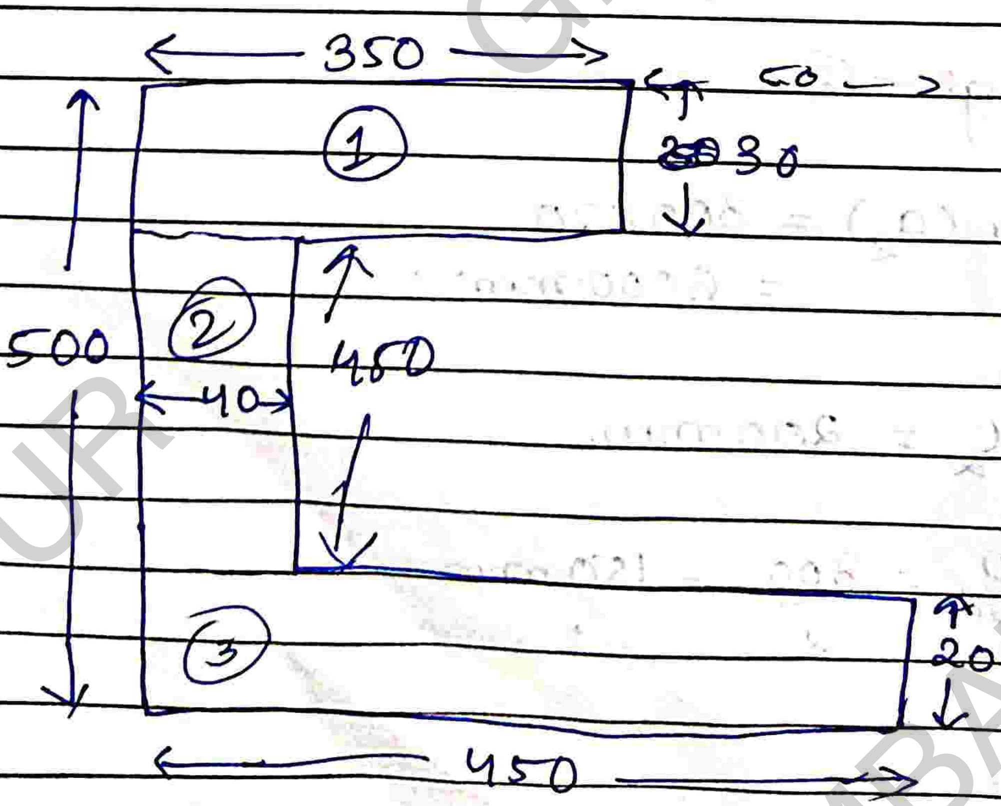
$$= \frac{12000 \times 200 + 6000 \times 200}{12000 + 6000}$$

$$= 200\,mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2}{a_1 + a_2}$$

$$= \frac{12000 \times 315 + 6000 \times 150}{12000 + 6000}$$

$$= \cancel{215}\ 260\,mm$$



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Rectangle - 1

area $= 350 \times 30$
$(a_1)$ $= 10500 \, mm^2$.

$x_1 = \frac{350}{2}$

$= 175 \, mm$

$y_1 = 20 + 450 + \frac{30}{2}$.

$= 20 + 450 + 15$
$= 485 \, mm$

Rectangle - 2

area $= 450 \times 40$
$(a_2)$ $= 18000 \, mm^2$.

$x_2 = \frac{40}{2} = 20 \, mm$

$y_2 = 20 + \frac{450}{2}$.

$= 245 \, mm$

Rectangle - 3.

area $= 450 \times 20$
$(a_3)$ $= 9000 \, mm^2$.

$x_3 = \frac{450}{2} = 225 \, mm$

$y_3 = \frac{20}{2} = 10 \, mm$

$\bar{x} = \frac{10500 \times 175 + 18000 \times 20 + 9000 \times 225}{10500 + 18000 + 9000}$

$= 112.6 \, mm$.

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$$\bar{y} = \frac{\cancel{100}\ 10500 \times 485 + 18000 \times 245 + 9000 \times 10}{10500 + 18000 + 9000}$$

$$= 255.8\ mm$$

<u>Moment of inertia</u>

~~Retta~~

MI of Rectangle ① :-

$$I_{xx_1} = I_x + Al^2$$

$$= \frac{350 \times 30^3}{12} + 10500 \times 485^2$$

$$= \cancel{4}\ 2.47 \times 10^9\ mm$$

MI of Rectangle ①.

$$I_{xx_2} = I_x + Al^2.$$

$$= \frac{40 \times 450^3}{12} + 18000 \times 245^2.$$

$$= 1.38 \times 10^9\ mm$$

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MI of Rectangle ③

$$I_{xx_3} = I_x + Al^2.$$

$$= \frac{450 \times 20^3}{12} + 9000 \times 10^2.$$

$$= 1.2 \times 10^6 mm.$$

$$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$$

$$= (2.47 \times 10^9 + 1.39 \times 10^9 + 1.2 \times 10^6) mm.$$

$$= 3.86 \times 10^9 mm$$

Ans)

$$I_{yy} = (I_{y_1} + Al^2) + (I_{y_2} + Al^2) + (I_{y_3} + Al^2)$$

$\cancel{= 30x}$

$$\text{Rec-①} = I_{y_1} + Al^2.$$

$$= \frac{30 \times 350^3}{12} + 10500 \times 175^2$$

$$= \cancel{109.02 \times 10^6 mm} \quad 428.75 \times 10^6 mm$$

$$\text{Rec-②} = I_{y_2} + Al^2. \qquad = 9.6 \times 10^6 mm$$

$$= \frac{450 \times 40^3}{12} + 18000 \times 20^2.$$

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Rectangle-3

$I_{xy_3} = I_{y_3} + Al^2$

$= \frac{20 \times 450^3}{12} + 9000 \times 225^2$.

$= 607.5 \times 10^6$ mm

$I_{yy} = I_{y_1} + I_{y_2} + I_{y_3}$

$= 428.75 \times 10^6 + 9.6 \times 10^6 + 607.5 \times 10^6$ mm

$= 1.04 \times 10^9$ mm.

Q1 Find the CG of the T-Section with flange 150mm × 50mm and web as 150 × 50 mm. [5

Q2 Find the CG of an I-Section for the given data [5

Data:-

Top flange = 300 × 30 mm
Bottom flange = 200 × 20 mm
web = 350 × 10 mm

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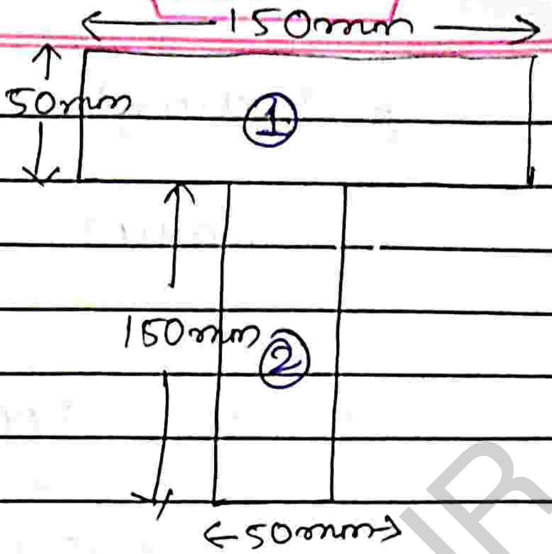
## Assignment




Rectangle - ①

$\text{area } (a_1) = (150 \times 50)\ mm^2$
$= 7500\ mm^2$

$x_1 = \frac{150}{2} = 75\ mm$

$y_1 = 150 + \frac{50}{2}$
$= 150 + 25$
$= 175\ mm$

Rectangle - ②

$\text{area } (a_2) = 150 \times 50\ mm^2$
$= 7500\ mm^2$

$x_2 = 50 + 25 = 75\ mm$

$y_2 = \frac{150}{2} = 75\ mm$

$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2}$

$= \frac{7500 \times 75 + 7500 \times 75}{7500 + 7500}$

$= 75\ mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$

$= \frac{7500 \times 175 + 7500 \times 75}{7500 + 7500}$

$= 125\ mm$

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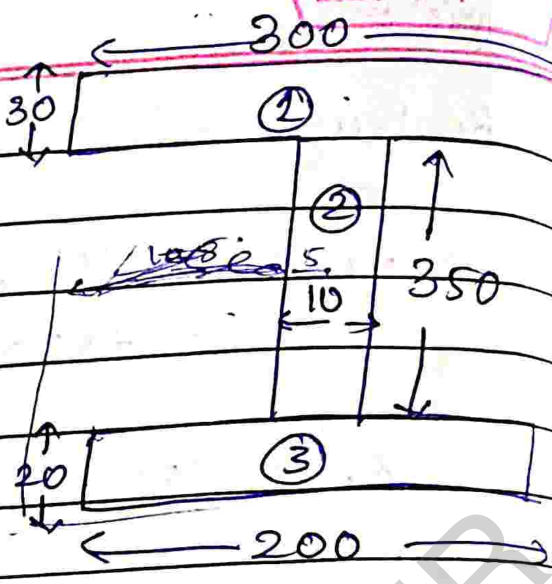


Q2 Rectangle - ①

area $(a_1) = 300 \times 30$
$= 9000 mm^2$.

$x_1 = \frac{300}{2} = 150 mm$

$y_1 = 20 + 350 + 15 = 385 mm$

Rectangle - ②

area $(a_2) = 350 \times 10$
$= 3500 mm^2$.

$x_2 = 150 mm$

$y_2 = 20 + \frac{350}{2}$
$= 20 + 175$
$= 195 mm$

Rectangle - ③

area$(a_3) = 200 \times 20 = 4000 mm^2$.

$x_3 = \frac{200}{2} = \cancel{100 mm}$ 150 mm

$y_3 = \frac{20}{2} = 10 mm$

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$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{9000 \times 150 + 3500 \times 150 + 4000 \times 150}{9000 + 3500 + 4000}$$

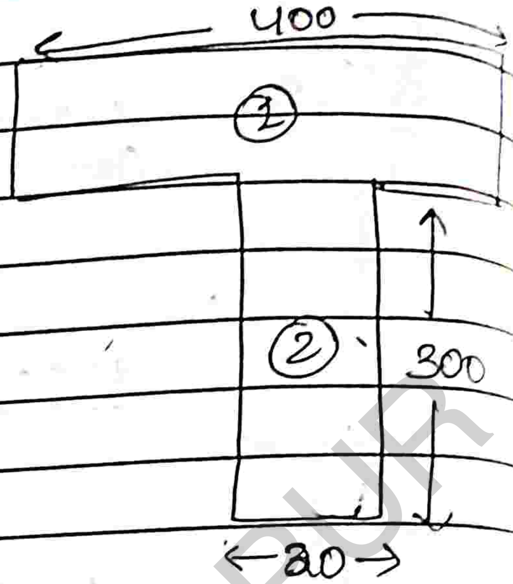
$= 150$ mm

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{9000 \times 385 + 3500 \times 195 + 4000 \times 10}{9000 + 3500 + 4000}$$

$= 253.78$ mm.

Assign. MI of T section




From Rectangle-1

area $(a_1) = 400 \times 30$
$= 12000\ mm^2$.

$y_1 = 300 + 15$
$= 315\ mm$

Rectangle-2

area $(a_2) = 300 \times 20$
$= 6000\ mm^2$.

$y_2 = \frac{300}{2}$
$= 150\ mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$.

$= \frac{12000 \times 315 + 6000 \times 150}{12000 + 6000}$

$= 260\ mm$

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MI of Rectangle - 1

$$I_{xx_1} = I_x + Al^2$$

$$= \frac{400 \times 30^3}{12} + 12000 \times (315 - 260)^2.$$

$$= \cancel{1.56 \times 10^6\ mm^4} \quad 37.2 \times 10^6.$$

MI of Rectangle - 2

$$I_{xx_2} = I_x + Al^2$$

$$= \frac{20 \times 300^3}{12} + 6000 \times (260 - 150)^2.$$

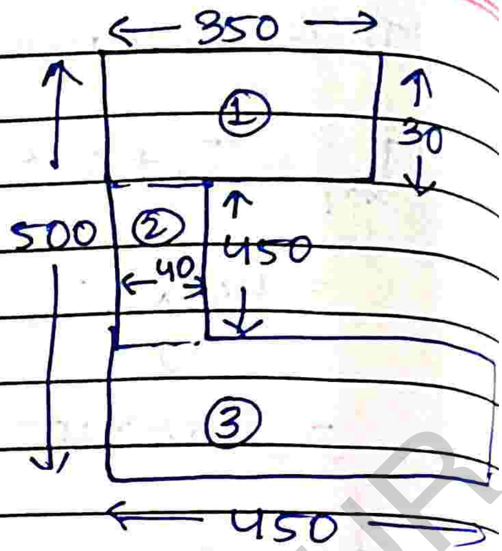
$$= \cancel{45.66 \times 10^6\ mm^4} \quad 117.6 \times 10^6.$$

$$I_{xx} = I_{xx_1} + I_{xx_2}.$$

$$= 1.56 \times 10^6 + 45.66 \times 10^6$$

$$= 47.22 \times 10^6\ mm^4$$

## MI of C-Section :-




Rectangle ①

$area(a_1) = 350 \times 30$
$= 10500\ mm^2$.

$x_1 = \frac{350}{2} = 175\ mm$

$y_1 = 20 + 450 + \frac{30}{2}$
$= 485\ mm$

Rectangle ②

$area(a_2) = 450 \times 40$
$= 18000\ mm^2$.

$x_2 = \frac{40}{2} = 20\ mm$

$y_2 = 20 + \frac{450}{2}$
$= 245\ mm$

Rectangle ③

$area(a_3) = 450 \times 20$
$= 9000\ mm^2$

$x_3 = 225\ mm$

$y_3 = 10\ mm$

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$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3}{a_1 + a_2 + a_3}$$

$$= \frac{10500 \times 175 + 18000 \times 20 + 9000 \times 225}{10500 + 18000 + 9000}$$

$$= 112.6\,mm$$

$$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{a_1 + a_2 + a_3}$$

$$= \frac{10500 \times 485 + 18000 \times 245 + 9000 \times 10}{10500 + 18000 + 9000}$$

$$= 255.8\,mm$$

$$I_{xx_1} = I_x + Al^2.$$

$$= \frac{350 \times 30^3}{12} + 10500 \times (485 - 255.8)^2$$

$$= \cancel{3.19 \times 10^6\,mm}.\ 552.38 \times 10^6\,mm^4$$

$$I_{xx_2} = I_x + Al^2.$$

$$= \frac{40 \times 450^3}{12} + 18000 \times (255.8 - 245)^2$$

$$\cancel{= 3089.84 \times 10^6\,mm.}$$

$$= 305.84 \times 10^6\,mm^4.$$

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$I_{xx_3} = I_{x_3} + Al^2$

$= \dfrac{450 \times 20^3}{12} + 9000 \times (255.8 - 10)^2$ 225.8

$= 544.05 \times 10^6 \, mm^4$

$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$

$= 552.38 \times 10^6 + 305.84 \times 10^6 + 544.06 \times 10^6$

$= 1.402 \times 10^9 \, mm^4$

$I_{yy_1} = I_{y_1} + Al^2$

$= \dfrac{30 \times 350^3}{12} + 10500 \times (175 - 112.6)^2$

$= 148.07 \times 10^6 \, mm^4$

$I_{yy_2} = I_{y_2} + Al^2$

$= \dfrac{450 \times 40^3}{12} + 18000 \times (112.6 - 20)^2$

$= 156.74 \times 10^6 \, mm^4$

$I_{yy_3} = I_{y_3} + Al^2$

$= \dfrac{20 \times 450^3}{12} + 9000 \times (225 - \cancel{186}\ 112.6)^2$

$= 265.57 \times 10^6 \, mm^4$

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$I_{YY} = I_{YY_1} + I_{XX_2} + I_{YY_3}$

$= (148.07 \times 10^6 + 156.74 \times 10^6 + 265.57 \times 10^6)$ $mm^4$

$= 570.38 \times 10^6 \, mm^4.$

MI of Rectangle - ① $I_{XX_1} = I_x + Ae^2.$

Assign

$= \frac{150 \times 50^3}{12} + 7500 \times (175 - 125)^2$

$= 20.31 \times 10^6 \, mm^4.$

Rect - ② $I_{XX_2} = I_x + Ae^2.$

$= \frac{50 \times 150^3}{12} + 7500 \times (125 - 75)^2.$

$= 32.81 \times 10^6 \, mm^4.$

$I_{XX} = I_{XX_1} + I_{XX_2}.$

$= 20.31 \times 10^6 + 32.81 \times 10^6 \, mm^4$

$= 53.12 \times 10^6 \, mm^4$

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$$I_{xx_1} = I_{x} + Al^2.$$

$$= \frac{50 \times 150^3}{12} + 7500 \times (25-2$$

$$= 14.06 \times 10^6 \ mm^4$$

$$I_{yy_2} = I_{y_2} + Al^2.$$

$$= \frac{150 \times 50^3}{12} + 7500(0)^2.$$

$$= 1.56 \times 10^6 \ mm^4.$$

$$I_{xy} = I_{yy_1} + I_{xx_2}$$

$$= 14.06 \times 10^6 + 1.56 \times 10^6 \ mm^4$$

$$= 15.62 \times 10^6 \ mm^4$$

I-section

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MI of Rectangle (1)

$I_{xx_1} = I_{x_1} + Ae^2$

$= \frac{300 \times 30^3}{12} + 9000 \times (385 - 253.78)^2$

$= 155.64 \times 10^6 mm^4$

$I_{xx_2} = I_{x_2} + A_2 e_2^2$

$= \frac{10 \times 350^3}{12} + 3500 \times (253.78 - 195)^2$

$= 47.82 \times 10^6 mm^4$

$I_{xx_3} = I_{x_3} + Ae^2$

$= \frac{200 \times 20^3}{12} + 4000 \times (253.78 - 10)^2$

$= 237.84 \times 10^6 mm^4$

$I_{xx} = I_{xx_1} + I_{xx_2} + I_{xx_3}$

$= 155.64 \times 10^6 + 47.82 \times 10^6 + 15.62 \times 10^6$

$= 219.08 \times 10^6 mm^4$

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# Ch-4

## Columns & Structs

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**Structs :-** A member on structure or bar which carries an axial compressive load is known as structs.

**columns :-** If the structs is vertical i.e, at 90° to the horizontal, is known as column.

→ All columns are structs but all structs are not column.

**Difference between ~~crossing~~ crushing and buckling :-**

| Crushing | Buckling |
|---|---|
| (i) It is applicable to short column only. | (i) It is applicable to long column only |
| (ii) Stress $(\sigma) = \frac{P}{A}$ <br> unit = $N/m^2$. | (ii) $\boxed{P_e = \frac{\pi^2 EI}{l_e^2}}$ *** <br> where $P_e$ = Euler's load <br> $I$ = moment of inertia <br> $E$ = young's modulus of elasticity <br> $l_e$ = Effective length |

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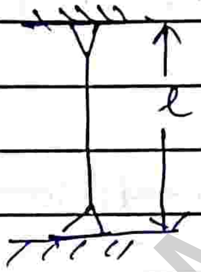
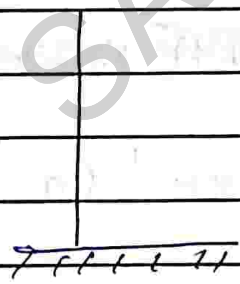
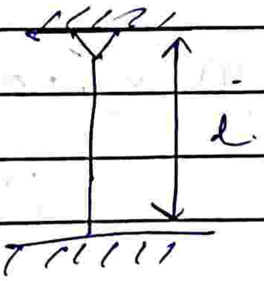
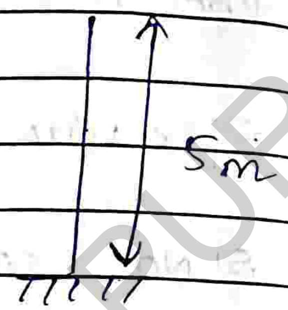
## Young's modulus of Elasticity :- (E)

The material which has ability to return its original shape after removing the load is known as elasticity

## Effective length of column :- ($l_e$)

| SL NO. | column condition | figure | Effective length ($l_e$) |
|---|---|---|---|
| 1. | Both ends are Hinged/pinned |  | $l_e = l$ |
| 2. | One end fixed and other end free |  | $l_e = 2l$ |
| 3. | one end is fixed and another end is hinged |  | $l_e = \frac{l}{\sqrt{2}}$ |
| 4. | Both ends are fixed |  | $l_e = \frac{l}{2}$ |

Q A Steel rod is 5m long and 50mm diameter is used as column with one end fixed and other free. Determine the Buckling load by using Euler's formula. Take E = 200 GPa




$E = 200\ GPa$

$I = \dfrac{\pi d^4}{64}$

$= \dfrac{3.14 \times (50 \times 10^{-3})^4}{64}$

$l_e = 2l$

$= 2 \times 5$

$= 10\,m$

$= \cancel{306.64 \times 10^{3}}\ \dfrac{3.14 \times (0.05)^4}{64}$

$\cancel{P_e = (3.14)^2 \times 200 \times 306.64} \quad = 3.06 \times 10^{-7}\ m^4$

$E = 200 \times 10^9\ Pa$

$P_e = \dfrac{(3.14)^2 \times 200 \times 10^9 \times \cancel{306.64 \times 10^3}\ 3.06 \times 10^{-7}}{(10)^2}$

$= \cancel{6.04 \times 10^{15}}$

$= 6.03 \times 10^3$

$= 6.03\ kN$

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1 Pa = 1 N/m$^2$.

1 KPa = $10^3$ N/m$^2$.

1 MPa = $10^6$ N/m$^2$

1 GPa = $10^9$ N/m$^2$.

Q2 Assign A steel rod of 60mm diameter and 15 m length is used as column. Determine the buckling load by using Euler's formula. Take E = 250 GPa

(i) Both ends are fixed
(ii) ~~Both~~ one end is fixed and as other end is Hinged

$$E = 250\ GPa$$
$$= 250 \times 10^9\ N/m^2.$$

$$I = \frac{\pi d^4}{64} = \frac{3.14 \times (0.06)^4}{64}$$
$$\cancel{= 1.76 \times 10^{-4}\ m^4}\ .6.35 \times 10^{-7}.$$

$$L_e = \frac{l}{2} = \frac{15}{2} = 7.5\ m$$

$$P_e = \frac{\pi^2 E I}{l_e^2}.$$

$$= \frac{(3.14)^2 \times 250 \times 10^9 \times \cancel{1.76 \times 10^{-4}}\ 6.35 \times 10^{-7}}{(7.5)^2}.$$

$$= \cancel{7.71 \times 10^6\ N} = 27.85\ KN$$

Teacher's Signature............

$E = 250\ GPa$

$= 250 \times 10^9\ N/m^2$.

$I = \frac{\pi d^4}{64}$

$= \frac{3.14 \times (0.06)^4}{64}$

$= \cancel{1.76 \times 10^{-4}\ mm^4}$. $6.35 \times 10^{-7}$.

$l_e = \frac{15}{\sqrt{2}}$

$= 10.06\ m$

$P_e = \frac{\pi^2 E I}{l_e^2}$.

$= \frac{(3.14)^2 \times 250 \times 10^9 \times 1.76 \times 10^{-4}}{(10.06)^2}$.

$= \cancel{4.28 \times 10^6\ N}$

$= 15.48\ kN$.

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## Assumptions made in Euler's Formula:- [5 M]

1)- Material should be Elastic, isotropic & homogeneous.

2)- The column should be perfectly straight and uniform

(3)- Load is applied axially and passes through the its centroid.

4. These is valid for long column only

## Slenderness ratio :- [2 Mark]

- The ratio of Effective length to minimum Radius of Gyration is called Slenderness ratio

$$SR = \frac{\text{Effective length } (l_e)}{\text{Min}^m \text{ Radius of gyration}}$$

$$\boxed{SR \text{ or } \lambda = \frac{l_e}{k}}$$

## Minimum Radius of Gyration :- [2 M]

It may be defined as the square root of the ratio of moment of inertia of a body to its cross-sectional area.

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$$\boxed{K = \sqrt{\frac{I}{A}}}$$

It is unitless

$$SR = \frac{le}{\frac{\sqrt{\frac{BD^3}{12}}}{BD}}$$

$$= \frac{le}{\sqrt{\frac{BD^{3\,2}}{12} \times \frac{1}{BD}}}$$

$$= \frac{le\sqrt{12}}{D}$$

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SHORT COLUMN and Long column :-

(1) Short column:- Columns having their length ~~less~~ < 8 times of their respective diameter or Slenderness ratio < 32 & are known as Short column.

(2) Long column:- The columns having their length greater than 30 times of the diameter or slenderness ratio greater than 120 are known as long column.

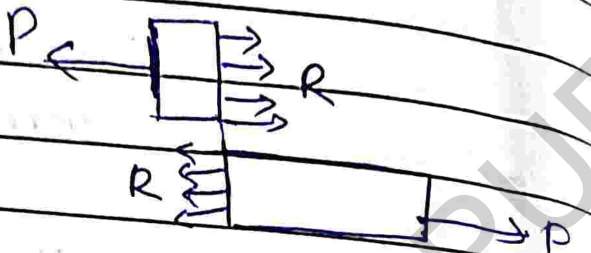
Between < 32 and > 120 is known as Intermediate column.

2 Marks

# Simple Stresses and Strains

**Stress** - It is the resistance offered by a body against the deformation is known as stress.

$$\boxed{\sigma = \frac{P}{A}}$$




It is denoted by the symbol Sigma ($\sigma$)

where P = Force or Load
A = ~~A~~ Cross-sectional area

unit of stress - $N/m^2$

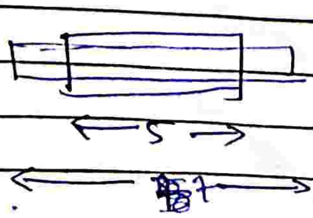
**Strains** :- It is the ratio of change in diamension to original diamen[sion]

- It is denoted by symbol 'e'

$$\boxed{e = \frac{\Delta L}{L}}$$

$\Delta L$ → change in length
$L$ → original length.

- It is a unitless quantity




$= \frac{2}{5}$ ← change in length / original length

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Types of Strains :-

(i) Longitudinal Strain
(ii) Lateral Strain
(iii) Volumetric Strain

(i) Longitudinal Strain :-

$$\boxed{e = \frac{\Delta L}{L}}$$

(ii) Lateral strain :-

$$\boxed{e = \frac{\Delta d}{d}}$$

(iii) volumetric strain :-

$$\boxed{e = \frac{\Delta V}{V}}$$

Hooke's law :- [2 Marks]

Statement :-

It states that "when a material is loaded such that the ratio of intensity of stress to the Strain is a constant"

Mathematically

$$\boxed{\frac{\sigma}{e} = \text{constant}}$$

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$\boxed{\sigma \propto e}$

$$\boxed{E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e}}$$

E = young's modulus of Elasticity

unit = $N/m^2$

A load of 5 kN is to be applied to develop 100 Mpa stress Find the diameter of the wir

$$\sigma = \frac{P}{A}$$

$$100 \times 10^6 \frac{N}{m^2}$$

$$P = 5\,kN = 5 \times 1000 = 5000\,N$$

$$A = \frac{P}{\sigma}$$

$$= \frac{5000}{100 \times 10^6}$$

$$= 5 \times 10^{-5} m^2.$$

$$A = \frac{\pi}{4} \times D^2.$$

$$D = \sqrt{\frac{4 \times 5 \times 10^{-5}}{3.14}}$$

$$= 7.97 \times 10^{-3} \times 1000 = 7.97\,mm$$

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$\sigma \propto e$

$$\frac{P}{A} \propto \frac{\Delta L}{L}$$

$$\frac{P}{A} = C\,\frac{\Delta L}{L}$$

$$\frac{P}{A} = \frac{E \times \Delta L}{L}$$

***

$$\boxed{\Delta L = \frac{PL}{EA}}$$

Q1 A Steel rod of ~~500~~ 500 mm long and 20 mm × 10 mm in cross-section is subjected to axial load of 300 kN. If the modulus of Elasticity $E = 2\times10^{5}\ N/m^2$, calculate the elongation and strain.

$L = 500\times10^{-3}\ m$

$E = 2\times10^{5}\ N/m^2$

$P = 300\times10^{3}\ N$

$A = \cancel{20\times10 = 200\ mm}$

$A = 20\times10^{-3}\times10\times10^{-3}\ m$

$= 2\times10^{-4}\ m^2$

$$\Delta L = \frac{300\times10^{3}\times500\times10^{-3}}{2\times10^{5}\times2\times10^{-4}}$$

$\Delta L = 3.75\times10^{3}\ m$

$= 3.75\ mm$

$$\sigma = \frac{3.75}{500}$$

$= 7.5\times10^{-3}$

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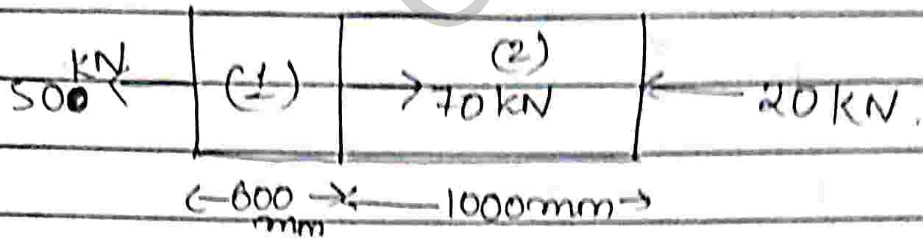
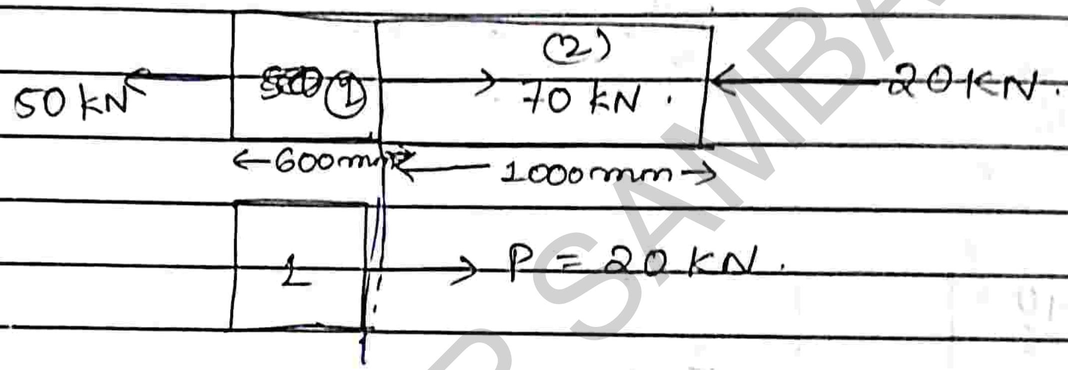
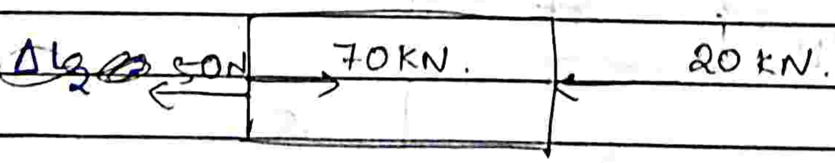



A Bar having cross-sectional area of $1000\,mm^2$ is subjected to an axial force given in the figure. Find the change in [length] of the bar take $E = 1.05 \times 10^5\ N/mm^2$.




$$\Delta l_1 = \frac{20 \times 10^3 \times 0.6}{1.05 \times 10^5 \times 10^{-3}}$$

$$= \cancel{0.114\,m}\ \cancel{0.285\,mm}\quad 285.71$$




$$\Delta l_2 = \frac{20 \times 10^3 \times 1}{1.05 \times 10^5 \times 10^{-3}}$$

$$= \cancel{190.476\,m} \qquad = 95.234\,m$$

$\cancel{\Delta l = 0.114 - 0.190}$ $= 0.094\,m$

$$\Delta L_1 = \frac{50 \times 10^3 \times 600}{1.05 \times 10^5 \times 1000}$$

$$= 0.285 \text{ mm}$$

$$\Delta L_2 = \frac{20 \times 10^3 \times 1000}{1.05 \times 10^5 \times 1000}$$

$$= 0.190$$

$$\Delta L = 0.095 \text{ mm}$$

# Shear Stress Distribution in Rectangular cross-section [5 − 10] (N) (T&N)



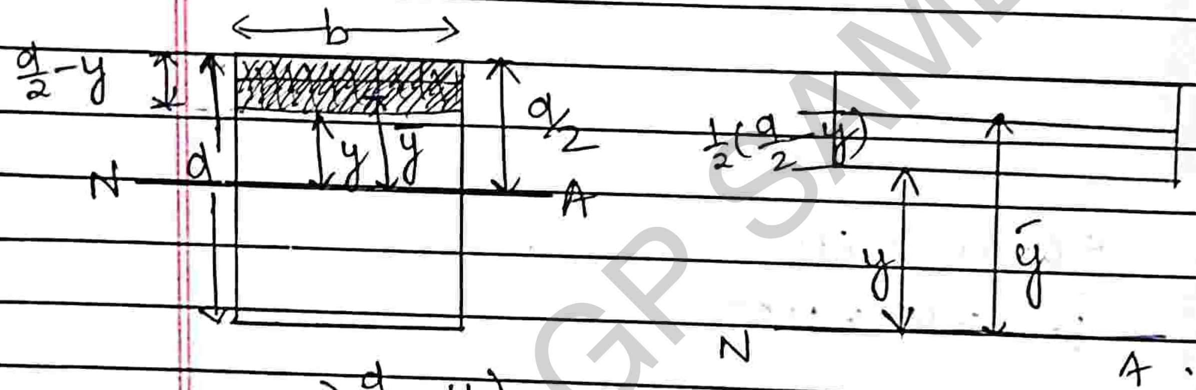
## Shear Stress :-

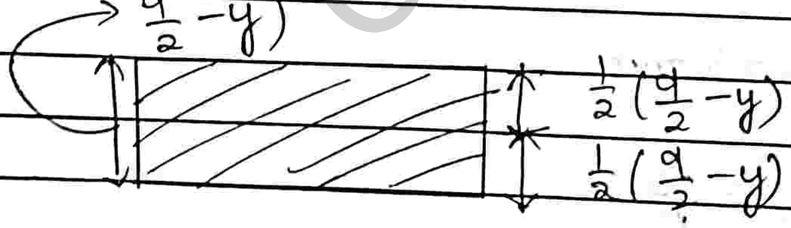
- Force acting parallel to the ~~ea~~ cross-sectional area

- It causes resistance to the shearing member

- Direction is opposite to the applied load

- It is denoted by Symbol ($\tau$) ~~Tea~~




$$\text{Area} = b \times \left(\frac{d}{2} - y\right)$$

$$I = \frac{bd^3}{12}$$

$$\bar{y} = \frac{1}{2}\left(\frac{d}{2} - y\right)$$

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b = width of rectangle
d = depth of rectangle
NA = Neutral axis
A = Area of strip
$\bar{y}$ = CG of the strip to the neutral axis
y = Distance of the strip to the neutral axis
I = Moment of inertia

$$\tau = F \times \frac{A\bar{y}}{I \cdot b}$$

$$= \frac{F \times b \times \left(\frac{d}{2} - y\right) \times \left\{y + \frac{1}{2}\left(\frac{d}{2} - y\right)\right\}}{\frac{bd^3}{12} \times b}$$

At the NA

$$y = 0$$

$$\tau = \frac{F \times \frac{bd}{2} \times \frac{d}{4}}{\frac{b^2d^3}{12}}$$

$$= \frac{F \times \frac{bd^2}{8}}{\frac{b^2d^3}{12}}$$

$$= F \times \frac{bd^2}{8} \times \frac{12}{b^2d^3} = \frac{3F}{2bd} = \frac{3}{2} \frac{F}{bd}$$

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***

$$\boxed{\tau = \frac{3}{2}\frac{F}{bd}}$$

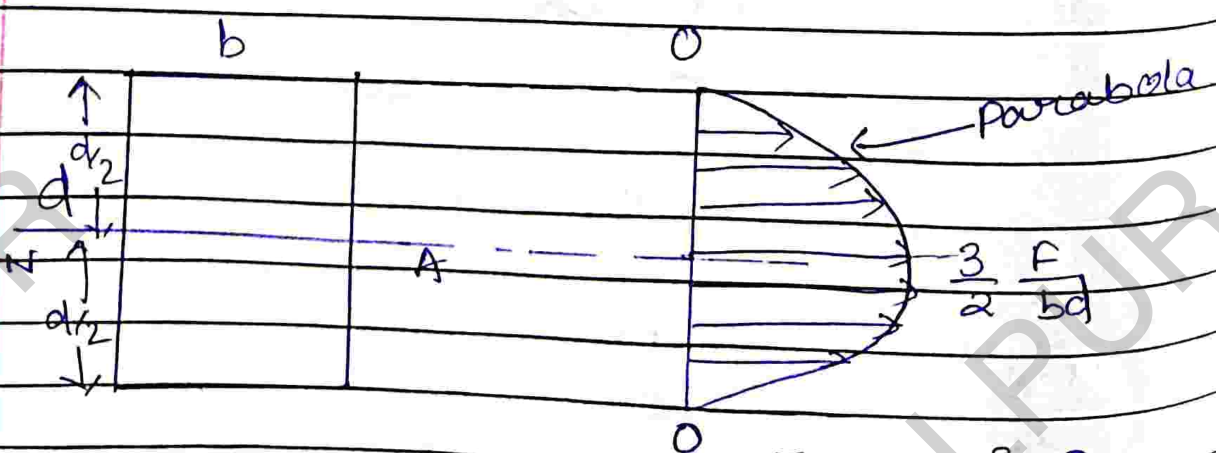
At outermost fiber

$$y = \frac{d}{2}$$

$$\tau = \frac{F \times b \times \left(\frac{d}{2} - y\right) \times \left\{y + \frac{1}{2}\left(\frac{d}{2} - y\right)\right\}}{\frac{bd^3}{12} \times b}$$

$$\tau = \frac{F \times b \times \cancel{\left(\frac{d}{2} - \frac{d}{2}\right)}^{\,0} \times \left(\frac{d}{2} + \cancel{\frac{1}{2}\left(\frac{d}{2} - \frac{d}{2}\right)}^{\,0}\right)}{\frac{bd^3}{12} \times b}$$

$$\tau = F \times 0 = 0$$



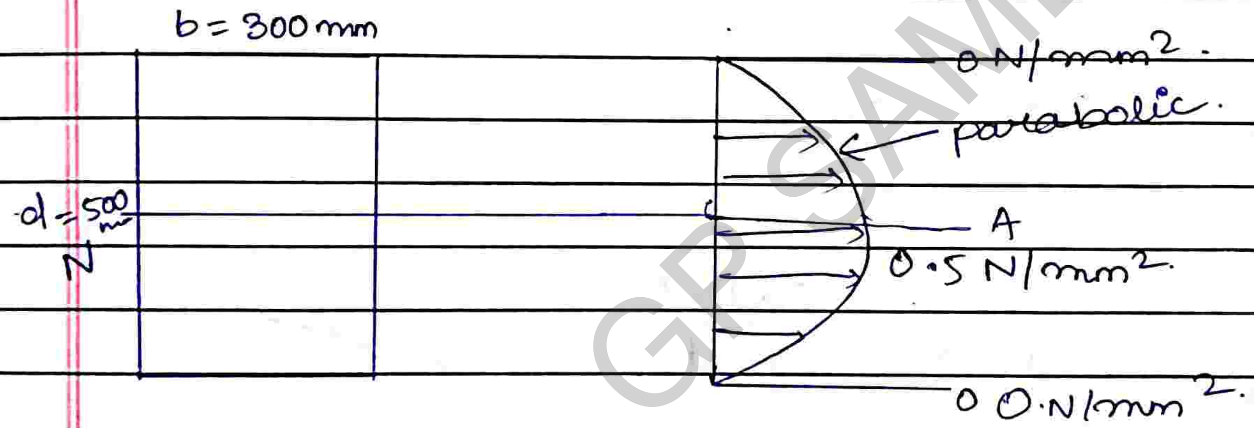

$$\tau_{max} = \frac{3}{2}\frac{F}{bd} = 1.5\frac{F}{A}.$$

Q) A rectangular beam ~~a~~ cross-section is 300mm × 500mm is subjected ~~by~~ to a maximum shear force of 50KN find the maximum shear stress and draw the distribution.

$$\tau = \frac{3}{2} \times \frac{50 \times 10^3}{300 \times 500}$$

$$= \frac{1}{2}$$

$$= 0.5 \text{ N/mm}^2.$$




Circular beam :-

$$\tau_{max} = \frac{4}{3} \frac{F}{A}$$

$$\boxed{\tau_{max} = 1.33 \frac{F}{A}}$$

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A rectangular beam 10cm wide is subjected to the maximum shear force of 50 KN, the corresponding maximum shear stress being $3 N/mm^2$. Calculate the depth of beam

→ width (b) = 10cm
= 100mm

Shear force F = 50 KN

Shear stress $\tau_{max} = 3 N/mm^2$.

d = ?

$$\tau = \frac{3}{2}\frac{F}{bd}$$

$$3 = \frac{3}{2} \times \frac{50 \times 10^3}{100 \times d}$$

$$3 = 1.5 \times \frac{50 \times 10^3}{100 \times d}$$

$$d = \frac{1.5 \times 50 \times 10^3}{100 \times 3}$$

d = 250mm.

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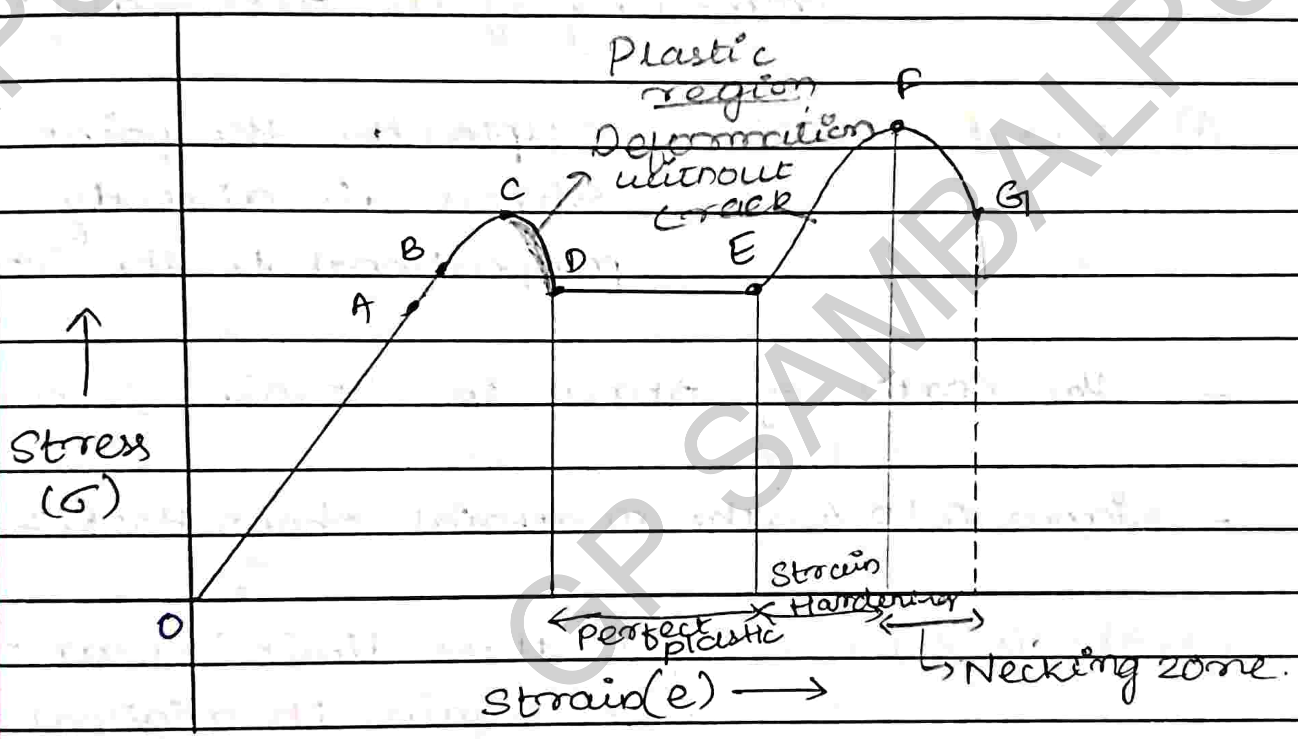
## Stress - Strain curve for mild steel bar.

when a ductile material like mild steel is Subjected to tensile force, it undergoes different Stages.

Stress - strain curve is the graphical representation of these stages.




A → proportional limit
B → Elastic limit
C → upper yield point
D → lower yield point
E → Strain Hardening
F → ultimate stress point
G → Failure point or Breaking point

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Elasticity :- It is the property of a material which allows to return its original shape after removing the external force.

Plasticity :- It is a property of a material which does not allows to return its original shape even after removing of the External force

(A) Proportional limit :- upto the ~~the~~ point A the Stress is directly proportional to the Strain

- The ratio of stress to Strain is constan

- from O to A the material obeys HOOKE's law

(B) Elastic limit :- upto these limit (B) material will regain its original Shape after removing the external load

(C) & (D) upper yield point and lower yield point :-

The upper yield point is the stress level just ~~below~~ before yielding start. ~~The~~

The lower yield point is the stress level at which yield maintained.

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(E) Strain Hardening :-

- In these point the material starts offering resistance against deformation

- These is because of change in the structure of mild steel.

(F) Ultimate Stress :- (Point F)

- These is the maximum stress that a material can bear

(G) Failure point :- (point G)

- In these point the Stress & Strain curve where material fails is known as Breaking point.

- Stress corresponding to these point is known failure stress or Breaking Stress

# Shear Force and Bending moment Diagram.

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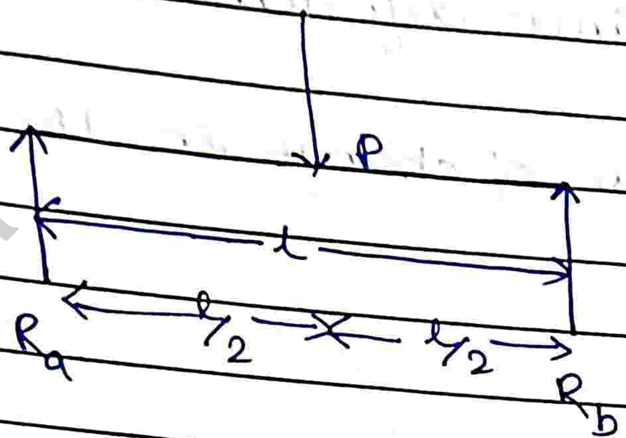
Simple Supported beam with point load at mid Section.

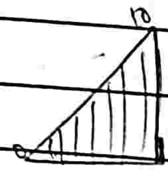




point load


UDL (2kN

uniformly distributed Load




uniformly Varing load

$\Sigma F_y = 0$

$R_a + R_b - P = 0.$

$\Sigma M_A = 0$

$-P \times \frac{l}{2} + R_b \times \cancel{\frac{l}{2}}\, l = 0$

$R_b \times l - P \times \frac{l}{2} + R_a \times 0 = 0.$

$R_b l - \frac{Pl}{2} = 0$

$R_b \times l = P \times \frac{l}{2}$

$R_b \times l = \frac{Pl}{2}$

$R_b = \frac{P\cancel{l}}{2\cancel{l}}$

$R_b = \frac{P}{2}$

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$R_a + R_b - P = 0.$

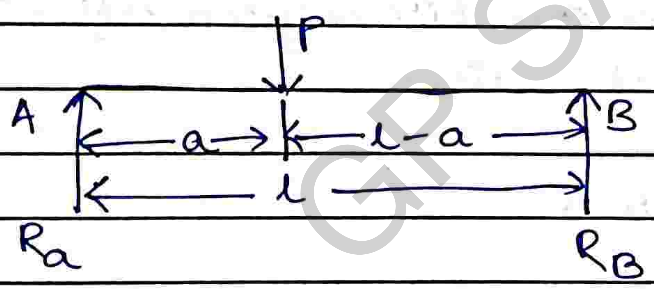
$R_a + R_b = P$

$R_a + \frac{P}{2} = P$

$R_a = P - \frac{P}{2}$

$R_a = \frac{P}{2}.$

Simply Supported with a point load at a distance 'a' from the left hand support :-




$\Sigma f_y = 0.$

$R_a + R_B - P = 0$

$\Sigma M_A = 0$

$R_B \times l - P \times a = 0.$

$R_B = \frac{Pa}{l}.$

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$$R_a + R_b - P = 0$$

$$R_a = P - R_b$$

$$R_a = P - \frac{Pa}{l}$$

$$R_a = \frac{lP - Pa}{l}$$

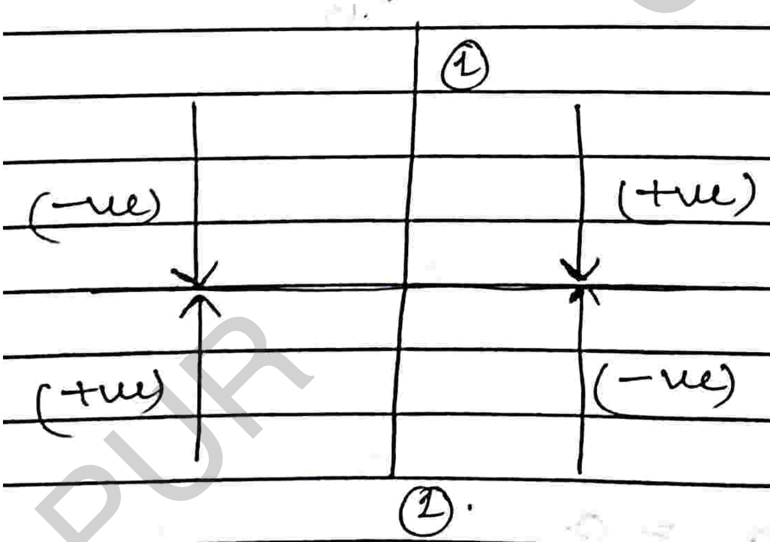
$$R_a = \frac{P(l-a)}{l}$$

2

## Shear Force

Shear force may be defined as the net sum of all the forces including the reaction acting at that section either from left side or Right side of that section.

Sign convention

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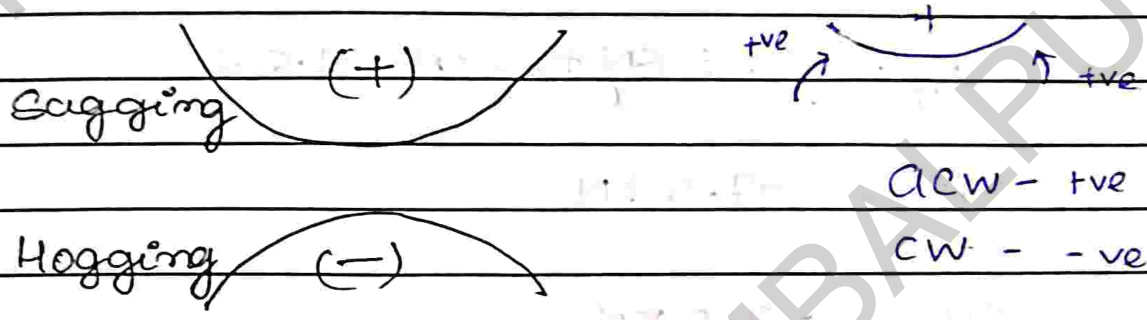
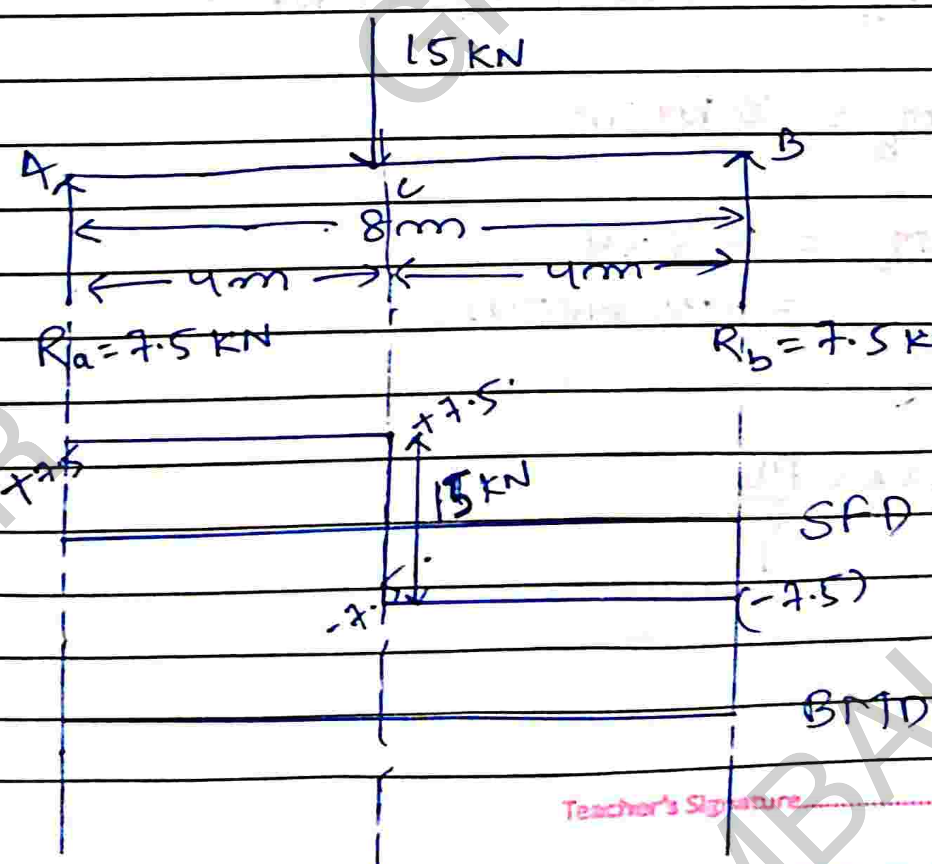
## Bending ~~Force~~ moment :-

Bending moment at a section may be defined as the net sum of all the moment either from left side or from Right side.

Sign convention




A simply supported beam of 8m span length, having a point load of 15 KN at the centre of the beam. Draw the SFD and BMD.



when load is applied at that point we should calculate two time.

$R_a = \frac{P}{2} = \frac{15}{2} = 7.5\ KN$

$R_b = \frac{P}{2} = \frac{15}{2} = 7.5\ KN$

SFD

$SF_A = -7.5\ KN + 15\ KN = +7.5\ KN$

$SF_B = -7.5\ KN$

$SF_C = -7.5\ KN$
$= -7.5\ KN + 15.$
$= +7.5\ KN.$

BMD

$BM_A = 7.5 \times 8 - 15 \times 4 = 0\ KN\text{-}m$

$BM_B = 0\ KN\text{-}m$

$BM_C = 7.5 \times 4$
$= +30\ KN\text{-}m.$

$Max = \frac{PL}{4}$

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$x^0 \rightarrow$

$x^1 \rightarrow$

$x^2 \rightarrow$ parabolic

$x^3 \rightarrow$ cubic

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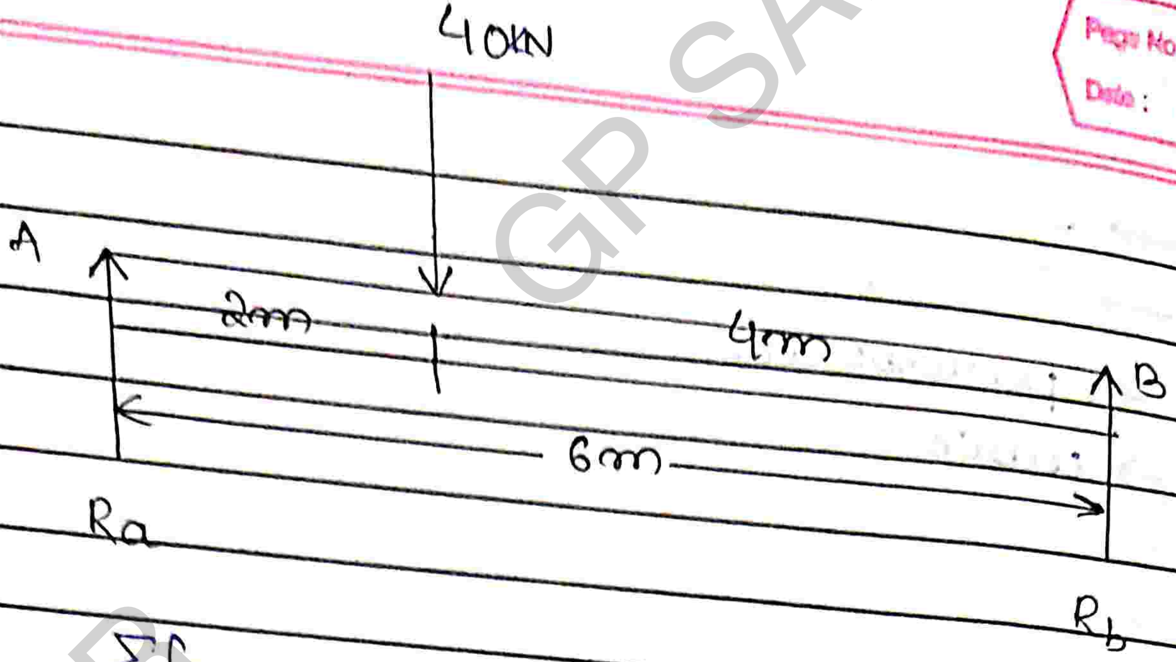



$\Sigma fy = 0$

$\Rightarrow R_a + R_b - 40 = 0$

$\Rightarrow R_a + R_b = 40\ kN$

$M_A = 0$

$R_b \times 6 - 40 \times 2 = 0$

$R_b \times 6 - 80 = 0$

$R_b \times 6 = 80$

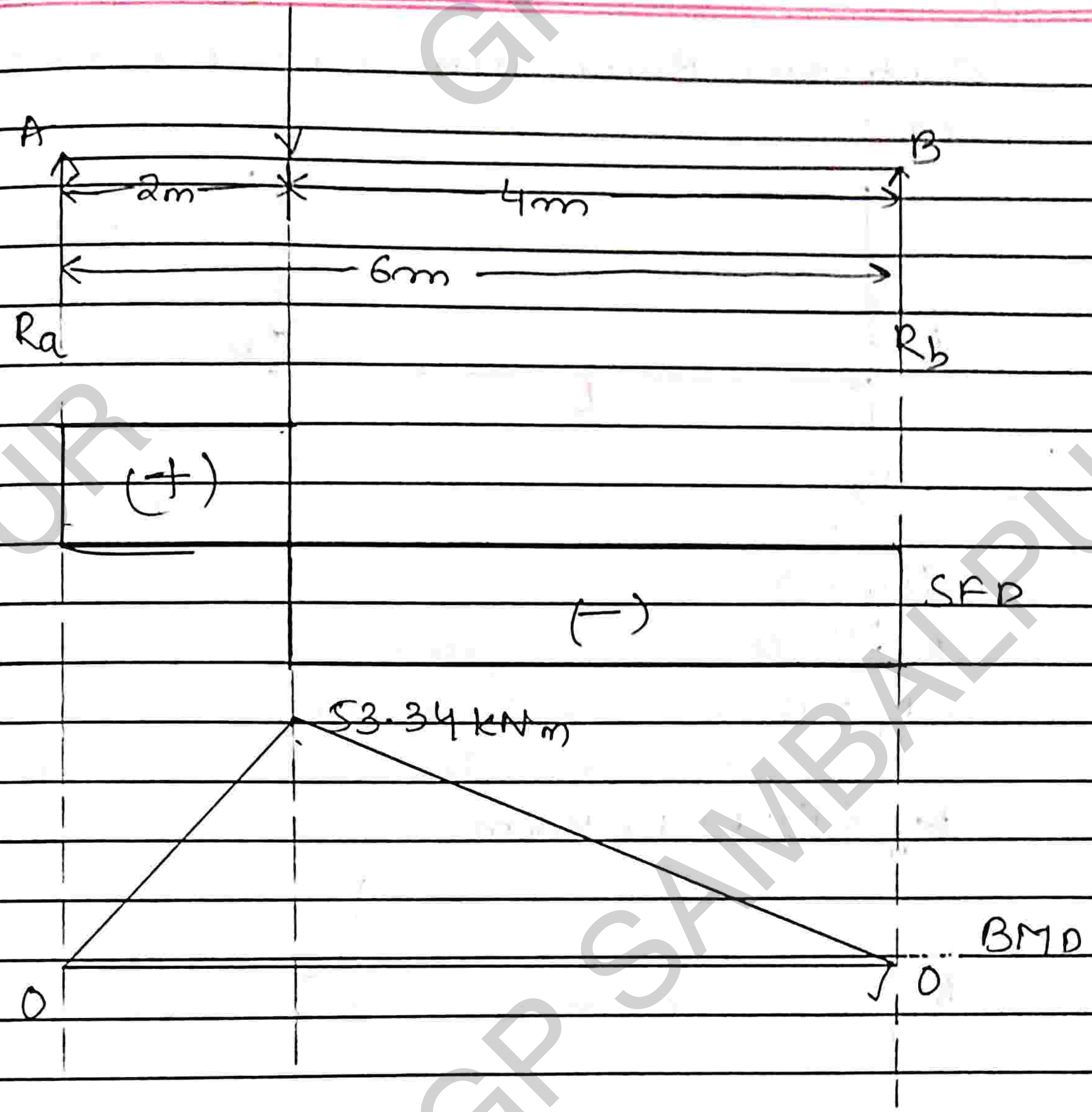
$R_b = \frac{80}{6}$

$R_b = 13.33\ kN$

$R_a + 13.33 = 40$

$R_a = 40 - 13.33$

$R_a = 26.67\ kN.$




**SFD** | **BMD**

$SF_A = -13.33 + 40$
$= 26.67$ kN.

$SF_B = -13.33$ kN

$SF_C = -13.33 + 40 = 26.7$ kN (with load)

$= -13.33$ kN (without load.

$M_A = 0$ KN-m
$M_B = 0$ KN-m

$M_C =$ ~~53.34~~
$26.67 \times 2$
$= 53.34$ kNm.

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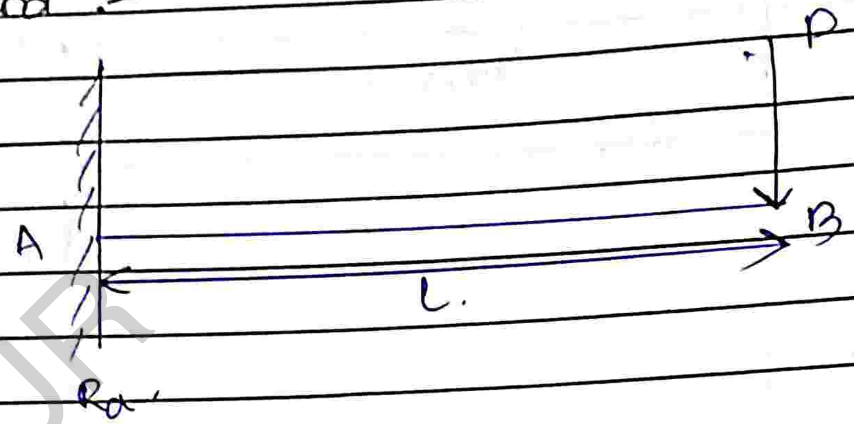
Cantilever Beam with a point load at free end :-

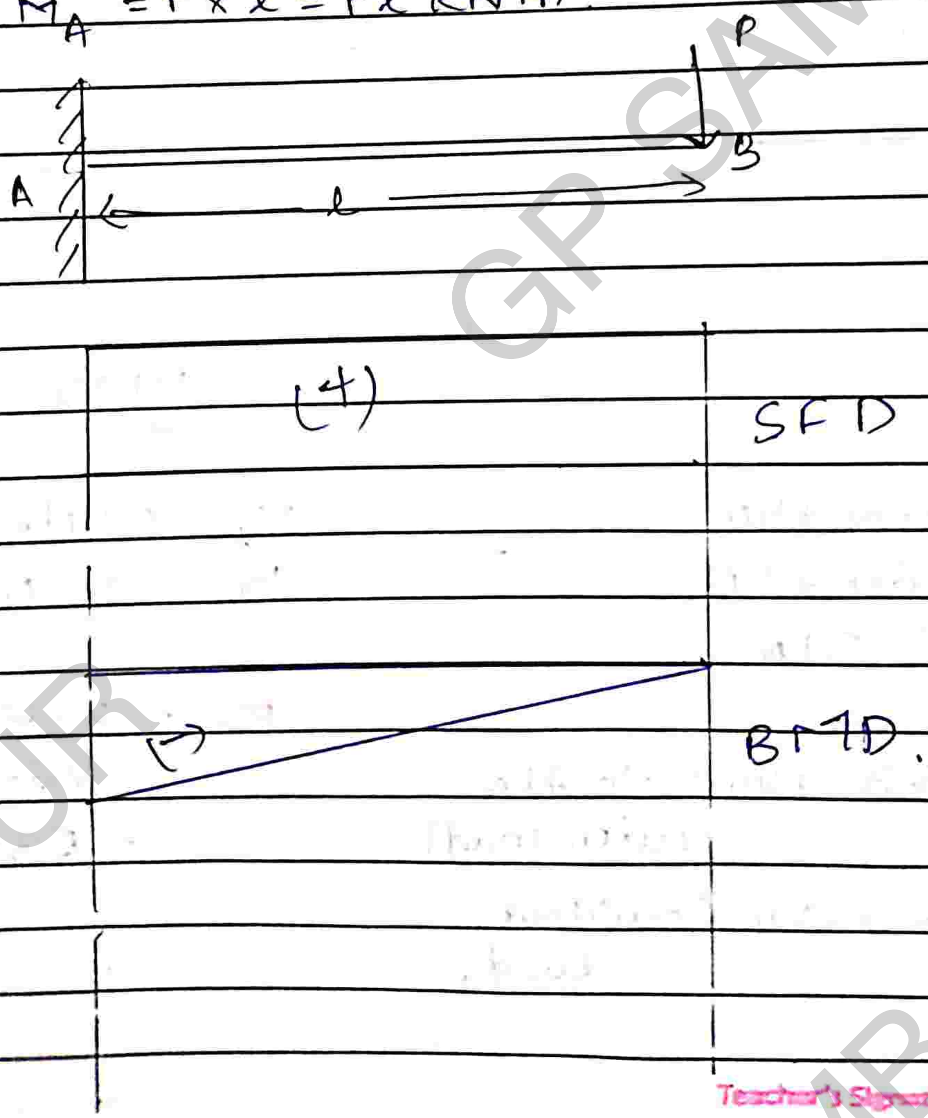



$\Sigma f_y = 0$

$R_a - P = 0$

$R_a = P$

$M_A = P \times l = Pl$ KNm

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## SFD

$SF_A = +P$

$SF_B = +P$

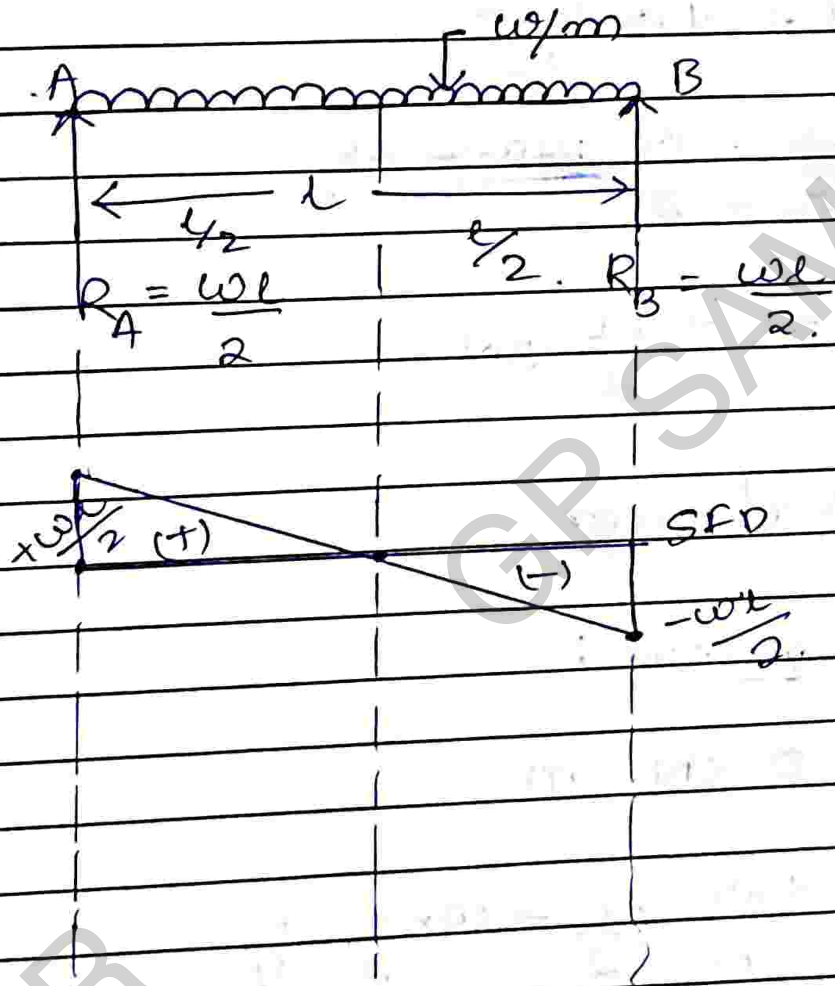
## BMD

$M_A = -Pl \ KNm$

$M_B = 0.$

Simply Supported Beam with U.D.L throughout the length.

UDL = uniformly distributed load.



Point = load × dis

Shear force.

$SF_B = \frac{-wL}{2}$ kN.

$SF_C = \frac{-wl}{2} + \frac{wl}{2}$.

$= 0$ KN

$SF_A = \frac{-wl}{2} + wl$

=

$= \frac{+wl}{2}$ KN

$l \to l^1$ power.

Bending moment :-

$BM_B = 0$ KN-m.

$BM_C = \frac{+wl}{2} \times \frac{l}{2} - w \times \frac{l}{2} \times \frac{l}{4}$.

$= \frac{wl^2}{4} - \frac{w \times l^2}{8}$.

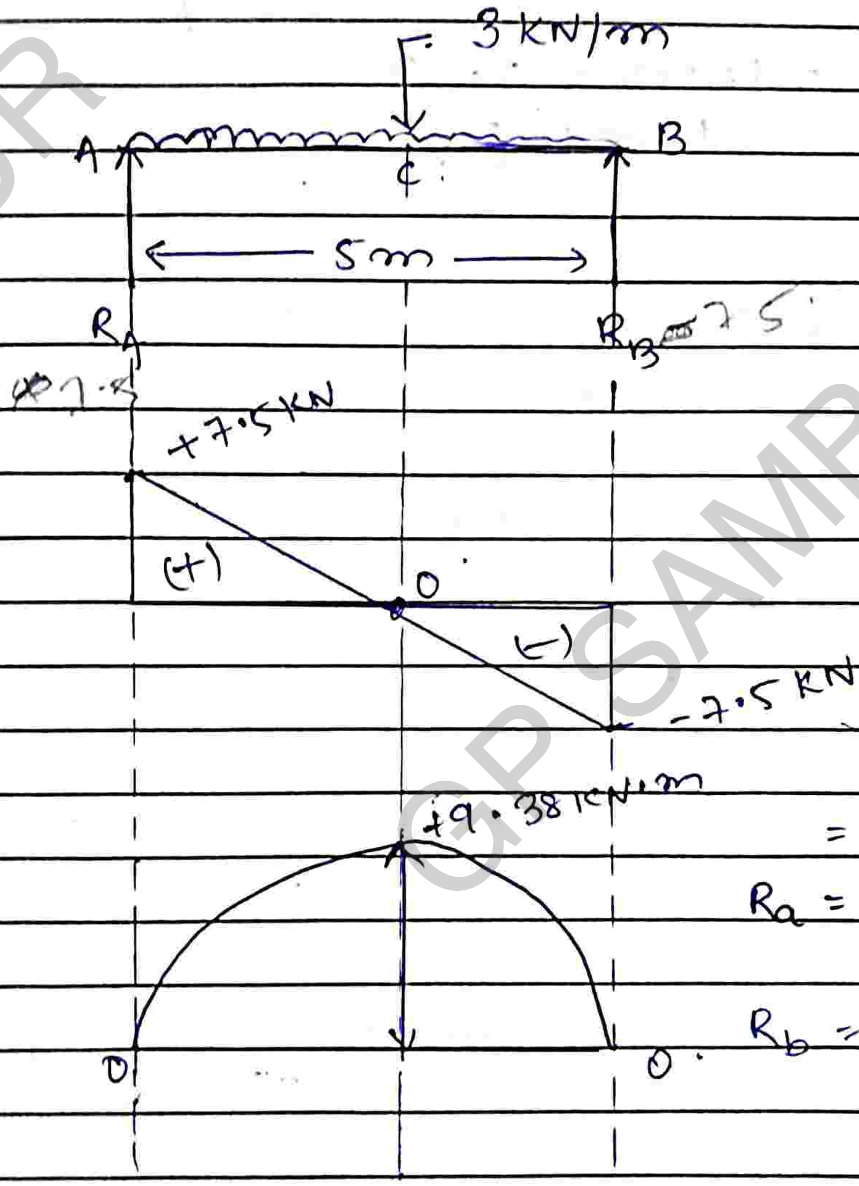
$= \frac{2wl^2 - wl^2}{8}$

$= \frac{wl^2}{8}$.

Teacher's Signature

$BM_A = 0$ KN.

Draw the shear force and BMD of a simply Support beam having 5m Span. and subjected UDL of 3KN/m.




$= \frac{wl}{2} = \frac{3 \times 5}{2} = \frac{15}{2}$

$R_a = \frac{15}{2} = 7.5$ KN

$R_b = \frac{15}{2} = 7.5$ KN

Shear force

$SF_B = -3 \times 2.5 = -7.5$ KN.

$SF_C = -3 \times 2.5 + 3 \times 2.5 = 0.$

$SF_A = -3 \times 2.5 + 3 \times 5 = +7.5$ KN.

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$BM_B = 0$

$BM_A = 0$.

$BM_C = 7.5 \times 2.5 - 7.5 \times \frac{5}{4}$

$= 18.75 - 9.37$

$= 9.38$ KN/m



Teacher's Signature

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Reactions

$R_a = \frac{wl}{2} = \frac{3\times5}{2} = \frac{15}{2} = 7.5 KN.$

$R_b = \frac{wl}{2} = \frac{3\times5}{2} = \frac{15}{2} = 7.5 KN.$

SFD

$SF_B = -\frac{wl}{2} = -\frac{3\times5}{2} = -\frac{15}{2} = -7.5 KN.$

$SF_A = -7.5 + 15 = +7.5.$

$SF_C = -7.5 + 7.5 = 0 KN.$

BMD

$BM_B = R_b \times 0 = 0 KN\text{-}m$

$BM_C = +7.5\times2.5 - 3\times2.5\times1.25$

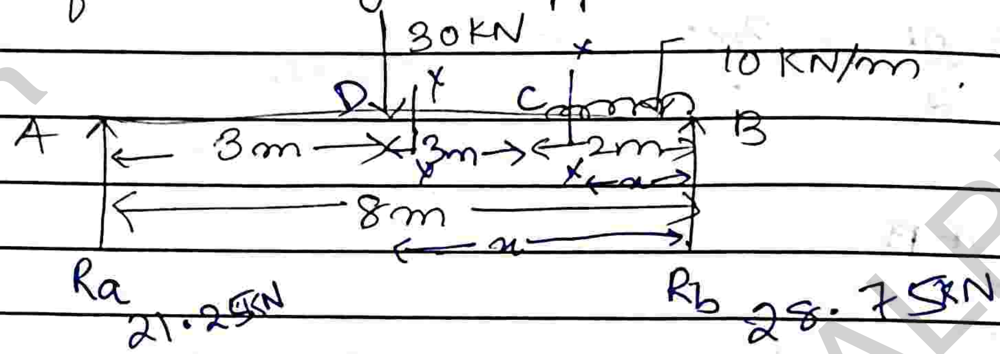
$= 9.375 KN\text{-}m$

$BM_A = 7.5\times5 - 3\times5\times2.5.$

$= 0 KN\text{-}m.$

Teacher's Signature..........

Draw the SFD and BMD of a Simply Supported beam having 8m span length Subjected to 30KN point load at a distance 3m from left hand support and UDL of 10KN/m of 2m from the right support.




$\Sigma f_y = 0.$

$R_a + R_b - 30 - 20 = 0.$

$R_a + R_b - 50 = 0$

$R_a + R_b = 50$

$\Sigma M_A = 0$

$R_b \times 8 - 20 \times 7 - 30 \times 3 = 0.$

$R_b \times 8 - 140 - 90 = 0.$

$8R_b - 230 = 0.$

$R_b = \frac{230}{8}$

$= 28.75 \text{ kN}$

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$R_a + R_b = 50.$

$R_a = 50 - 28.75.$

$= 21.25\ KN.$

Shear force Equation :-

Portion BC (x from B)

$S_x = -28.75 + 10x =$

$S_B = -28.75\ KN$

$S_C = -28.75 + 20$

$= -8.75\ KN$

portion CD (x from D)

$S_x = -28.75 + 20$

$= -8.75\ KN$

$S_C = -8.75\ KN.$

$S_B = -8.75\ KN.$

Portion DA (x from D)

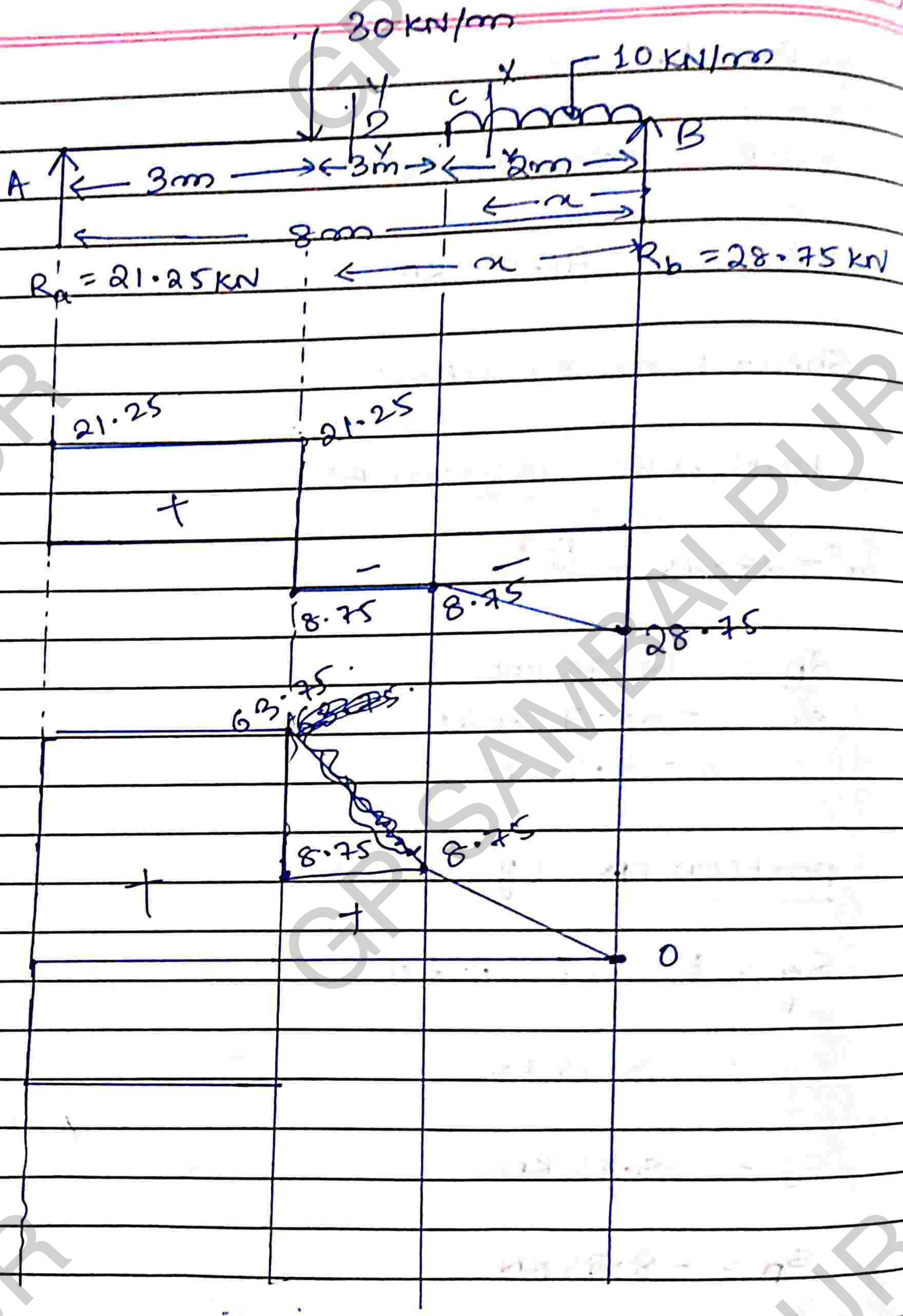
$S_x = -28.75 + 10 \times 2 + 30$

$= 21.25\ KN$

$S_D = 21.25\ KN$

$S_A = 21.25\ KN$

Teacher's Signature.........................


30 kN/m
10 kN/m
A
B
C
D
Y
3m
3m
2m
x
8m
$R_a$ = 21.25 KN
$R_b$ = 28.75 kN
21.25
21.25
+
-
-
8.75
8.75
28.75
63.75
8.75
8.75
+
+
0

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Bending moment

Portion BC (x from B)

$\boxed{M_B = 0 \text{ KNm}}$

~~$BM_E = 28.75 \times 1 - 10 \times 0.5$~~
~~$= 23.75$~~

$M_C = 28.75 \times 2 - 20 \times 1$

$\boxed{M_C = 37.5 \text{ KNm}}$

Portion CD (x from B)

~~$M_C = 8.75 \text{ KNm}$~~

$M_D = 28.75 \times 5 - 20 \times (4)$

$\boxed{M_D = 63.75 \text{ KNm}}$

Portion DA

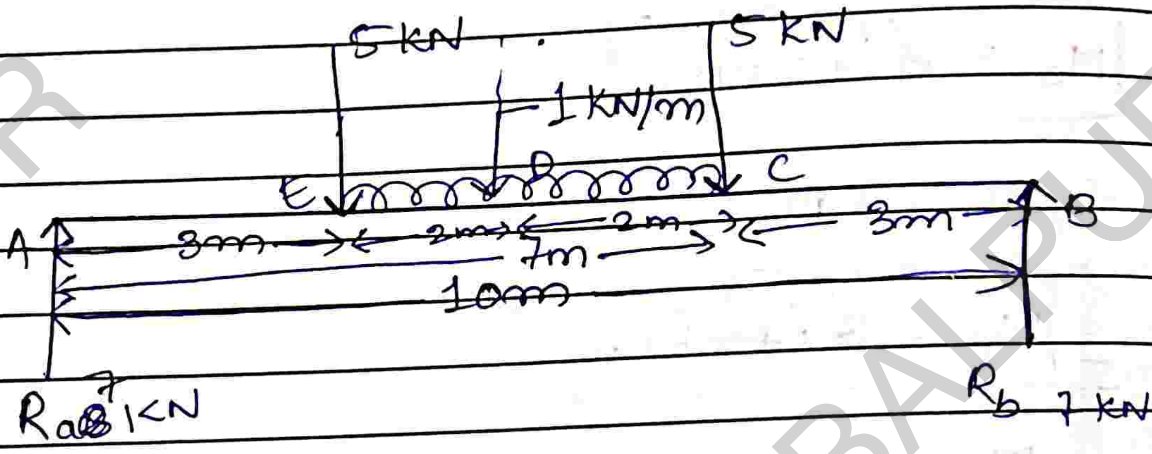
$M_D = 28.75 \times 5 - 20 \times 4$
$= 63.75 \text{ KNm}$

$M_A =$ ~~$28.75 \times 5 - 20 \times 4 - 30 \times 3$~~ $0.$

~~$= 26.25 \text{ KNm}$~~

Teacher's Signature.........................

A beam AB 10m long has supports at its ends A and B. It carries a point load 5KN at 3m from A and a point load of 5KN at 7m from A and uniformly distributed load of 1KN/m between the point load. Draw SFD and BMD for the beam.




$\Sigma f_y = 0$

$R_a - 5 - 5 + R_b = 0$

$R_a + R_b - 10 = 0$

$R_a + R_b = 10$ ——— (1)

$\Sigma M_A = 0$

~~$R_b \times 10 - 5 \times 7 - 5 \times 3 = 0$~~

~~$10R_b - 35$~~

$R_b \times 10 - 5 \times 7 - 4 \times 5 - 5 \times 3 = 0$

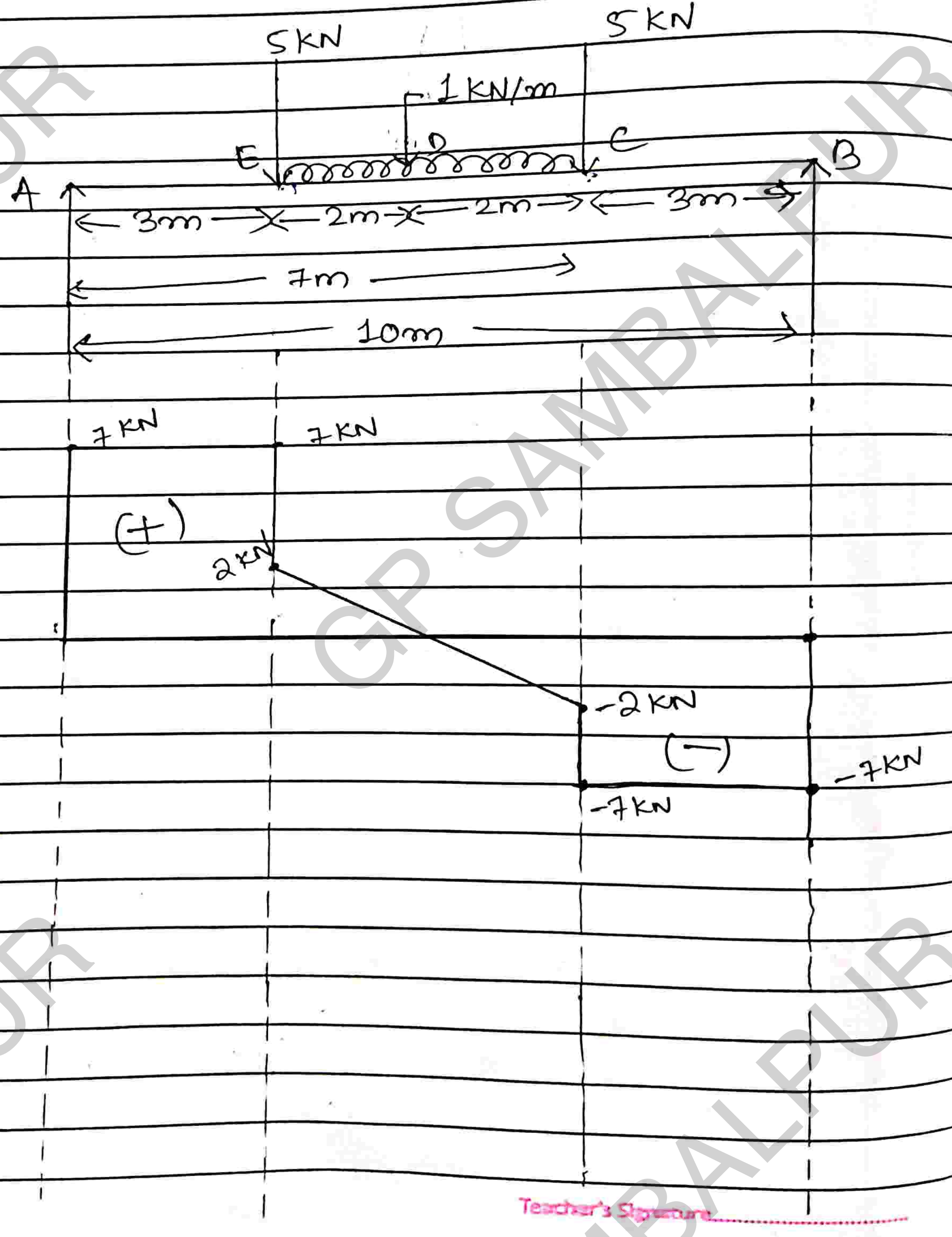
$10R_b - 35 - 20 - 15 = 0$

$R_b = \frac{70}{10}$

$R_b = 7KN$

Teacher's Signature

A beam AB 10m long has supports at its ends A and B. It carries a point load 5 KN at 3m from A and a point load of 5 KN at 7m from A and a uniformly distributed load of 1 KN/m betn the point load. Draw SFD and BMD of the beam.



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$\Sigma f_y = 0$

$R_a + R_b - 5 - 5 - (1 \times 4) = 0.$

$R_a + R_b - 5 - 5 - 4 = 0$

$R_a + R_b = 14$ ——— (1)

$\Sigma M_A = 0$

$R_b \times 10 - 5 \times 7 - (1 \times 4) \times 5 - 5 \times 3 = 0$

$10 R_b - 35 - 20 - 15 = 0.$

$10 R_b = 70$

$\boxed{R_b = 7 \text{ KN}}$

$R_a + R_b = 14$

$R_a + 7 = 14$

$\boxed{R_a = 7 \text{ KN}}$

Calculation for SFD

$SF_B = -7 \text{ KN}$

$SF_C = (-7 + 5) \text{KN} = -2 \text{ KN}$

$SF_E = \cancel{7} - 2 + (1 \times 4) = 2 \text{ KN}$

$SF_E = 2 + 5 = 7 \text{ KN}$

$SF_A =$ [illegible] KN

Teacher's Signature..............................

Bending Moment :-

$BM_B = 0$ KNm.

$BM_C = 7 \times 3 = 21$ KNm

$BM_E = 7 \times 7 - 5 \times 4 - 4 \times 2 = 21$ KNm

$BM_A = 7 \times 10 - 5 \times 7 - 4 \times 5 - 5 \times 3 = 0$ KNm

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# Mechanical properties of materials [2 Marks]

(1) Hardness – The property of material by which it resist the local deformation ~~when~~ when undergoes on impact.

- It is the state of material, being for which it can withstand the load.

(2) Creep – These is the permanent change in shape and size of a material under the application of load or temperature.

- It is the temperature dependent (time).

- creep at different temperature for different materials

(3) Rigidity – These is defined as the property possessed by a solid body to change its shape

- These is the property of material to resist the bending.

(4) Fatigue :– These is the property of a material subjected to repeated cyclic load to resist the deterioration.

(5) Durability :– The ability of a material to remain serviceable during the useful time without damaging the material.

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- It represents how long the material works.

(6.) Elasticity & plasticity

when the force is applied on a material it undergoes deformation but after ~~more~~ removing the force it return back to its original shape. This property of material is called as Elasticity

when the force is ~~f~~ applied on a material

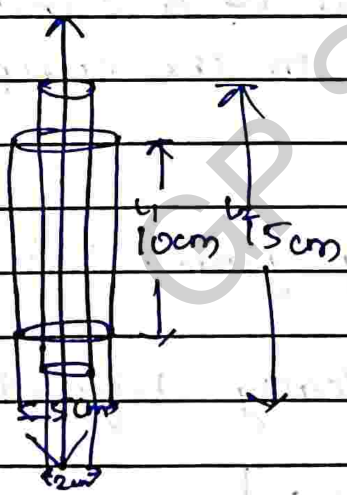
## Poisson's Ratio :-

- Poisson's Ratio is defined as "the ratio between the lateral strain to the longitudinal strain"

- It is denoted by symbol ($\mu$)

$$\boxed{\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}}$$

$$\mu = \left| \frac{\text{lateral strain}}{\text{longitudinal strain}} \right|$$




$$\mu = \left| \frac{-3}{5} \right|$$

$$= \frac{3}{5}$$



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## MODULUS OF RIGIDITY :-

1. — It is also known as Shear modulus

2. — It is the ~~major~~ measure of the rigidity of body, given by the ratio of shear stress to shear strain

3. — It is denoted by the symbol 'G'

$$\text{Shear modulus} = \frac{\text{Shear stress}}{\text{Shear strain}} \quad \frac{\tau}{e}$$

## Bulk modulus

- The Bulk modulus of a Substance is a major of how resistant to the compression of the substance

- It is denoted by Symbol 'K'

- It is mathematically defined as

$$\boxed{K = -V\left(\frac{dP}{dV}\right)}$$

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Relationship between :-

$$\boxed{E = 2G(1+\mu)}$$

$$\boxed{E = 3K(1-2\mu)}$$

Q A cylindrical bar is 20mm diameter and 750mm long. During test, longitudinal strain was found 3 times of the lateral strain calculate the modulus of rigidity and Bulk modulus. If the $E = 2\times10^5\ N/mm^2$.

Lateral strain $= x$

longitudinal strain $= 3x$

$$\mu = \frac{x}{3x}$$

$$\mu = \frac{1}{3}$$

$$\mu = 0.33$$

$$2\times10^5 = 2G(1+0.33)$$

$$2G = \frac{2\times10^5}{1.33}$$

$$G = \frac{2\times10^5}{1.33\times2}$$

$$G = 75.18\times10^3\ N/mm^2$$

$$2\times10^5 = 3K(1-2\times\cancel{0.33}\tfrac{1}{3})$$

$$3K = \frac{2\times10^5}{\cancel{0.34}\frac{1}{\cancel{3}}\times\cancel{3}}$$

$$K = \cancel{196.07\times10^3}\ N/mm^2$$

$$2\times10^5\ N/mm^2$$

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(ii) Also find the change volume when volumetric stress of 100 $N/mm^2$ is applied.

$$K = \frac{\text{Volumetric Stress}}{\text{Volumetric strain}}$$

$$2\times10^5 \, N/mm^2 = \frac{100 \, N/mm^2}{\frac{dv}{v}}$$

$$dv = \frac{100}{2\times10^5} \times v$$

$$dv = \frac{100}{2\times10^5} \times l \times A \quad \left[A = \frac{\pi}{4}\times 20^2\right]$$

$$= 314.159$$

$$dv = \frac{100}{2\times10^5} \times 750 \times 314.159$$

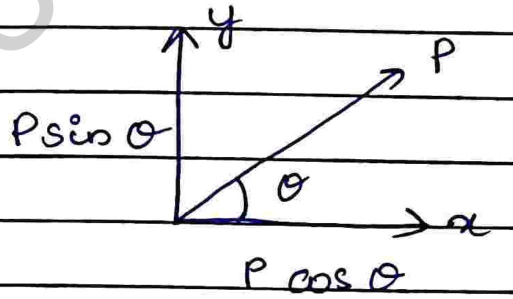
$$dv = 117.80 \, mm^3$$

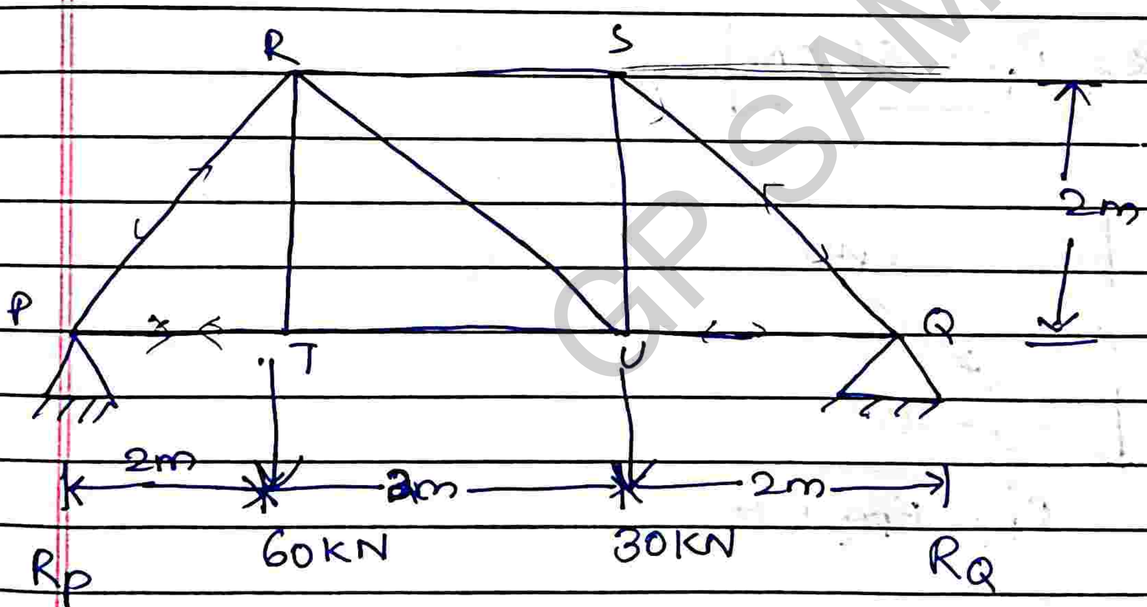
## Analysis of trusses :-

### Define truss :-

A framework, typically consisting of posts struts & Supporting a roof or bridges.

### Resolution of forces :-




$$\Sigma f_y = 0$$

$$\rightarrow R_p - RT + RQ - SU = 0$$

$$R_p + R_Q - 60 - 30 = 0$$

$$R_p + R_Q = 90$$

Take $M_P = 0$

$(-R_Q \times 6) + (SU \times 4) + (RT \times 2)$

$(-RQ \times 6) + (30 \times 4) + (60 \times 2)$

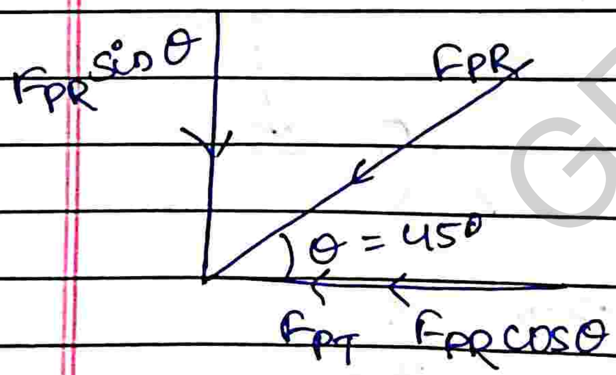
$-6RQ + 120 + 120$

$RQ = \frac{240}{6}$

$\boxed{RQ = 40\,KN}$

$R_P + RQ = 90\,KN$

$\cancel{RQ}\ \boxed{R_P = 50\,KN}$




$\Sigma f_y = 0$

$\Rightarrow +R_P - F_{PR}\sin\theta = 0$

$\Rightarrow R_P = F_{PR}\sin\theta$

$\Rightarrow 50 = F_{PR}\sin 45°$

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$\Rightarrow \frac{50}{\sin 45} = F_{PR}$.

$\Rightarrow \boxed{70.71 KN = F_{PR}}$. [Tension]

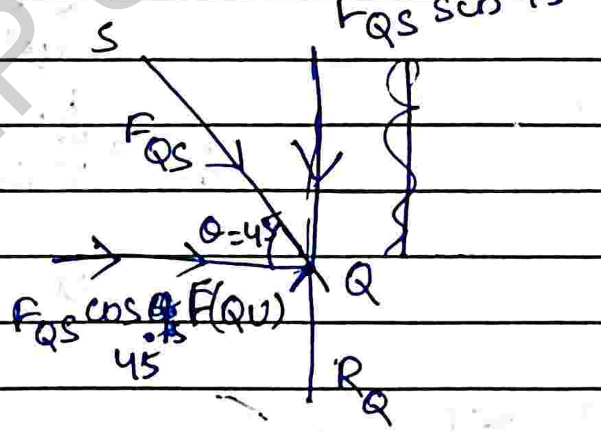
$\Sigma f_x = 0$.

$F_{PT} + F_{PR} \cos\theta =$

$F_{PT} + 70.71 \times \frac{1}{\sqrt{2}}$

$\boxed{F_{PT} = (-) 49.99 \ (c)}$

FBD of point Q




$\Sigma f_y = 0$

$R_Q - F_{QS} \sin 45 = 0$

$R_Q = F_{QS} \sin 45°$

$F_{QS} = \frac{40}{\sin 45°}$

$F_{QS} = 40\sqrt{2}$.

$\boxed{F_{QS} = 56.56 \text{ kN}}$ (Tension)

Teacher's Signature

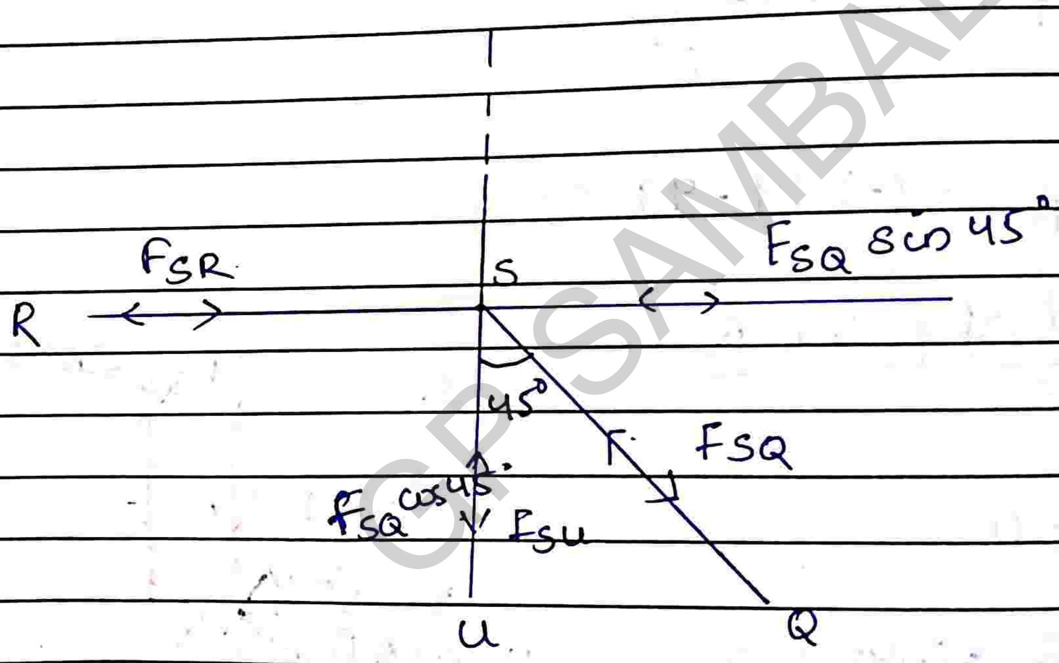
$\Sigma f_x = 0.$

$F_{QS} \cos 45 + F_{QU} = 0.$

$F_{QU} = -F_{QS} \cos 45.$

$= -56.56 \times \frac{1}{\sqrt{2}}.$

$\boxed{F_{QU} = -39.99}$ (compression)




$\Sigma f_x = 0$

$\Rightarrow F_{SR} - F_{SQ} \sin 45 = 0$

$\Rightarrow F_{SR} = F_{SQ} \sin 45°$

$= 56.56 \times \frac{1}{\sqrt{2}}.$

$\boxed{F_{SR} = 39.99 \text{ KN (Tension)}}$

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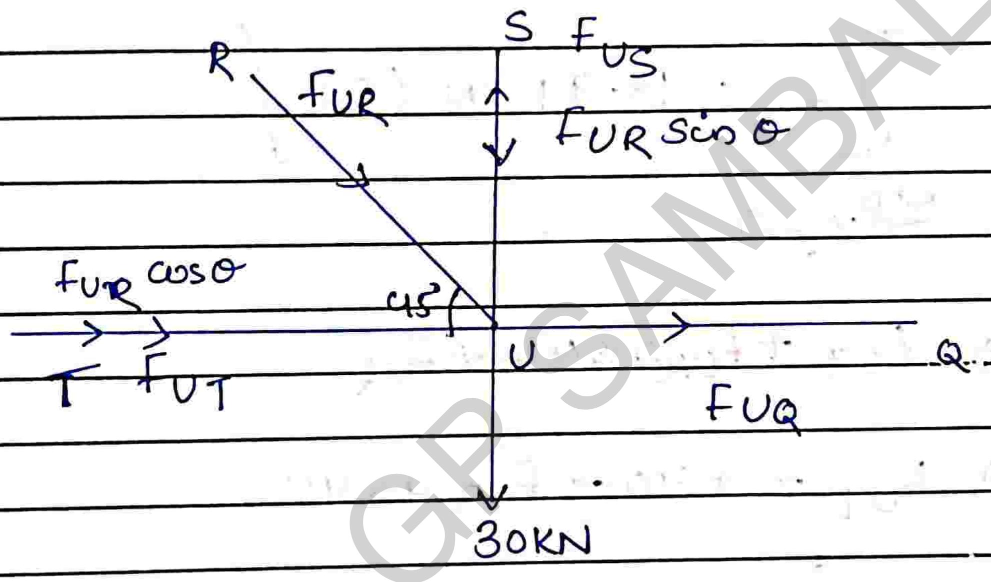
$\Sigma f_y = 0$

$F_{SU} + F_{SQ} \cos\theta = 0$

$F_{SU} = -F_{SQ} \cos 45^\circ$

$= -56.56 \times \cos 45^\circ$

$\boxed{F_{SU} = -39.99 \text{ kN}}$ (compression)




~~$\Sigma f_x = 0$~~

~~$F_{UT} + F_{UR} \cos 45 + F_{UQ} = 0$.~~

$F_{UT} =$

Teacher's Signature

$\Sigma f_y = 0$

$\Rightarrow f_{US} + f_{UR} \sin\theta - 30$

$\Rightarrow +30 + F_{UR} \sin 45^\circ + 40 = 0$

$\Rightarrow F_{UR} \sin 45 = -70$

$\Rightarrow F_{UR} = \dfrac{-70}{\sin 45}$

$\boxed{F_{UR} = -98.99 \text{ KN (C)}}$

$\Sigma f_x = 0$

$\Rightarrow F_{UT} + f_{UR} \cos\theta = + f_{UQ}$

$\Rightarrow F_{UT} = +40 - f_{UR} \cos 45^\circ$

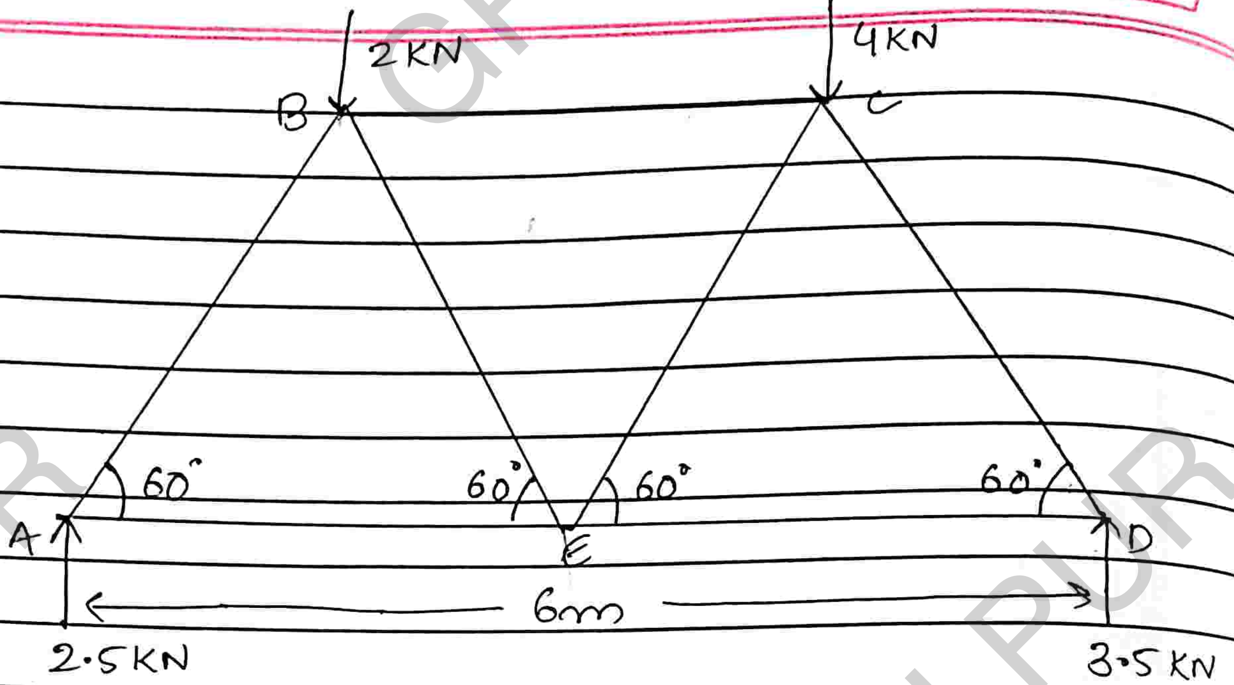
$= \cancel{40} (\cdot 40 - (-98.99 \times \cos 45^\circ)$

$= 40 + 98.99 \times \cos 45$

$\boxed{f_{UT} = 109.99 \text{ KN (T)}}$

Find out forces in all the members with their nature as tensile or compressive as shown in fig. below [10]






$\Sigma f_y = 0$

$R_A + R_D - 2 - 4 = 0$

$R_A + R_D - 6 = 0$

$R_A + R_D = 6\,KN \quad \text{---(1)}$

$M_A = 0$

$R_D \times 6 - 4 \times 4.5 - 2 \times 1.5 = 0$

$6R_D - 21 = 0$

$R_D = \frac{21}{6}$

$R_D = 3.5\,KN$

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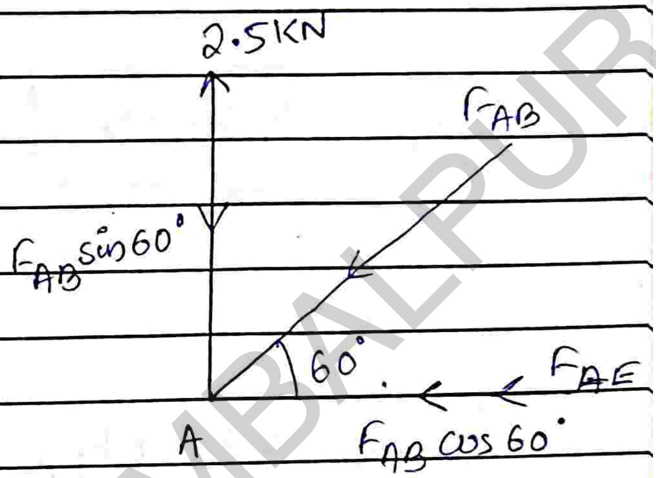
$R_A + R_D = 6\ KN$

$R_A + 3.5 = 6\ KN$

$R_A = 6 - 3.5$

$R_A = 2.5\ KN.$

For point A :-




$\Sigma f_y = 0$

$= -f_{AB}\sin 60° + 2.5 = 0$

$F_{AB}\sin 60 = 2.5$

$F_{AB} = \frac{2.5}{\sin 60°}$

$\boxed{F_{AB} = 2.88\ KN}$ (T)

$\Sigma f_x = 0$

$-F_{AB}\cos 60° - F_{AE} = 0$

$F_{AE} = -f_{AB}\cos 60°$

$F_{AE} = -2.88 \times \frac{1}{2}.$

$\boxed{F_{AE} = -1.44\ KN}$ (C)

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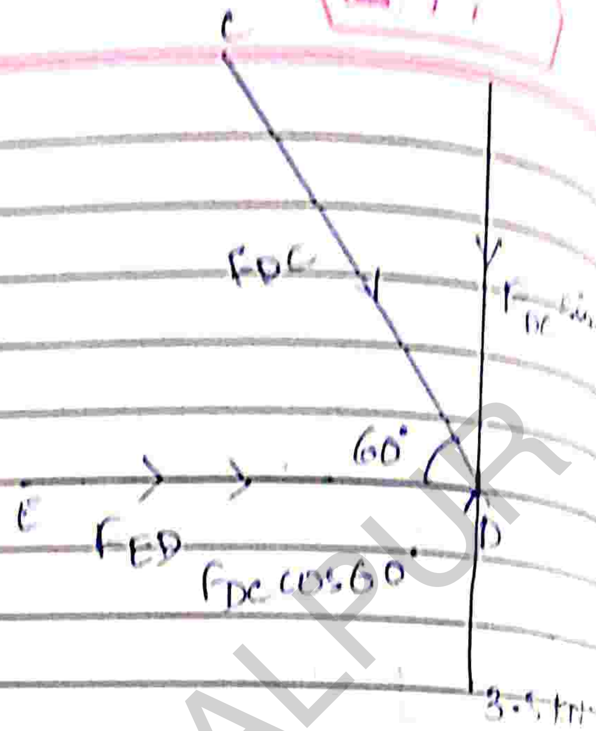


$\Sigma F_y = 0$

$3.5 - F_{DC} \sin 60° = 0$

$F_{DC} = \dfrac{3.5}{\sin 60°}$

$\boxed{F_{DC} = 4.04 \text{ kN}}$ (T)

$\Sigma F_x = 0$

$F_{DC} \cos 60° + F_{ED} = 0$

$F_{ED} = -F_{DC} \cos 60$

$= -4.04 \times \cos 60°$

$\boxed{F_{ED} = -2.02 \text{ kN}}$ (C)

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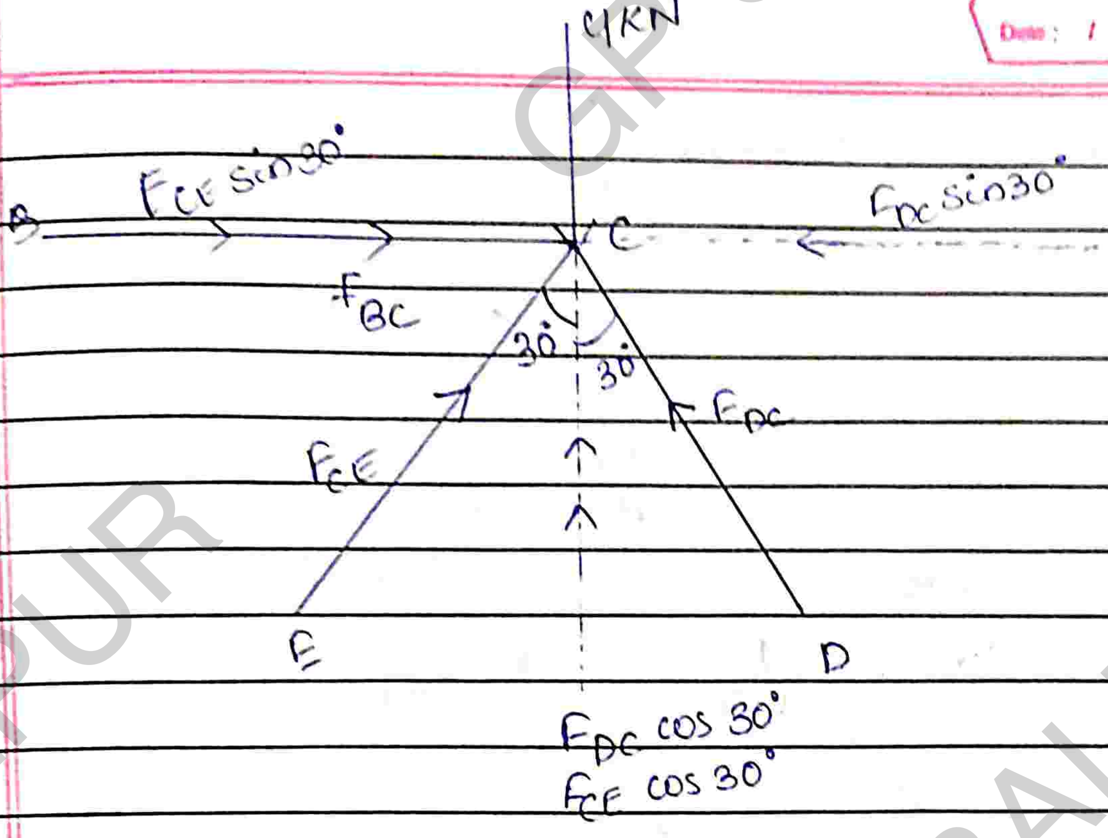


$\Sigma f_y = 0$

$F_{DC}\cos 30° + F_{CE}\cos 30° - 4 = 0$

$F_{CE}\cos 30° = 4 - F_{DC}\cos 30°$

$= 4 - 4.04 \times \cos 30°$

$= 4 - 3.49$

$= \dfrac{0.51\ KN}{\cos 30°}$

$\boxed{F_{CE} = 0.588\ KN}$

$\Sigma f_x = 0.$

$F_{BC} + F_{CE}\sin 30° - F_{DC}\sin 30° = 0.$

$F_{BC} + 0.588 \times \sin 30° - 4.04 \times \sin 30° = 0$

$F_{BC} + 0.294 - 2.02 = 0.$

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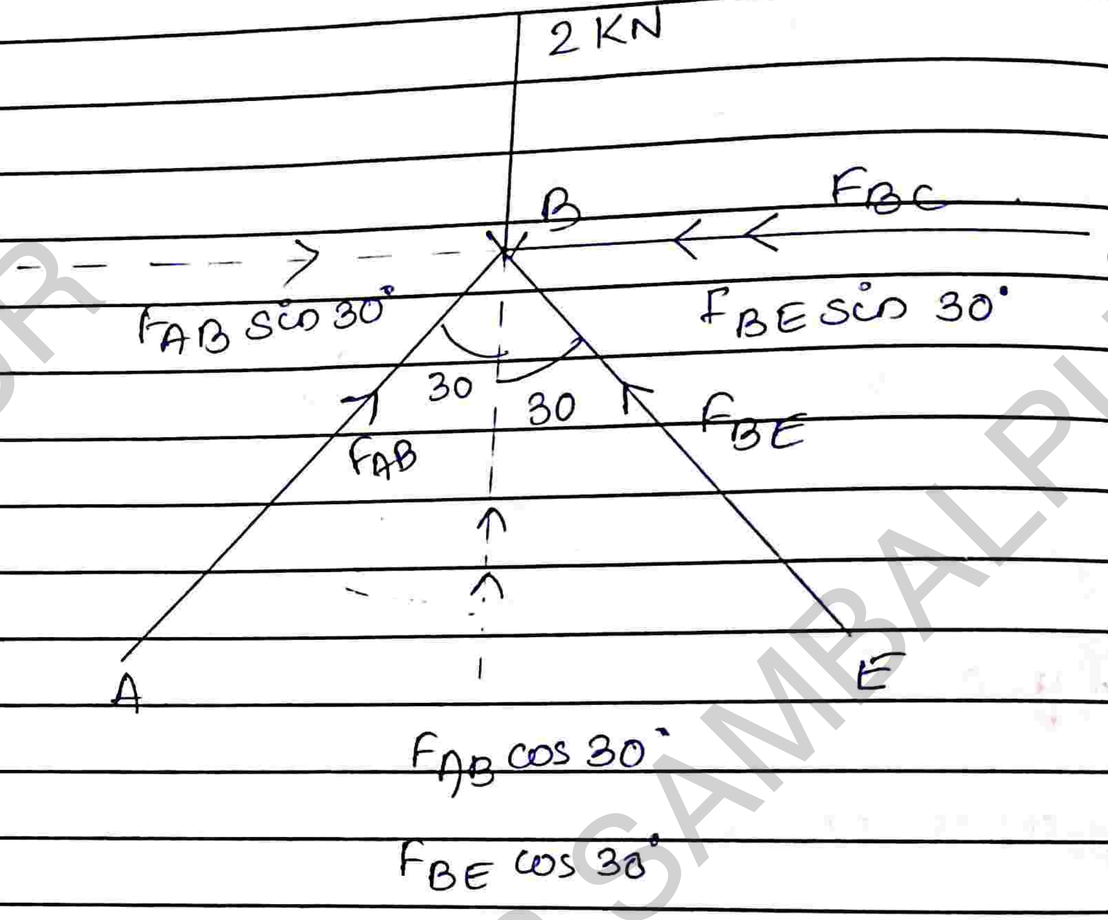
$\boxed{F_{BC} = 1.726\,KN}$ (T)




$\Sigma f_y = 0$

$F_{AB}\cos 30° + F_{BE}\cos 30° - 2 = 0$

$F_{BE}\cos 30° = 2 - 2.88 \times \cos 30°$

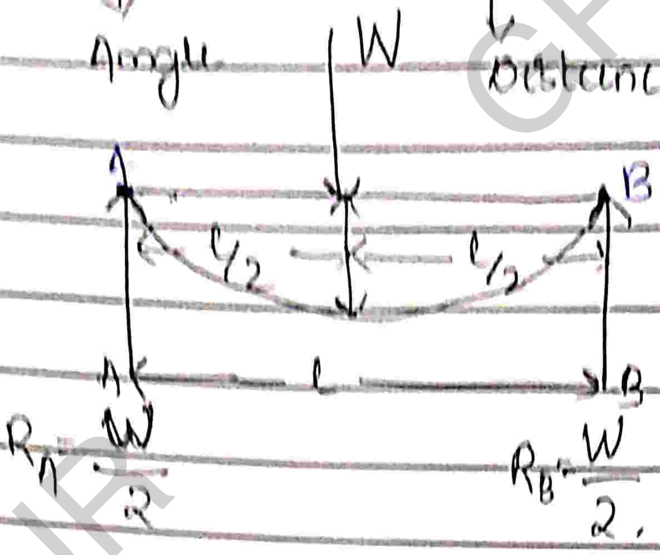
$= \dfrac{-0.49}{\cos 30}$

$\boxed{F_{BE} = -0.57\,KN}$ (C)

## Slope and deflection :-

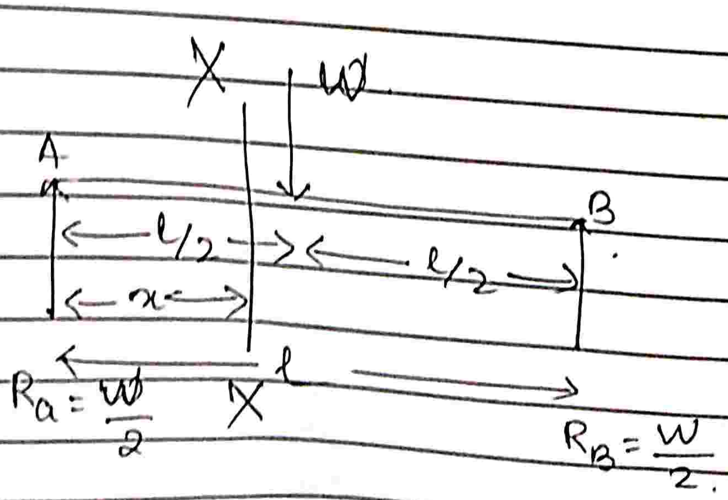
Angle

Distance.




| Slope | Deflection. |
| --- | --- |
| $A = \theta_A \ (x = 0)$ | $A = 0 \quad (x = 0)$ |
| $B = \theta_B \ (x = l)$ | $B = 0 \quad (x = l)$ |
| $C = 0$ $(x = \frac{l}{2})$ $(x = \frac{l}{2})$ | $C = y_{max} \ (x = \frac{l}{2})$ |

let take a section $X X'$ at a distance of '$x$' from A



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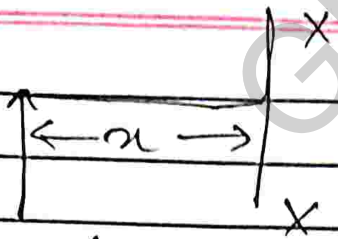


$R_a = \frac{W}{2}$

$$M_{xx} = F \times D$$

$$= \frac{W}{2} \times x$$

$$= \frac{Wx}{2}$$

We know,

$$\boxed{EI\frac{d^2y}{dx^2} = M_{xx}}$$ Slope curvature Eqⁿ.

$$EI\frac{d^2y}{dx^2} = \frac{W}{2}x \quad \text{———} (1)$$

Integrate the Eqⁿ -(1) w.r.t dx.

$$EI\int\frac{d^2y}{dx^2} = \frac{W}{2}\int x$$

$$EI\frac{dy}{dx} = \frac{W}{2}\frac{x^2}{2}.$$

$$EI\frac{dy}{dx} = \frac{Wx^2}{4} + C_1.$$

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$$\Rightarrow EI\frac{dy}{dx} = \frac{w}{4}x^4 + c_1$$

$$EI\,\cancel{\frac{dy}{dx}}\;0 = \frac{w}{4}x^2 + c_1$$

$$\cdot 0 = \frac{w}{4} \times \left(\frac{l}{2}\right)^2 + c_1$$

$$\cdot 0 = \frac{w}{4}\,\frac{l^2}{4} + c_1$$

$$0 = \frac{wl^2}{16} + c_1$$

$$\boxed{c_1 = -\frac{wl^2}{16}}$$

At point C

$$EI\frac{dy}{dx} = \frac{wx^2}{4} - \frac{wl^2}{16}$$

$$EI\frac{dy}{dx} = \frac{w}{4}\left(\frac{l}{2}\right)^2 - \frac{wl^2}{16}.$$

$$\frac{dy}{dx} = \frac{wl^2}{16} - \frac{wl^2}{16}.$$

$$\boxed{\theta_C = \frac{dy}{dx} = 0.}$$

Deflection :-

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B

$$EI \times \int \frac{dy}{dx} = \int \frac{wx^2}{4} - \frac{wl^2}{16}$$

$$EI.y = \frac{w}{4} \cdot \frac{x^3}{3} - \frac{wl^2}{16}x$$

$$EI.y = \cancel{\frac{w}{4} \cdot \frac{x^3}{3}} \frac{wx^3}{12} - \frac{wl^2}{16}x + c_2$$

Section -A

$x = 0$

$$EI.y = \frac{w}{12} \times (0)^3 - \frac{wl^2}{16} \times 0 + c_2$$

$$EI.y = 0 - 0 + c_2$$

$$0 = 0 - 0 + c_2$$

$$\boxed{c_2 = 0}$$

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At point (C)

$$EI \times y = \frac{w}{4} \times \frac{x^3}{3} - \frac{wl^2}{16} x$$

$$EI \times y = \frac{w}{12} \left(\frac{l}{2}\right)^3 - \frac{wl^2}{16} \times \frac{l}{2}$$

$$= \frac{w}{12} \times \frac{l^3}{8} - \frac{wl^3}{32}$$

$$= \frac{wl^3}{96} - \frac{wl^3}{32}$$

$$= \frac{wl^3 - 3wl^3}{96}$$

$$= -\frac{\cancel{2}wl^3}{\cancel{96}\,48}$$

$$\boxed{y_{max} = \frac{-wl^3}{48EI}}$$ Deflection

$$\boxed{\theta_A \Rightarrow -\frac{wl^2}{16EI} = \theta_B}$$ slope.

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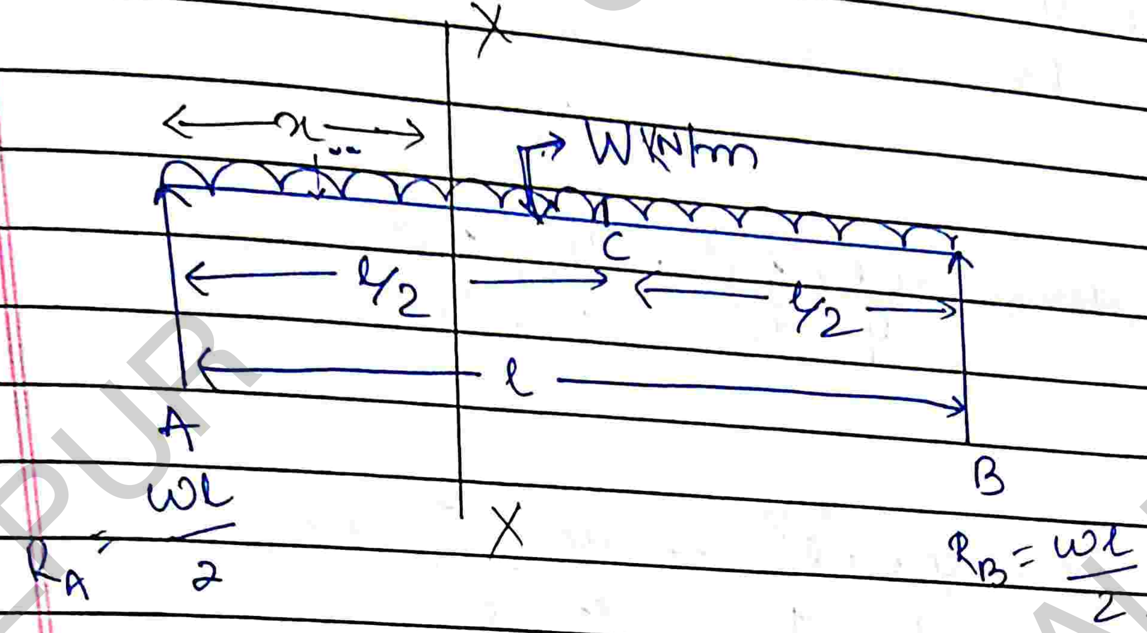
Simply supported beam with UDL throughout the length :—




Let choose a Section XX at a distance of xm from end 'A'

$$M_{xx} = \frac{wl}{2} \times x - wx \cdot \frac{x}{2}$$

$$= \frac{wlx}{2} - \frac{wx^2}{2}$$

we know $\boxed{EI \frac{d^2y}{dx^2} = M_{xx}}$

$$EI \frac{d^2y}{dx^2} = M_{xx}$$

$$EI \int \frac{d^2y}{dx^2} = \int \frac{wl}{2} x - \int \frac{wx^2}{2}$$

$$EI \frac{dy}{dx} = \frac{wl}{2} \frac{x^2}{2} - \frac{w}{2} \frac{x^3}{3} + C_1$$

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$$EI\frac{dy}{dx} = \frac{wlx^2}{4} - \frac{wx^3}{6} + C_1$$

$$EI\,0 = \frac{wl}{4}\left(\frac{l}{2}\right)^2 - \frac{w}{6}\left(\frac{l}{2}\right)^3 + C_1$$

$$0 = \frac{wl}{4}\cdot\frac{l^2}{4} - \frac{w}{6}\frac{l^3}{8} + C_1$$

$$0 = \frac{wl^3}{16} - \frac{wl^3}{48} + C_1$$

$$0 = \frac{3wl^3 - wl^3}{48} + C_1$$

$$0 = \frac{\cancel{2}wl^3}{\cancel{48}} + C_1$$

$$C_1 = -\frac{\cancel{2}wl^3}{\underset{24}{\cancel{48}}}$$

$$\boxed{C_1 = -\frac{wl^3}{24}}$$

$$\cancel{EI \cdot \frac{dy}{dx} = \frac{wl^3}{16} - \frac{wl^3}{48} - \frac{wl^3}{24}}$$

$$EI \frac{dy}{dx} = \cancel{\frac{wlx^2}{4}}^{\,0} - \cancel{\frac{wx^3}{6}}^{\,0} - \frac{wl^3}{24}$$

$$EI \frac{dy}{dx} = -\frac{wl^3}{24}$$

$$\boxed{\theta_A = \frac{-wl^3}{24EI}}$$

$$\boxed{\theta_B = \frac{-wl^3}{24EI}}$$

<u>Deflection</u>

$$EI \cdot \int \frac{dy}{dx} = \int \frac{wlx^2}{4} - \int \frac{wx^3}{6} - \int \frac{wl^3}{24}.$$

$$EI \cdot y = \frac{wl}{4} \cdot \frac{x^3}{3} - \frac{w}{6} \frac{x^4}{4} - \frac{wl^3}{24} x + c_2$$

$$EI\, y = \frac{wl\, x^3}{12} - \frac{wx^4}{24} - \frac{wl^3 x}{24} + c_2.$$

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$\cancel{0} = 0.$

$$\boxed{C_2 = 0}$$

$$EI.y = \frac{wl x^3}{12} - \frac{wx^4}{24} - \frac{wl^3 x}{24}$$

$$EI.y = \frac{wl(l/2)^3}{12} - \frac{w.\left(\frac{l}{2}\right)^4}{24} - \frac{wl^3\left(\frac{l}{2}\right)}{24}$$

$$EI.y = \frac{wl\, l^3}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}.$$

$$EI.y = \frac{wl^4}{96} - \frac{wl^4}{384} - \frac{wl^4}{48}$$

$$EI.y = \frac{4wl^4 - wl^4 - 8wl^4}{384}$$

$$EI.y = \frac{-5wl^4}{384}$$

$$\boxed{y_{max} = \frac{-5wl^4}{384\, EI}.}$$

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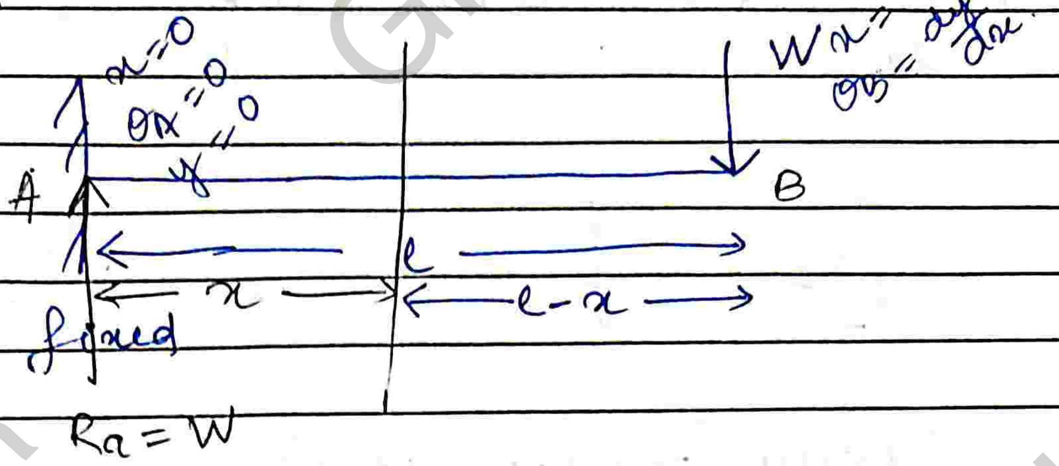
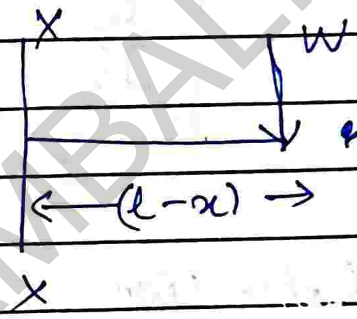
Cantilever beam with a point load at its free end :-




Let take a section XX from A at a distance of $x$




$M_{xx} = -W(l-x)$

we know

$$EI\frac{d^2y}{dx^2} = M_{xx}$$

$$EI\frac{d^2y}{dx^2} = -w(l-x)$$

$$EI\int\frac{d^2y}{dx^2} = -wl\int 1\,dx + w\int x\,dx$$

$$EI\frac{dy}{dx} = -wlx + \frac{wx^2}{2} + C_1$$

$$\cancel{EI\frac{dy}{dx} = \frac{2wlx - wx^2}{2}}$$

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$EI\frac{dy}{dx}$ $EI\times 0 = -wl\times 0 + \frac{wx(0)^2}{2} + C_1$

$$\boxed{C_1 = 0}$$

$EI\times\theta_B = -wl\times l + w$

$= \frac{-2wl^2 + wl^2}{2}$

$$\boxed{\theta_B = \frac{-wl^2}{2EI}}$$

$EI\times\frac{dy}{dx} = wlx - \frac{wx^2}{2}$

$$\cancel{\frac{dy}{dx} = \frac{2wlx - wx^2}{2EI}}$$

$$\Rightarrow \cancel{\frac{dy}{dx} = \frac{2wl^3 - wl^2}{2EI}}$$

Deflection :-

$EI\int\frac{dy}{dx} = \int wl\,x\,dx - \frac{w}{2}\int x^2\,dx.$

$EI.y = -wl\frac{x^2}{2} + \frac{w}{2}\frac{x^3}{3} + C_2$

$EIy = -\frac{wlx^2}{2} + \frac{wx^3}{6} + C_2.$

$$\cancel{EIy = \frac{wl^3}{2} - \frac{w l^3}{6} + C_2.}$$

$\cancel{EIy =}$

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$\therefore C_2 = 0$.

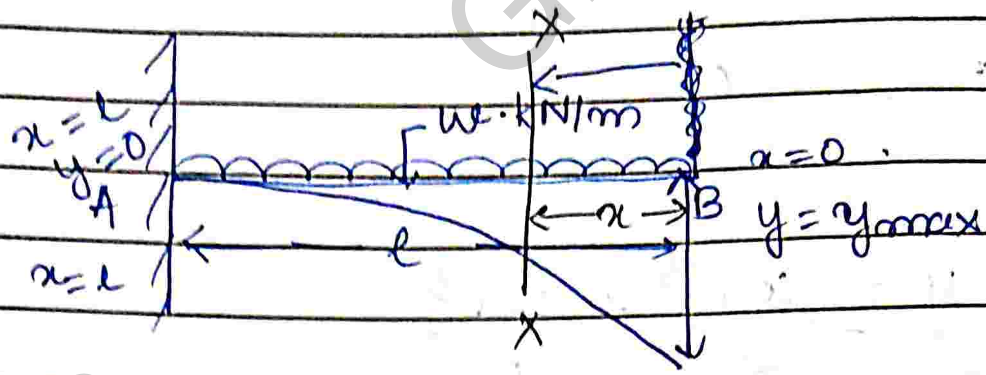
$$EI\,y = -\frac{wl^3}{2} + \frac{wl^3}{6}$$

$$EI.y = \frac{-3wl^3 + wl^3}{6}$$

$$EI\,y = \frac{-\cancel{2}wl^3}{\cancel{6}_3}$$

$$\boxed{y_{max} = \frac{-wl^3}{3EI}}$$

Cantilever beam subjected with UDL throughout the length :-




Let take a section XX ~~from~~ at the distance of x from end B.

$$M_{xx} = -wx \cdot \frac{x}{2}$$

$$= -\frac{wx^2}{2}$$

We know,

$$EI \frac{d^2y}{dx^2} = \frac{-wx^2}{2}$$

Integrating on both side

$$EI \int \frac{d^2y}{dx^2} = -\frac{w}{2} \int x^2$$

$$EI \frac{dy}{dx} = -\frac{w}{2} \frac{x^3}{3} + C_1$$

$$EI \frac{dy}{dx} = \frac{-wx^3}{6} + C_1$$

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~~$EI\theta_B$~~

~~$EI \times 0 = 0 + c_1$~~

$$EI\frac{dy}{dx} = -\frac{wx^3}{6} + c_1$$

~~$\boxed{c_1 = 0}$~~

~~$$EI\frac{dy}{dx} = \frac{-wx^3}{6}$$~~

$$EI \times 0 = \frac{-wl^3}{6} + c_1$$

$$\boxed{c_1 = \frac{wl^3}{6}}$$

$$EI\frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}$$

$$EI\frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}.$$

At point $\frac{dy}{dx} = \theta_B$ and $x = 0$.

$$\frac{dy}{dx} \quad EI\frac{dy}{dx} = 0 + \frac{wl^3}{6}.$$

$$\boxed{\theta_B = \frac{wl^3}{6EI}}$$

$$\boxed{\theta_A = 0}$$

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$$EI\frac{dy}{dx} = \frac{-wx^3}{6} + \frac{wl^3}{6}.$$

Integrating both side.

$$EI\int\frac{dy}{dx} = \frac{-w}{6}\int x^3 + \frac{wl^3}{6}\int 1\,dx \cancel{dx}.$$

$$EI\cdot y = \frac{-w}{6}\frac{x^4}{4} + \frac{wl^3x}{6} + C_2.$$

$$EI\cdot y = \frac{-wx^4}{24} + \frac{wl^3x}{6} + C_2.$$

$$\cancel{\boxed{C_2 = 0}}$$

$$\cancel{EI\cdot y = \frac{-wx^4}{24} + \frac{wl^3x}{6}.}$$

$$\cancel{EI\,y_{max} = \frac{-wx^4}{24}}$$

$$0 = \frac{-wl^4}{24} + \frac{wl^4}{6} + C_2.$$

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$$0 = \frac{-wl^4 + 4wl^4}{24} + C_2.$$

$$0 = \frac{3wl^4}{24} + c_2$$

$$c_2 = -\frac{\cancel{3}wl^4}{\cancel{24}\,8}$$

$$\boxed{c_2 = -\frac{wl^4}{8}}$$

$$EI \cdot y = -\frac{wx^4}{24} + \frac{wl^3x}{6} - \frac{wl^4}{8}.$$

$$\boxed{y_{max} = -\frac{wl^4}{8EI}}$$



Signature

# Stresses in shaft due to torsion :

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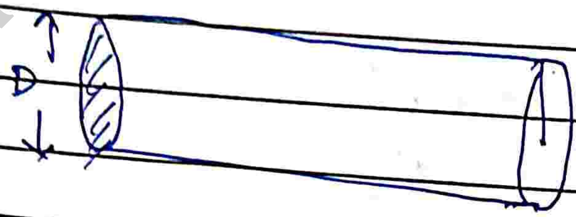
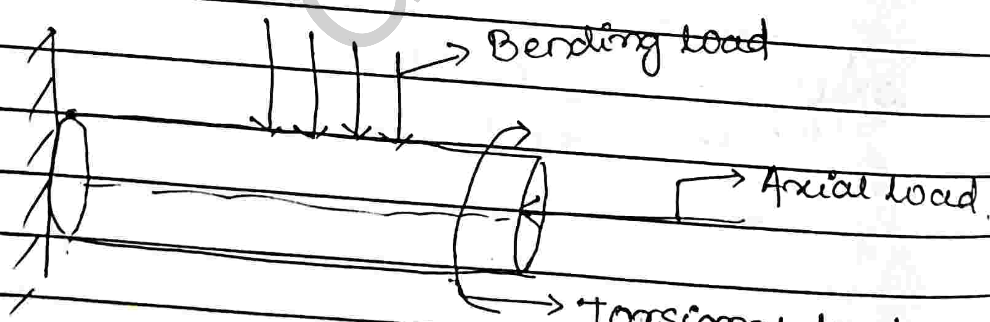
Shaft - The shafts are usually cylinderical in Section & Solid or hollow.

- They are made of mild steel and copper alloy




- Shaft may be subjected to following loads :-

(a) Torsional load
(b) Bending load
(c) Axial load
(d) combination of the above three.




- Shaft is a member like tunnel, pipe for pumping water, conveyance for the menor Material.

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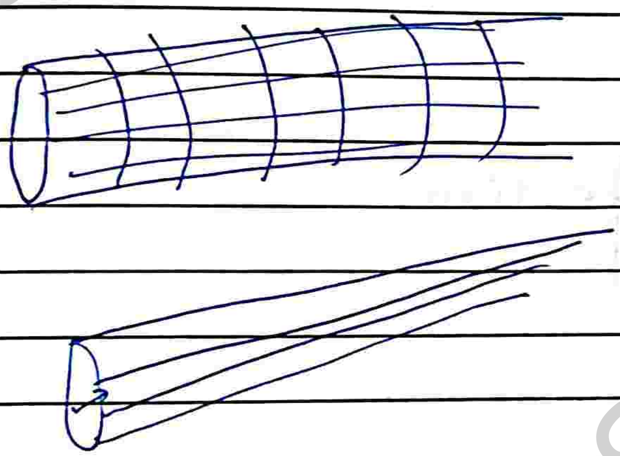
## Torsion :-

- Torsion is a moment that twist or deforms a member about its longitudinal axis.

- By observation, if angle of rotation is small, length of the shaft and its longitudinal axis remain unchanged.



After Deformation,

- Only ~~two~~ longitudinal lines becomes twisted and remain unchanged

Shear Strain due to torsion :-

The angular displacement of the one Surface of the shaft from its original position due to the applied of torque.

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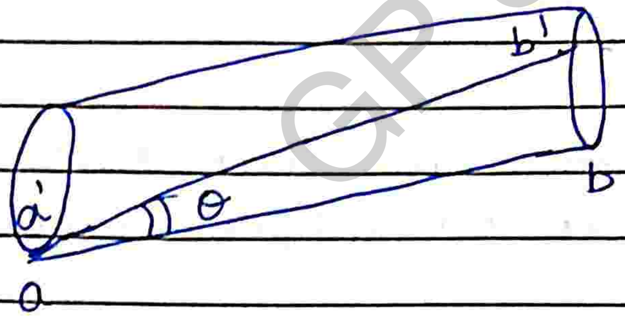


– θ can be measured radian between the final and original position of the bar.

$$180^\circ = \pi$$

$$\boxed{1^\circ = \frac{\pi}{180^\circ}}$$ radian

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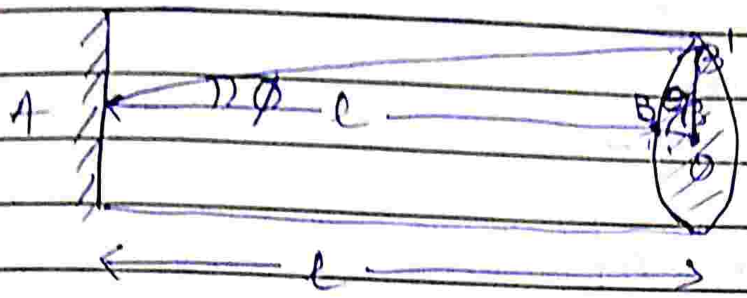
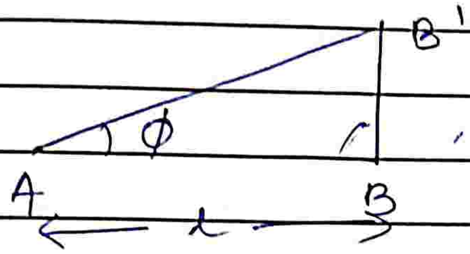
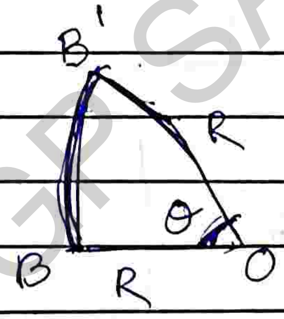
## Torsional Equation for shaft :-




$\theta$ = angle of twist
$\phi$ = angle of shear stress




$$\tan\phi = \frac{BB'}{l}$$




length of Arc BB'
$= R \times \theta$

putting the value of Arc in $\tan\phi$

~~$\tan\phi =$~~

$$\tan\theta = \frac{R\theta}{l}$$

$$\tan\phi = \frac{BB'}{l} = \frac{R\theta}{l}$$

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$\phi$ is very very small. ~~then~~

i.e, $\boxed{\tan\phi = \phi}$

$$\boxed{\phi = \cancel{BB}\,\frac{R\times\theta}{l}} \quad \text{——— (1)}$$

$$G = \frac{\text{Shear stress}}{\text{shear strain}} = \frac{\tau}{\phi}$$

$$G = \frac{\tau}{\phi}$$

$$G\times\phi = \tau$$

$$\boxed{\phi = \frac{\tau}{G}} \quad \text{——— (2)}$$

from (1) and (2)

$$\boxed{\frac{R\times\theta}{L} = \frac{\tau}{G}}$$

from Eqn -(1) and (2) we get that

$$\boxed{\frac{R\times\theta}{L} = \frac{\tau}{G}}$$

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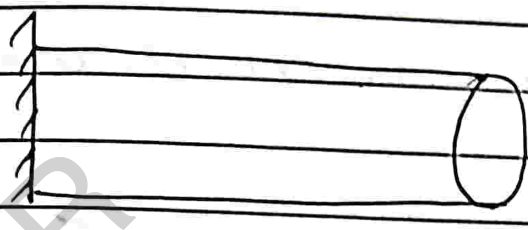
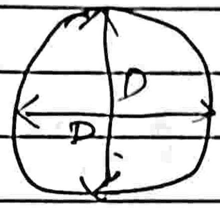
$$\boxed{\frac{G\theta}{l} = \frac{\tau}{R}}$$



We know that MI of a circle $= \frac{\pi D^4}{64}$.




Polar moment of inertia :-

$$I_p = I_{xx} + I_{yy}$$

$$= \frac{\pi D^4}{64} + \frac{\pi D^4}{64}$$

$$= \frac{\pi D^4 + \pi D^4}{64}$$

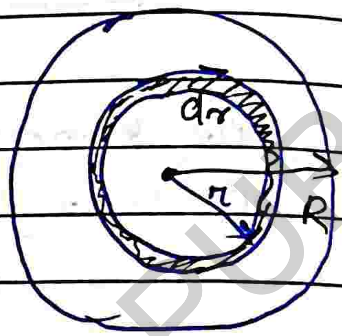
$$= \frac{\cancel{2}\pi D^4}{\cancel{64}\,32}$$

$$\boxed{I_p = \frac{\pi D^4}{32}}$$

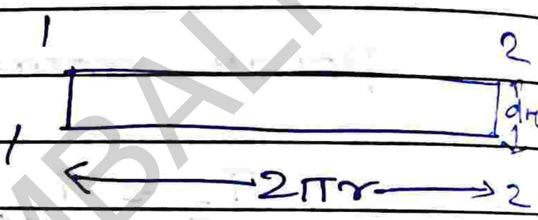
It is denoted by Ip or J

$$\frac{\tau}{R} = \text{constant}$$

Lets choose a strip from the circular section
thickness = $dr$




Area of the small strip
$= l \times \text{Thickness}$
$= 2\pi r \times dr$.




$$\tau_1 = \frac{F}{A}$$

$$\tau_1 = \frac{F}{2\pi r \times dr}$$

$$\frac{\tau}{R} = \text{constant}$$

$$\boxed{\frac{\tau}{R} = \frac{\tau_1}{r}}$$

$$\tau_1 = \frac{\tau}{R} \cdot r$$

$$\frac{\tau}{R} \times r = \frac{F}{2\pi r \times dr}$$



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$\frac{\tau}{R} \times r \times 2\pi r \times dr = F$

Torque = Force x Radial distance

$T = F \times r$

$T = \frac{\tau}{R} \times r \times 2\pi r \times dr \times r$

$T = \frac{\tau}{R} \times r^3 \times 2\pi \times dr$

$T = \int_0^R \frac{\tau}{R} \times r^3 \times 2\pi \times dr$

$T = \frac{\tau 2\pi}{R} \int r^3 \, dr$

$\frac{2\pi\tau}{R} \left[\frac{r^4}{4}\right]_0^R + C$

$= \frac{2\pi\tau}{R} \left[\frac{R^4}{4} - \frac{0^4}{4}\right] + C$

$T = \frac{2\pi\tau}{\cancel{R}} \frac{R^{\cancel{4}3}}{4} + C$

$T = \frac{\cancel{2}\pi\tau R^3}{\cancel{4}2} \Rightarrow T = \frac{\pi\tau R^3}{2}$

$$T = \frac{\tau\pi}{2} \times R^3 \times \cancel{\frac{\tau\pi}{2}}$$

$$= \frac{\tau\pi}{2} \times \left(\frac{D}{2}\right)^3 \times \cancel{\frac{\tau\pi}{2}}$$

$$\boxed{T = \frac{\tau\pi D^3}{16}} \times \cancel{\frac{\tau\pi}{2}}$$

$$\Rightarrow \tau = \frac{16T}{\pi D^3}$$

$$\frac{G\theta}{l} = \frac{\tau}{R}$$

$$\frac{G\theta}{l} = \tau \times \frac{1}{R}$$

$$\frac{G\theta}{l} = \frac{16T}{\pi D^3} \times \frac{1}{R}$$

$$\frac{G\theta}{l} = \frac{16T}{\pi D^3 R}$$

$$\frac{G\theta}{l} = \frac{16T}{\pi D^3 \times \frac{D}{2}}$$

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$$\frac{G\theta}{l} = \frac{16T}{\pi D^3 \times D} \times 2$$

$$\frac{G\theta}{l} = \frac{32T}{\pi D^4}$$

$$\frac{G\theta}{l} = \frac{T}{I_p}$$

$$\boxed{\frac{T}{I_p} = \frac{G\theta}{l} = \frac{\tau}{R}}$$

* * *

# Numericals :-

A solid steel shaft 5m long is stretched at 80mpa when twisted through 4°. use G = 83 GPa compute the shaft diameter :-

$l = 5m$

$G = 83 \times 10^9 \, N/m^2$.

$\tau = 80 \times 10^6 \, N/m$

$\theta = 4°$

$= \frac{4 \times \pi}{\cancel{180}_{45}}$

$= 0.02\pi$

$$\frac{G\theta}{l} = \frac{\tau}{R}$$

$$R = \frac{\tau l}{G\theta}$$

$$= \frac{80 \times 10^6 \times 5}{83 \times 10^9 \times 0.02\pi}$$

$\cancel{=2262} \quad \cancel{0.076m}$, $= \frac{80 \times 10^6 \times 5}{83 \times 10^9 \times 0.069}$

$\cancel{D = 2 \times 0.076}$

$\cancel{= 0.152}$

$R = 0.069$

$D = 2 \times 0.069$

$= 0.138m$

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# Bending Stress



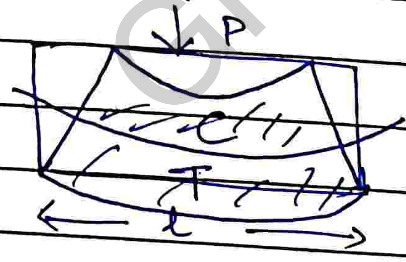
When some external load on a beam, the shear force and bending moments are set up at all sections of the beam.

Due to shear force and bending moment, the beam undergoes certain deformation the material of the beam will offer resistance or stress against these deformation.

The stress introduced by the bending moment are known as bending stress.

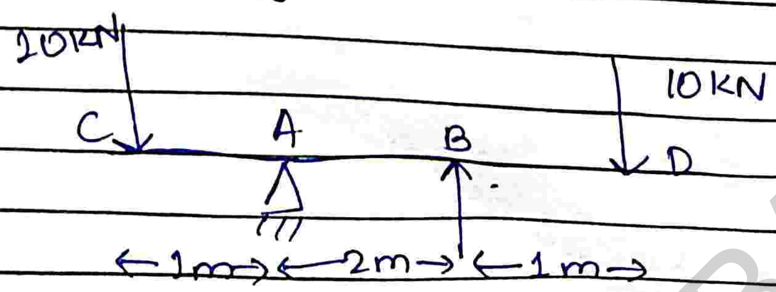
Neutral axis :- (NA)

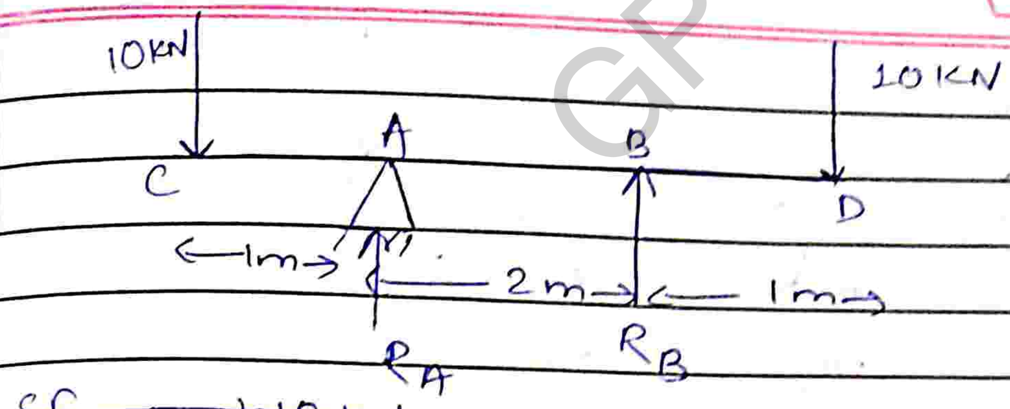
↳ Neutral axis is the line of intersection which divides the beam or member into two zone i.e, compression zone and tension zone




↳ Neutral axis remain unchanged.

Pure Bending






~~$SF_D$ = +10 KN~~

$R_A + R_B - 10 - 10 = 0$

$R_A + R_B - 20 = 0$

$R_A + R_B = 20$ ——(1)

$M_A = 0$

$-10 \times 3 + R_B \times 2 - 10 \times 1 = 0$

$-30 + 2R_B - 10 = 0$

$2R_B - 40 = 0$

$R_B = \frac{40}{2}$ 20

$R_B = 20$ KN

$R_A + R_B = 20.$

$R_A = 20 - 20$

$R_A = 0$

$SF_D = +10$ KN

$SF_B = -20 + 20 = 10$ KN

$SF_A = 10$ KN

$SF_C = +10 - 20 - 0 = 0$

$+10 - 10 - 10$

$-10$ KN.

$BM_D = 0$ KN/m

$BM_B = -10 \times 1 = 10$ KN/m

$BM_A = -10 \times 3 + 20 \times 2 + 0$

$= -30 + 40 = 10$ KN/m

$BM_C = -10 \times 4 + 20 \times 3 + 0 \times 1 + 10 \times 0$

$= -40 + 60$

$= -20$ KN/m.

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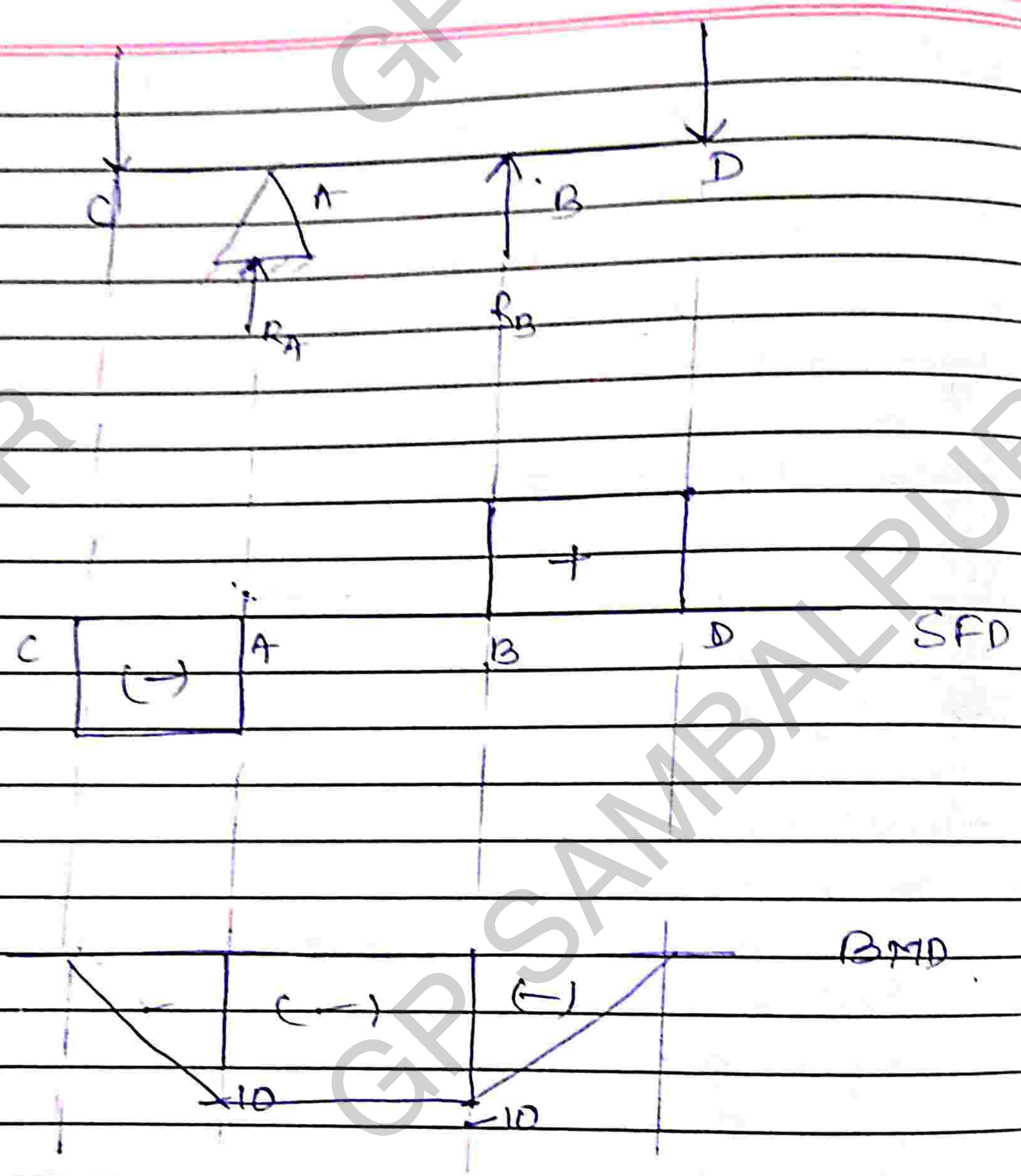


Simple bending 1. SF = 0
2. BM = constant

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Pure Bending :- (Simple Bending)

→ when the beam is loaded with some external loads, but there is no shear between a section of a beam but constant bending moment observed on that section. This condition of the beam, is known as pure Bending or Simple Bending



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A B

$M_A = M_B$

$-10\times3+R_b\times2-10\times1 = -10\times1+R_a\times2-10\times3$

$\boxed{R_b = R_a}$

$\Sigma V = 0$

$10-R_a-R_b+10=0$

$\boxed{20 = R_a+R_b}$

$R_a+R_a=20$

$2R_a=20$

$R_a=\frac{20}{2}$

$R_a=10$

$R_b=10$

Important

## ** Assumption of pure Bending :- [5 marks]

Following are the assumption of pure Bending.

(a) Materials ~~are~~ is homogeneous

(b) Member is elastic and isotropic

(c) The member is straight throughout its length

(d) The transverse cross-sectional area of the beam is constant.

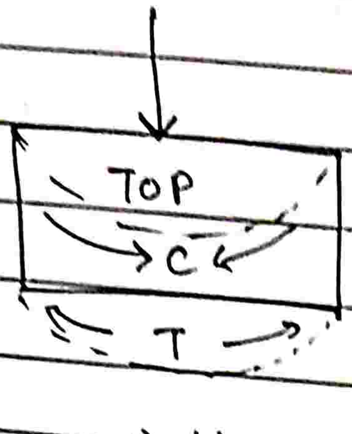
(e) It should obey the Hooke's law.

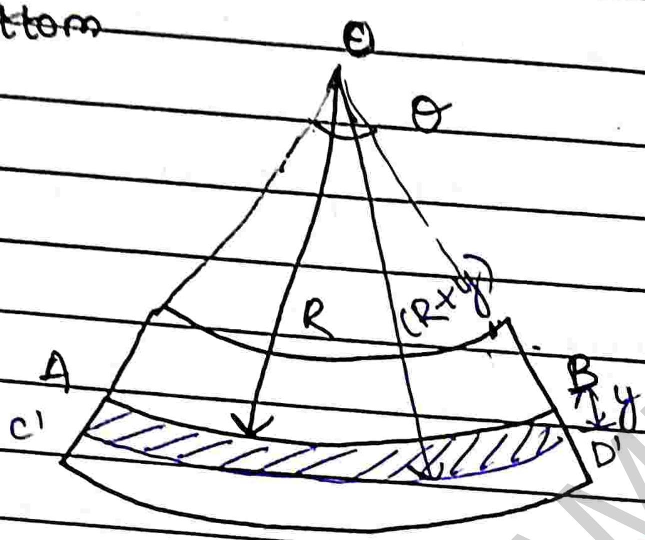
(f) The value of modulus of elasticity (E) is constant

(g) The transverse cross-section of the beam is same before the bending and after the bending

(h) Neutral axis divides the beam into two zone upper zone compression and lower zone tension.

# Derivation of Equation of pure Bending :-




In the above diagram,

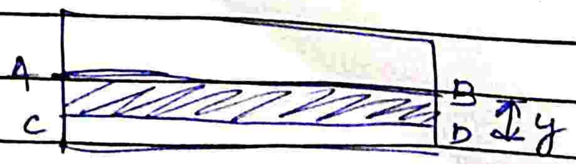
AB = Neutral axis
R = Radius of the curve upto neutral axis
θ = Angle at centre

length of Arc,

AB = Radius x Angle

$AB = R\theta$

Lets choose a small strip whose thickness is y



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C'D' > CD because CD is in Tension Zone.

Radius of C'D' fibre is $(R+y)$

length of arc $C'D' = (R+y) \times \theta$

| | | |
|---|---|---|
| CD | | C'D' |
| $\Downarrow$ | | $\Downarrow$ |
| AB [opposite side of rectangle]. | | $(R+y) \times \theta$. |
| $\Downarrow$ | | $R\theta + y\theta$. |
| $R\theta$ | | |

change in length $= C'D' - CD$
$= \cancel{R\theta} + y\theta - \cancel{R\theta}$.
$= y\theta$

Strain $= \dfrac{\text{change in length}}{\text{original length}}$.

$$\epsilon = \frac{y\cancel{\theta}}{R\cancel{\theta}} = \frac{y}{R}. \quad \text{—— (1)}$$

we know that $E = \dfrac{\sigma}{\epsilon}$

$$\boxed{\epsilon = \frac{\sigma}{E}} \quad \text{—— (2)}.$$

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from the $Eq^n$ - ① & ② we get that.

$$\boxed{\frac{y}{R} = \frac{\sigma}{E}}$$

$R\sigma = E \times y$

$$\boxed{\frac{\sigma}{y} = \frac{E}{R}}$$

A cantilever beam 2m long is subjected to an UDL of 5kN/m over its entire length. Find out the slope and Deflection of the beam at its free end. Take $EI = 2.5 \times 10^{12}\ N/mm^2$.

$l = 2 \times 10^3\ mm$

$w = 5\ kN/m$

$= 5 \times 10^3 / 10^3$

$= 5\ N/mm^2$

Slope $= \frac{wl^3}{6EI}$

$= \frac{5 \times (2 \times 10^3)^3}{6 \times 2.5 \times 10^{12}}$

$= 2.667 \times 10^{-10}$ radian

$y_{max} = \frac{-wl^4}{8EI}$

$= \frac{-5 \times (2 \times 10^3)^4}{8 \times 2.5 \times 10^{12}}$

$= -4\ mm$

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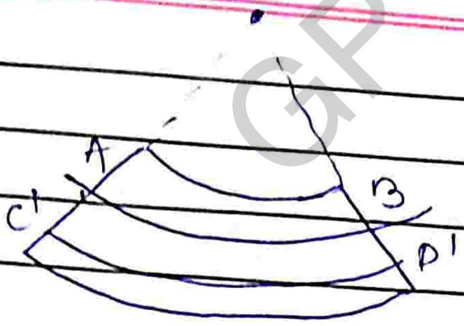


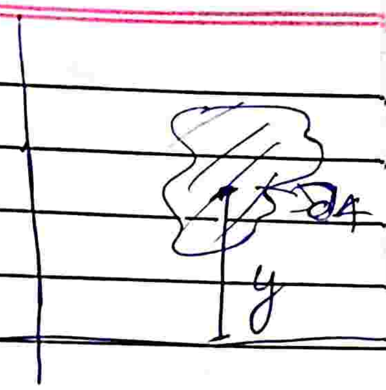


Moment of force = $F \times y$

Moment of area = $\delta A \times y$

Moment of moment of Area = $\delta A \times y \times y$

$= \delta A \times y^2$.

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{E}{R} \times y$$

We know that Stress, $\sigma = \frac{F}{A}$

$\Rightarrow F = \sigma \times A$

$$F = \sigma \times A$$
$$= \frac{E}{R} \times y \times \delta A$$

[$\delta A$ – Area of the strip.]

Moment (about neutral axis) = Force × $y$

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$$M = F \times y$$

$$= \frac{E}{R} \times y \times \delta A \times y$$

$$= \frac{E}{R} \times y^2 \times \delta A.$$

$$M = \int \frac{E}{R} \times y^2 \times \delta A$$

$$M = \frac{E}{R} \int y^2 \delta A$$

$$M = \frac{E}{R} I$$

$$\boxed{\frac{M}{I} = \frac{E}{R}}$$

$$\boxed{\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y}}$$

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A cantilever beam of 50mm wide and 150mm high and subjected with a moment of 6000 N·m. Find the stress developed on the beam.

$$I = \frac{bd^3}{12}$$

$$= \frac{50 \times 150^3}{12}$$

$$= 14.06 \times 10^6 \text{ mm}^4$$

$M = 6000 \text{ Nm} = 6000 \times 10^3 \text{ N/mm}$

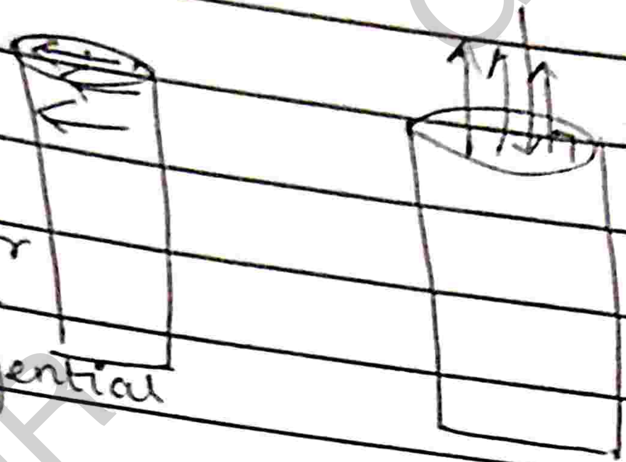
$y = 75 \text{ mm}$

$$\sigma = \frac{6000 \times 10^3 \times 75}{14.06 \times 10^6}$$

$\sigma = 32.005 \text{ N/mm}^2$.

# Complex Stress Analysis

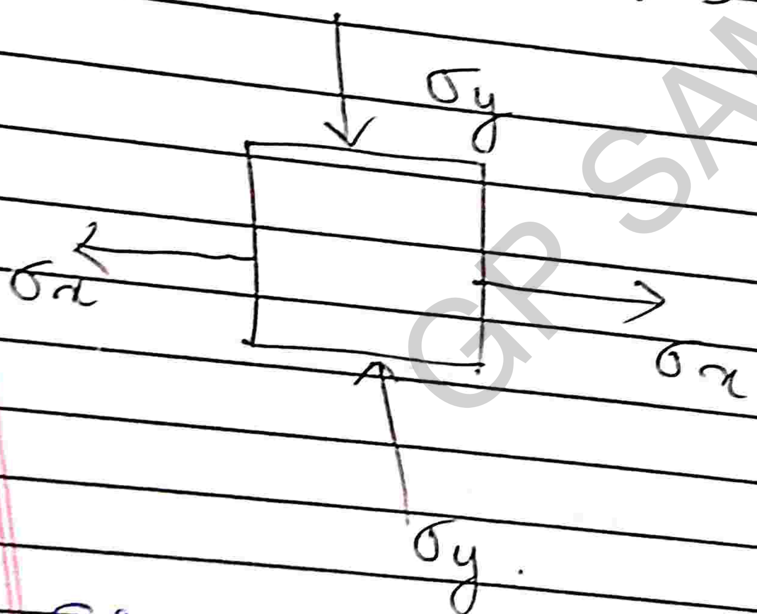
Shear or tangential



Normal.

3D ⟶ 2D

Thickness <<< l & B.




## Sign convention for shear.

Tension = +ve

compression = -ve

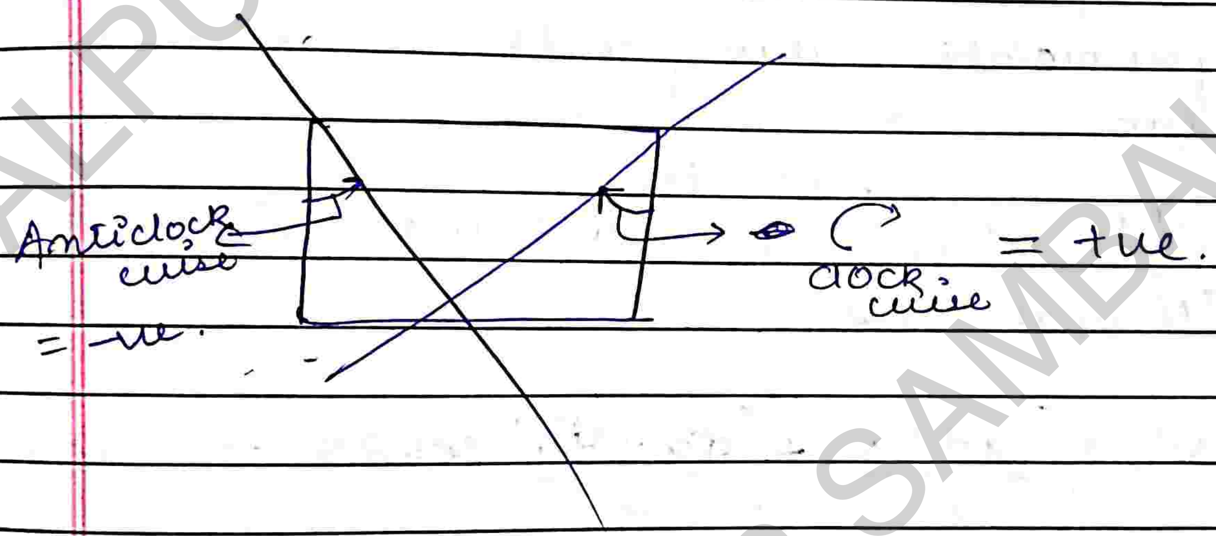
Anticlockwise = -ve

clockwise = +ve.

$\sigma_x, \sigma_y \rightarrow$ Normal
$\tau \rightarrow$ shear stress

principal ~~a~~ plane

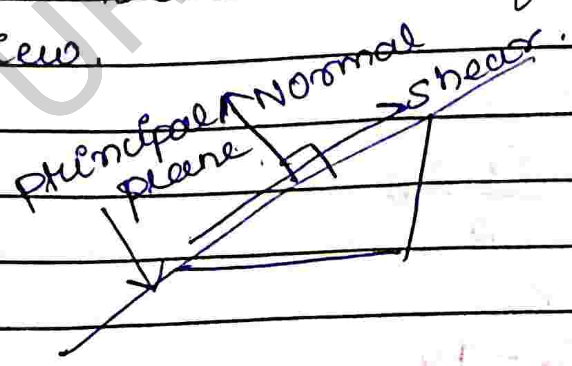
Normal Stress | Resultant Stress
Shear stress | Angle of Resultang.




principal stresses and plane.

→ A Structural member may be Subjected to different types of Stresses Simultaneously.

→ It is therefore necessary to find the region where the effect of those forces or stress will be critical from the design point of view.



↳ The plane which have no shear stress, are known as principal planes.

↳ The normal stress acting on a principal plane are known as principal stress.

↳ Therefore it is necessary to find out the greater of normal stress i.e, the maximum principal stress which shall not exceed the a permissible value for the safety of the design.

→ Major principal stress = 40
→ minor principal stress = 50°

Normal Stress

$$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

Tangential Stress

$$\sigma_t = \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta - \tau\cos 2\theta = 0$$

$$\Rightarrow \left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta = \tau\cos 2\theta.$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{\tau}{\frac{\sigma_x-\sigma_y}{2}}$$

$$\Rightarrow \tan 2\theta = \frac{\tau}{\frac{\sigma_x-\sigma_y}{2}}$$

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$$\Rightarrow 2\theta = \tan^{-1}\left(\frac{\tau}{\frac{\sigma_x - \sigma_y}{2}}\right)$$

$$\Rightarrow \theta = \frac{\tan^{-1}\left(\frac{\tau}{\frac{\sigma_x - \sigma_y}{2}}\right)}{2}.$$

Q. At a point in a Stressed body, the principal Stresses are 100 MN/m² (Tensile) & 600 MN/m² (compressive). calculate the normal and shear stress on a plane inclined at 50° to the plane carrying 100 MN/m².

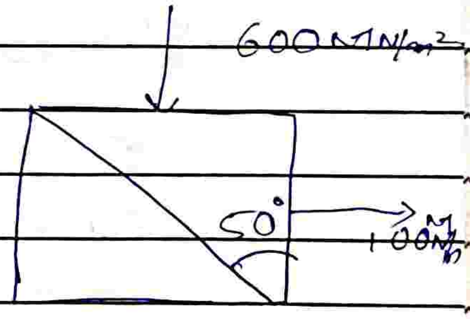
$\sigma_x = 100\,MN/m^2$

$\sigma_y = -600\,MN/m^2$.

$\theta = 50^\circ$

$\tau = \cancel{\frac{\sigma_x - \sigma_y}{2} \sin 2\theta}$.

$= 0$




$$\sigma_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$= \frac{100 - 600}{2} + \frac{100 + 600}{2} \cos 100$$

$$= -310.776\ MN/m^2.$$

$$\tau = \frac{100 + 600}{2} \sin 2 \times 50^\circ.$$

$$\tau = \frac{700}{2} \sin 100$$

$$= 344.68 \ MN/m^2$$

$$R = \sqrt{(\sigma_n)^2 + (\tau)^2}$$

$$= \sqrt{(-310.776)^2 + (344.68)^2}$$

$$= 149.07 \ MN/m^2$$

The principal stress of a point of 2 perpendicular stress are $75 \ N/mm^2 (t)$, $35 \ N/mm^2$ it oblique ma 20° ~~[illegible]~~ which measure the principal stress find the normal and shear stress resultant stress

Given data

$\sigma_x = 75 \ N/mm^2$
$\sigma_y = 35 \ N/mm^2$
$\theta = 20°$
$\tau = 0$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right) \cos 2\theta$$

$$= \left(\frac{75 + 35}{2}\right) + \left(\frac{75 - 35}{2}\right) \cos 40°$$

$$= \frac{110}{2} + \frac{40}{2} \cos 40°$$

$$= 70.32 \ N/mm^2$$

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$$\tau = \frac{\sigma_x + \sigma_y}{2} \sin 2\theta$$

$$= \frac{75+35}{2} \sin 2\times 20$$

$$= \frac{110}{2} \sin 40$$

$$= 35.35 \ N/mm^2$$

$$R = \sqrt{(\sigma_n)^2 + (\tau)^2}$$

$$= \sqrt{(70.32)^2 + (35.35)^2}$$

$$= 78.70 \ N/mm^2$$

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A body is subjected with tensile stresses of 200 MPa and 150 MPa which is mutually ⊥r to each other. It is also associated with a shear stress ~~200~~ 40 MPa.

(i) Find the magnitude of normal stress
(ii) Shear stress
(iii) Resultant stress
(iv) Direction of principal stress on a inclined plane at an angle of 30° with major tensile stress.

$\sigma_x = 200\,MPa$

$\sigma_y = 150\,MPa$

$\tau = 40\,MPa$

$$\theta = \frac{\tan^{-1}\left(\dfrac{40}{\frac{200-150}{2}}\right)}{2}$$

$$\theta = \frac{\tan^{-1}\left(\dfrac{40}{25}\right)}{2}$$

$\theta = 28.99$

$\approx 29°$

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$$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

$$= \left(\frac{200+150}{2}\right) + \left(\frac{200-150}{2}\right)\cos(2\times 29) + 40 \sin(2\times 2$$

$$= \frac{350}{2} + \frac{50}{2}\cos 58 + 40 \sin 58.$$

$$= 222.16 \text{ MPa}$$

$$\tau = \frac{\sigma_x+\sigma_y}{2} \sin 2\theta.$$

$$= \frac{200+150}{2} \sin(2\times 29)$$

$$= \frac{350}{2} \sin 58.$$

$$= \cancel{48}\ 148.40 \text{ MPa}.$$

$$R = \sqrt{(\sigma_n)^2 + (\tau)^2}.$$

$$= \sqrt{(222.16)^2 + (148.40)^2}$$

$$= 267.16 \text{ MPa}.$$

(iv) $\tau = \frac{200+150}{2} \sin(30^\circ \times 2)$

$$= 151.55 \text{ MPa}$$

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$$\theta = \tan^{-1}\left(\frac{151.55}{\frac{200-150}{2}}\right)$$

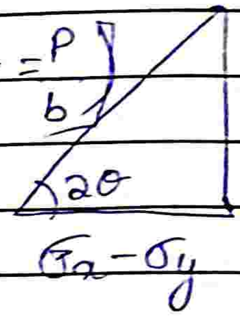
$$\overline{\quad 2 \quad}$$

$\theta = 40.31^{\circ}$

Major and minor principal stress

$$\tan 2\theta = \frac{\tau}{\frac{\sigma_x - \sigma_y}{2}}$$

$$\tan 2\theta = \frac{2\tau}{\sigma_x - \sigma_y} \qquad \left[\tan\theta = \frac{P}{b}\right]$$




$$H = \sqrt{P^2 + B^2}.$$

$$= \sqrt{(2\tau)^2 + (\sigma_x - \sigma_y)^2}$$

$$\cancel{= \sqrt{4\tau^2 + \sigma_x^2 + \sigma_y^2 - 2\sigma_x\sigma_y}}$$

$$\cos 2\theta = \frac{B}{H} = \frac{\sigma_x - \sigma_y}{\sqrt{(2\tau)^2 + (\sigma_x - \sigma_y)^2}}$$

$$\sin 2\theta = \frac{P}{H} = \frac{2\tau}{\sqrt{(2\tau)^2 + (\sigma_x - \sigma_y)^2}}$$

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau \sin 2\theta$$

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A body is subjected with tensile stress of 200 MPa and 150 MPa mutually ⊥r to each other A shear stress of 30 MPa is also developed on the body. Find the value of major and minor principal stress on a section inclined at an angle of 30° with major stress.

$$\tan 2\theta = \frac{30}{\frac{200-150}{2}}$$

$$= \frac{\cancel{30}\,6}{\cancel{25}\,5}$$

$$\tan 2\theta = \frac{6}{5} \left(\frac{P}{b}\right)$$

$$H = \sqrt{P^2 + b^2} = \sqrt{36^2 + 25^2} = 7.81$$

~~cos 2θ =~~

$$\cos 2\theta = \frac{b}{H}$$

$$= \frac{5}{7.81}$$

$$= 0.64$$

$$\sin 2\theta = \frac{P}{h}$$

$$= \frac{6}{7.81}$$

$$= 0.768.$$

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For major.

$$\sigma_n = \left(\frac{200+150}{2}\right) + \left(\frac{200-150}{2}\right) 0.64 + 30 \times 0.76...$$

$$= 214.04 \text{ MPa}$$

For minor.

$$\sigma_n = 135.96 \text{ MPa}$$

A rectangular beam of cross-section is 200mm wide and 350 mm deep if the section is subjected to a maximum shear of 80KN. Find the maximum stress develop and draw the shear distribution diagram

$b = 200mm$
$d = 350mm$
$~~\tau_{max}~~ = f = 30 KN$

$$A = b \times d = 70000 \text{ mm}$$

$$\tau_{max} = 1.5 \times \frac{f}{A}$$

$$= 1.5 \times \frac{30 \times 10^3}{70000}$$

$$= 0.642 \text{ N/mm}^2.$$

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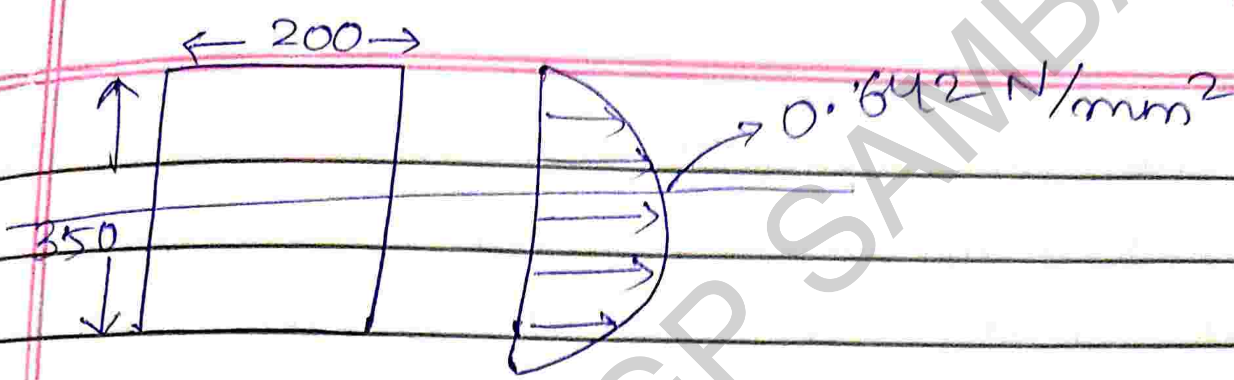
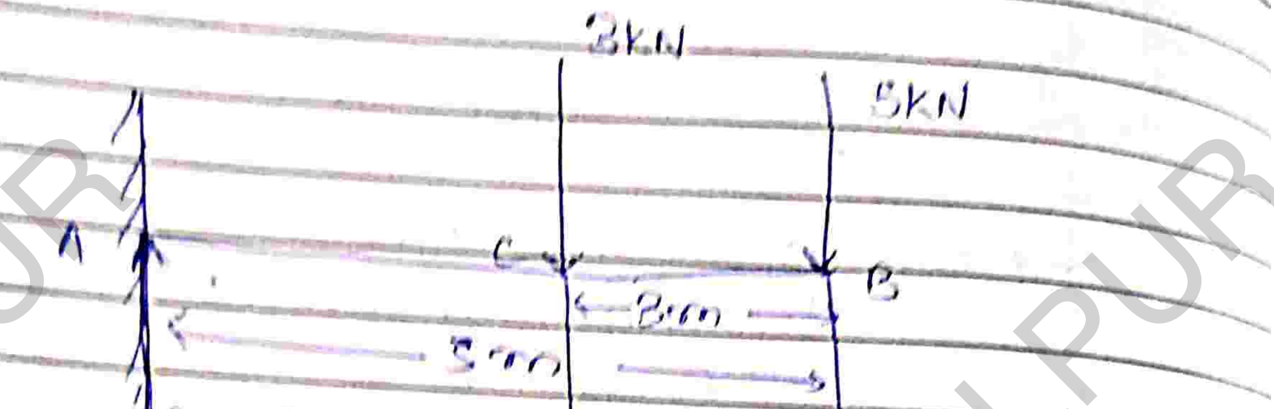


Q. Draw the SFD and BMD of a cantilever beam having span length 5m subjected with two point loads at its free end and other 3m from its free end. The free end load is 5KN and other one is 3KN

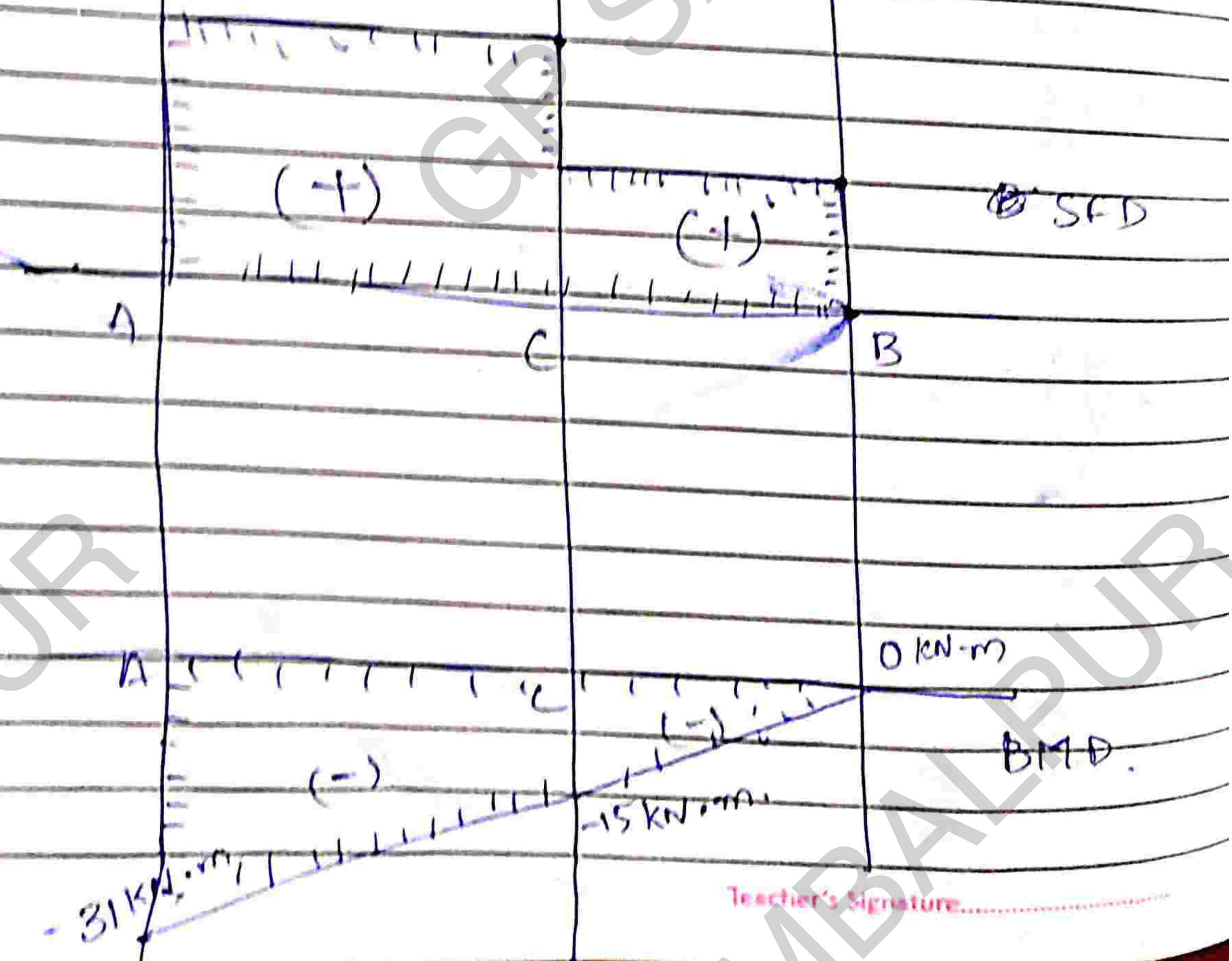



$R_A = 8KN$

$\Sigma F_y = 0$

$R_A - 3 - 5 = 0$

$R_A = 8\ KN$



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SFD :-

$SF_B = +5\,KN$

$SF_C = (+5 + 3)KN = +8\,KN$

$SF_A = +5 + 3 - 8 = 0\,KN$

BMD :-

$BM_B = 0\,KN.m$

$BM_C = -5 \times 3 = -15\,KN.m$

$BM_A = -5 \times 5 - 3 \times 2 = -25 - 6$

$= -31\,KN.m$

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A rectangular beam of 100mm wide and 200mm deep is simply supported over a span of 5m. If the beam is subjected to 20KN central load find the maximum bending stress and maximum shear stress. Also draw the shear distribution for the beam.

Area of the beam = $100 \times 200$

$= 20000\ mm^2$.

$F = 20KN = \frac{20}{2} = \cancel{10\ KN}$

$= 10000$

$\tau = \frac{3}{2} \frac{F}{A}$

$= \frac{3}{2} \times \frac{10000}{20000}$

$\cancel{= 7.5 \times 10^{-4}\ KN}$

$= \frac{3}{4}$.

$= 0.75\ N/mm^2$.

$$\frac{E}{R} = \frac{M}{I} = \frac{\sigma}{y}$$

$$\frac{\sigma}{y} = \frac{M}{I}.$$

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$$M = \frac{Wl}{4}$$

$$= \frac{\cancel{20 \times 5}}{\cancel{4}} \qquad \frac{\overset{5000}{\cancel{20000}} \times 5}{4}$$

$$= \cancel{25\ N.m}$$

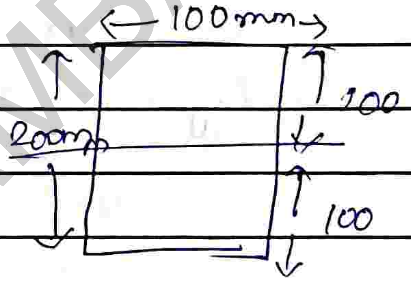
$$= 25000$$

$$= 25 \times 10^3\ N.m.$$

$$I = \frac{100 \times 200^3}{12}$$

$$= 66.67 \times 10^6\ mm^4.$$

$$y = 100mm$$




$$\frac{25 \times 10^3}{66.67 \times 10^6} \times 100 = \sigma.$$

$$\sigma = 37.49 \times 10^{-3}.$$

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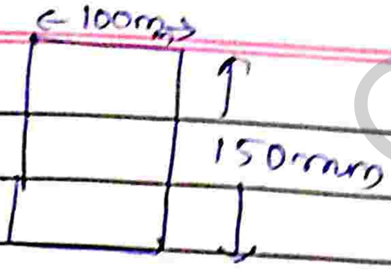
Q2




$$\tau_{max} = 1.5 \times \frac{F}{A}$$

$$2\ N/mm^2 = 1.5 \times \frac{F}{100 \times 150}$$

$$F = \frac{2 \times 100 \times 150}{1.5}$$

$$F = 20 \times 10^3$$

$$= 20\ kN.$$

$\cancel{f = wl}$ $\quad F = \frac{wl}{2}$

$\cancel{20 = w \times 2}$ $\quad \frac{20000}{\cancel{2}} \cdot 2 = \cancel{w}$ $\quad w = \frac{\cancel{2 \times 20000}}{\cancel{2000}}$

$\cancel{w = 10}$

$\boxed{\cancel{w = 20000\ N/mm}}$ $\quad w = 20\ N/mm$

$$\frac{M}{I} = \frac{\sigma}{y} \qquad I = \frac{bh^3}{12}$$

$$= \frac{100 \times 150^3}{12}$$

$$= 28.125 \times 10^6\ mm^4.$$

$$\frac{M}{28.125 \times 10^6} = \frac{28}{75}$$

$$M = \frac{28 \times 28.125 \times 10^6}{75}$$

$$M = 10.5 \times 10^6\ N.mm$$

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$M = \frac{w l^2}{8}$.

$\frac{10.5 \times 10^6 \times 8}{(2)^2} = w$

$w = 21 \times 10^6 N/mm$

$w = \frac{10.5 \times 10^6 \times 8}{(2000)^2}$.

$w = 21 \, N/mm$

Q1

$l = 5m$

$l_{eff} = 2 \times 5$
$= 10m$

$I = \frac{\pi D^4}{64}$

$= \frac{\pi \times (50 \times 10^{-3})^4}{64} = 3.06 \times 10^{-7} \, m^4$.

$E = 200 \times 10^9 \, N/m^2$.

$P = \frac{\pi^2 E I}{l_{eff}}$

$= \frac{\pi^2 \times 200 \times 10^9 \times 3.06 \times 10^{-7}}{10} = 60.40 \times 10^3$

$= 60.40 \, kN$.

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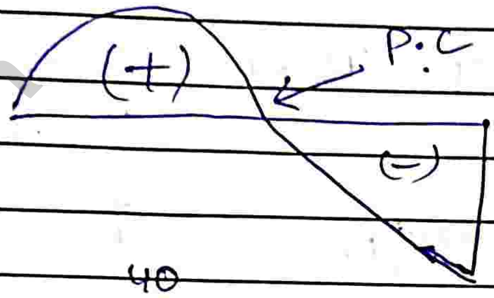
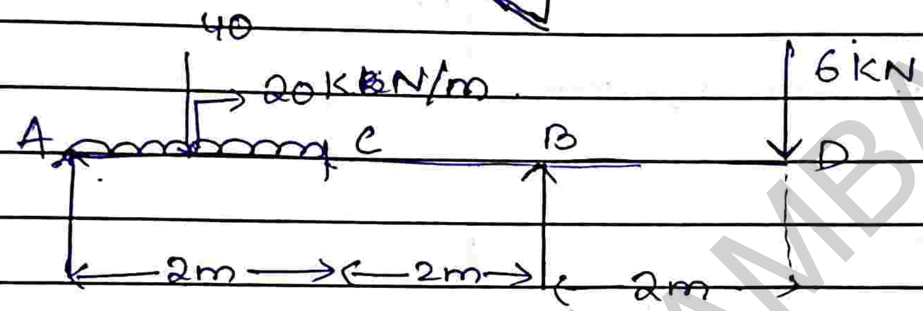
## Point of contraflexure :-

↳ where the Bending moment changes its sign is called as point of contraflexure.




$R_A + R_B - 6 \cancel{20} - 40$

$R_A + R_B = 46 \quad \text{——} (1)$

$M_A = 0$

$-6 \times 6 + R_B \times 4 - 40 \times 1 = 0$

$-36 + 4R_B - 40 = 0$

$4R_B = 36 + 40$

$R_B = \dfrac{76}{4}$

$\boxed{R_B = 19 \text{ kN}}$

$R_A + 19 = 46$

$\boxed{R_A = 27 \text{ kN}}$

$BM_D = -6 \times 0 = 0$ KN-m

$BM_B = -6 \times 2 = -12$ KN-m

$BM_C = -6 \times 4 + 19 \times 2 = 14$ KN-m

$BM_A = -6 \times 6 + 19 \times 4 - 40 \times 1 = 0$ KN-m