LECTURE NOTES

ON

FLUID MECHANICS

4TH SEMESTER

DIPLOMA IN MECHANICAL ENGINEERING

NM INSTITUTE OF ENGINEERING & TECHNOLOGY

BHUBANESWAR

UNIT 1 FLUID PROPERTIES

Fluids: Substances capable of flowing are known as fluids. Flow is the continuous deformation of substances under the action of shear stresses.

Fluids have no definite shape of their own, but confirm to the shape of the containing vessel. Fluids include liquids and gases.

Fluid Mechanics:

Fluid mechanics is the branch of science that deals with the behavior of fluids at rest as well as in motion. Thus,it deals with the static, kinematics and dynamic aspects of fluids.

The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called fluid dynamics.

Fluid Properties:

1. ***Density (or)Mass Density***:

Density or mass density of a fluid is defined as the ratio of the mass of the fluid to its volume. *Thus,* ***Mass per unit volume of a fluid is called density***.

$$\text{Mass density, } \rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

S.I unit of density is kg/m^3.

The value of density for water is 1000 kg/m^3.

2. ***Specific weight (or) Weight Density (w)***:

Specific weight or weight density of a fluid is the ratio between the weight of a fluid to its volume.

The weight per unit volume of a fluid is called specific weight or weight density.

$$\text{Weight density} = \frac{\text{Weight of fluid}}{\text{Volume of fluid}}$$

$$w = \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$w = \rho g$$

S.I unit of specific weight is N/m^3.

The value of specific weight or weight density of water is $9810 N/m^3$ or $9.81 kN/m^3$.

3. ***Specific Volume (v)***:

Specific volume of a fluid is defined as the volume of a fluid occupied by unit mass.

Volume per unit mass of a fluid is called Specific volume.

$$Specific\ volume = \frac{Volume\ of\ a\ fluid}{Mass\ of\ fluid} = \frac{1}{\rho}$$

Thus specific volume is the reciprocal of mass density. **S.I unit: m^3 /kg**.

4. *Specific Gravity (s)*:

Specific gravity is defined as the ratio of the specific weight of a fluid to the specific weight of a standard fluid.

$$Specific\ gravity = \frac{Specific\ weight\ of\ liquid}{Specific\ weight\ of\ water}$$

Specific gravity is also equal to Relative density. Relative density = ______________

5. *Viscosity*:

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over adjacent layer of the fluid.

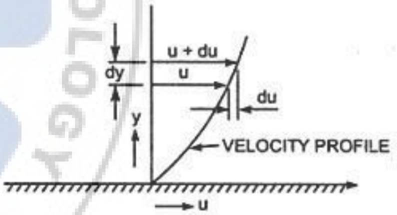
When two layers of a fluid, at distance 'dy' apart, move one over the other at different velocities, say u and u+du as shown in figure. The viscosity together with relative velocity causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y.

$$\tau \alpha \frac{du}{dy}$$

or $$\tau = \mu \frac{du}{dy}$$



Velocity variation near a solid boundary.

Fig.1. Velocity distribution curve

Thus the viscosity is also defined as the shear stress required to produce unit rate of shear strain.

S.I unit: Ns/m². It is still expressed in poise (P) as well as centipoises (cP).

$$\text{One poise} = \frac{1}{10}\frac{Ns}{m^2}; \qquad 1\ \text{centipoise} = \frac{1}{100}\ \text{poise}$$

Kinematic Viscosity (ν): It is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by the nu.

V = Viscosity/Density = μ/ρ

In MKS and SI unit of kinematic viscosity is metre^2/sec or m^2/sec while CGs units it is written as cm2/s. In CGS units kinematic viscosity is also known as stoke.

One stoke = cm^2/s = $(1/100)^2$ m^2/s = 10^{-4} m^2/s and centistokes means = 1/100 stoke

Newton's Law of Viscosity:

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

$$\tau = \mu \frac{du}{dy}$$

*6.**<u>Compressibility</u>***:

Compressibility is the reciprocal of the bulk modulus of elasticity, K, which is defined as the ratio of compressive stress to volumetric strain.

Compression of fluids gives rise to pressure with the decrease in volume.

If dv is the decrease in volume and dp is the increase in pressure, Volumetric Strain = $\frac{-dV}{V}$

(- ve sign indicate the volume decreases with increase of pressure)

$$\text{Bulk modulus, K} = \frac{\text{Increase of pressure}}{\text{Volumetric Strain}}$$

$$= \frac{dp}{\frac{-dV}{V}}$$

$$\text{Compressibility} = \frac{1}{K}$$

*7.**<u>Surface tension</u>***:

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

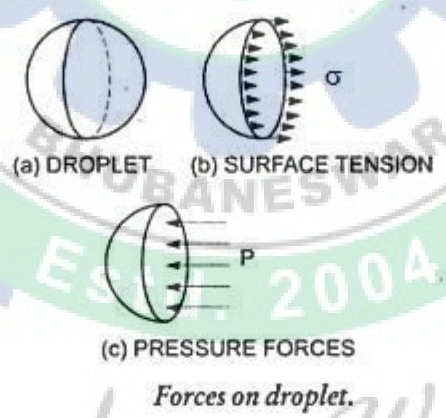


Fig.2. Forces on droplet

Surface Tension on Liquid Droplet:

Consider a small spherical droplet of a liquid of diameter 'd'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let σ = Surface tension of the liquid

p = Pressure intensity inside the droplet (in excess of the outside pressure intensity) d = Dia. of droplet.

Let the droplet is cut into two halves. The forces acting on one half will be

i) Tensile force (F_T)due to surface tension acting around the circumference of the cut portion as shown in fig. and this is equal to = σ x Circumference = $\sigma \times \pi d$

Pressure force (Fp) on the area C= $p \times (\pi/4) d^2$ as shown in the figure.

These two forces are equal under equilibrium conditions. i.e.,

$$p \times (\pi/4)\, d^2 = \sigma \times \pi\, d$$

$$P = (\sigma \times \pi\, d)/(\sigma \times \pi\, d) = 4\,\sigma/d$$

Surface Tension on a Hollow Bubble*:*

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces arc subjected to surface tension. In that case,

$$p\,(\pi/4)\, d^2 = 2(\sigma \times \pi\, d)$$

$$P = (2\sigma\, \pi\, d)/\,(\pi/4)\, d^2 = 8\,\sigma/d$$

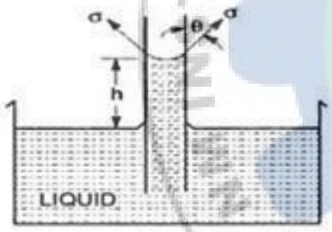
8. <u>Capillarity</u>:

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.

The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Expression for Capillary Rise:

Consider a glass tube of small diameter 'd' opened at both ends and is inserted in a liquid. The liquid will rise in the lube above the level of the liquid.



Capillary rise

Fig.3. Capillary Rise

Let, h = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height h is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Let, σ = Surface tension of liquid

θ= Angle of contact between liquid and glass lube.

The weight of liquid of height 'h' in the tube = (Area of tube x h) x ρ x g

where, ρ = density of liquid

Vertical component of the surface tensile force

= σ x Circumference x cos θ

= σ x πd x cos θ

Weight of liquid of height 'h' in the tube = Vertical component of the surface tensile force

$$\frac{\pi}{4}\, d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos\theta$$

$$h = \frac{\sigma \times \pi d \times \cos\theta}{\frac{\pi}{4} d^2 \times \rho \times g}$$

9.<u>Vapour pressure</u>:

***Vapour pressure** is the pressure of the vapor over a liquid which is confined in a closed vessel at equilibrium.* Vapour pressure increases with temperature. All liquids exhibit this phenomenon.

Types of fluid:

i. ***Ideal Fluid***: A fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

ii. ***Real Fluid***: A fluid, which possesses viscosity, is known as real fluid. All the fluids are real fluids in actual practice.

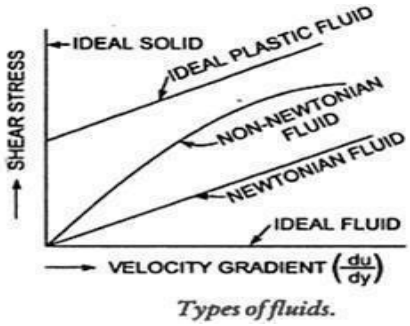


Fig.4. Types of Fluid

iii. ***Newtonian Fluid***: A real fluid, in which the shear stress is directly proportional to the rate of shear strain (or) velocity gradient, is known as a Newtonian fluid.

iv. ***Non-Newtonian Fluid:*** A real fluid, in which the shear stress is not proportional to the rate of shear strain (or) velocity gradient, is known as a Non-Newtonian fluid.

v. ***Ideal Plastic Fluid***: A fluid, in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain (or) velocity gradient, is known as ideal plastic fluid

Fluid Pressure

Fluid pressure is the force exerted by the fluid per unit area.

Fluid pressure or Intensity of pressure or pressure, = Fluids exert pressure on surfaces with which they are in contact.

Fluid pressure is transmitted with equal intensity in all directions and acts normal to any plane.
In the same horizontal plane the pressure intensities in a liquid are equal.

S.I unit of fluid pressure are N/m² or Pa, where 1 N/m² = 1 Pa.

Many other pressure units are commonly used:

1 bar = 10^5 N/m²
1 atmosphere = 101325 N/m² = 101.325kN/m²

Some Terms commonly used in static pressure analysis include:

Pressure Head***: The pressure intensity exerted at the base of a column of homogenous fluid of a given height in metres.***

Vacuum: A perfect vacuum is a completely empty space in which, therefore the pressure is zero.

Atmospheric Pressure***:*** The pressure at the surface of the earth exerted by the head of air above the surface.

At sea level the atmospheric pressure = 101.325 kN/m² = 101325 N/m² or pa
= 1.01325 bar
= 760 mm of mercury
= 10.336 m of water

Atmospheric pressure is measured by a device called a barometer; thus, the atmospheric pressure is often referred to as the barometric pressure.

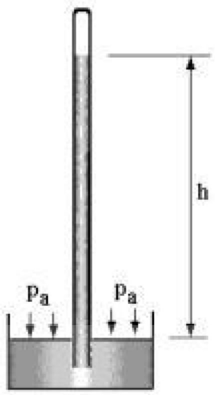
Gauge Pressure: The pressure measured by a pressure gauge above or below atmospheric pressure.

Vacuum pressure: The gauge pressure less than atmospheric is called Vacuum pressure or negative pressure.

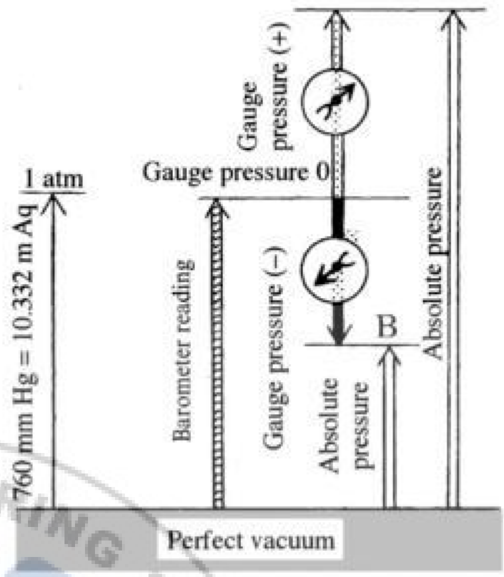
Absolute Pressure: The pressure measured above absolute zero or vacuum.

Absolute Pressure = Atmospheric Pressure + Gauge Pressure

Absolute Pressure = Atmospheric Pressure – Vacuum pressure



Barometer



Atmospheric, Gauge & Absolute pressure

Fig.5. Barometer, Atmospheric, Gauge and Absolute Pressure

Hydrostatic law

The **hydrostatic law** is a principle that identifies the amount of pressure exerted at a specific point in a given area of fluid.

It states that, "The rate of increase of pressure in the vertically downward direction, at a point in a static fluid, must be equal to the specific weight of the fluid."

Pressure Variation in static fluid

Consider a small vertical cylinder of static fluid in equilibrium.

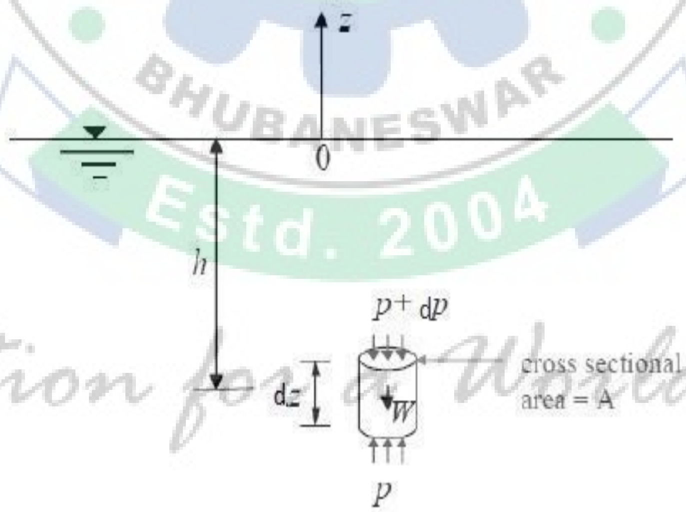


Fig.6. Pressure variation in static fluid

Assume that the sectional area is "**A**" and the pressure acting upward on the bottom surface is **p** and the pressure acting downward on the upper surface (dz above bottom surface) is (p + dp)dz.

Let the free surface of the fluid be the origin, i.e., $Z = 0$. Then the pressure variation at a depth $Z = -h$ below the free surface is governed by

$(p + dp) A + W = pA$

⇨ $dpA + \rho gAdz = 0$ [$W = w$ x volume $= \rho g\ Adz$] $dp = -\rho gdz$

⇨

$= - \rho g = - w$

Therefore, the hydrostatic pressure increases linearly with depth at the rate of the specific weight, $w = \rho g$ of the fluid.

If fluid is homogeneous, ρ is constant.

By simply integrating the above equation,

$\int dp = - \int \rho g\, dz \Rightarrow p = - \rho g Z + C$

Where C is constant of integration.

When $z = 0$ (on the free surface), $p = C = p_o$ = the atmospheric pressure.

Hence, $p = - \rho gZ + p_o$

Pressure given by this equation is called ABSOLUTE PRESSURE, i.e., measured above perfect vacuum.

However, it is more convenient to measure the pressure as gauge pressure by setting atmospheric pressure as datum pressure. By setting $p_o = 0$,

$p = -\rho gz+0 = -\rho gz = \rho gh$

$\mathbf{p = wh}$

The equation derived above shows that when the density is constant, the pressure in a liquid at rest increases linearly with depth from the free surface.

Here, **h** is known as ***pressure head*** or simply ***head*** of fluid.

In fluid mechanics, fluid pressure is usually expressed in height of fluids or head of fluids.

Hydrostatic force

Hydrostatic pressure is the force exerted by a static fluid on a plane surface, when the static fluid comes in contact with the surface. This force will act normal to the surface. It is also known as ***Total Pressure***.

The point of application of the hydrostatic or total pressure on the surface is known as ***Centre of pressure***.

The vertical distance between the free surface of fluid and the centre of pressure is called depth of centre of pressure or location of hydrostatic force.

Total Pressure on a Horizontally Immersed Surface

Consider a plane horizontal surface immersed in a liquid as shown in figure.

Let, w = Specific weight of the liquid, kN/m³

A = Area of the immersed surface in m²

= Depth of the horizontal surface from the liquid level in m

We know that,

Total pressure on the surface, **P** = Weight of the liquid above the immersed surface

P = Specific weight of liquid x Volume of liquid

= Specific weight of liquid x Area of surface x Depth of liquid P = *w*A kN

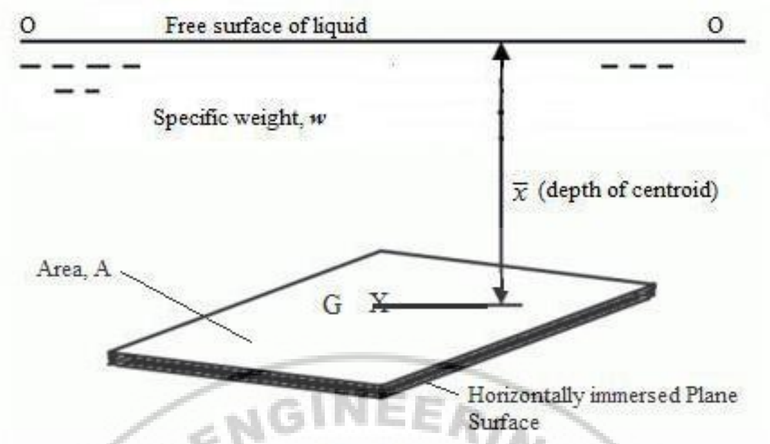


Fig:7. Horizontal Plane surface submerged in liquid

Total Pressure and depth of centre of pressure on a Vertically Immersed Surface

Consider an irregular plane vertical surface immersed in a liquid as shown in figure .
Let,
w = Specific weight of liquid
A = Total area of the immersed surface
= Depth of the center of gravity of the immersed surface from the liquid surface
Now. consider a strip of width ‘b’, thickness ‘dx’ and at a depth *x* from the free surface of the liquid

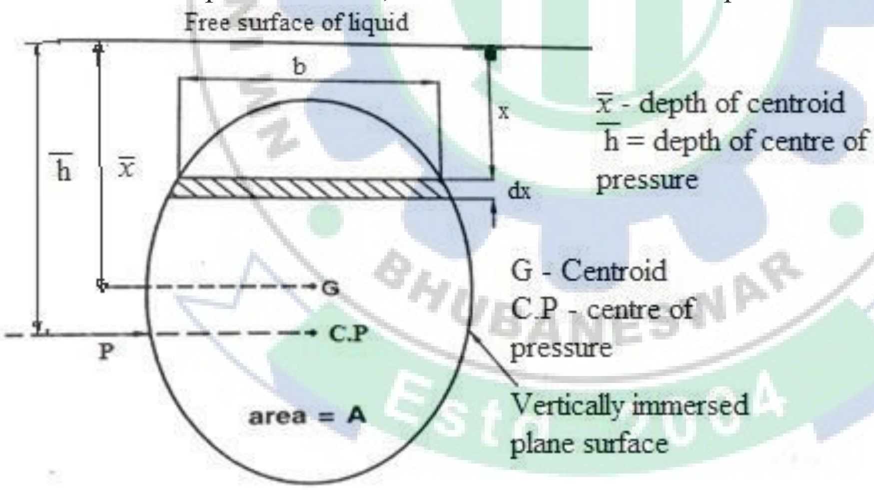


Fig: 9. Vertical Plan immersed in liquid

Moment of pressure on the strip about the free surface of liquid = w x b dx X x = w x^2 b dx Total moment on the entire plane immersed surface = $\int w\ x^2\ b\ dx$

M = $w \int$ 2

But, $\int$ 2 = second moment of area about free liquid surface = I_o

therefore, M = w Io

$I_o = I_G + A\ x^2$, according to parallel axis theorem.

Therefore, M = w (I_G + A x^2)---------------------------------- (1)

Also = x h = Awx x h ---------------------------------- (2)

Since equations 1 & 2 are equal,

A x h = (IG + A x²)

epth of centre of pressure, h= (IG + A x²) / w A

h* = [(I_G + A$\bar{h}^2$)/A$\bar{h}$] + $\bar{h}$

Total Pressure and depth of Centre of Pressure on an Inclined Immersed Surface

Consider a plane inclined surface, immersed in a liquid as shown in figure. Let,

w = Specific weight of the liquid

A = Total area of the immersed surface

x = Depth of the centroid of the immersed plane surface from the free surface of liquid.

θ = Angle at which the immersed surface is inclined with the liquid surface h= depth of centre of pressure from the liquid surface

b = width of the considered thin strip dx = thickness of the strip

O = the reference point obtained by projecting the plane surface with the free surface of liquid

x = distance of the strip from O

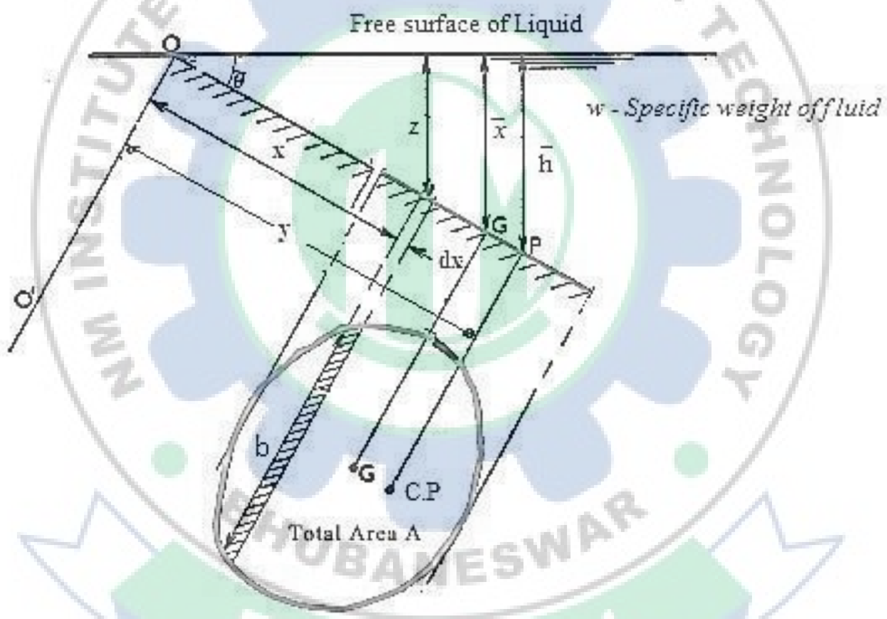


Fig: 10. Inclined Immersed Plain

Let the plane of the surface, if produced meet the free liquid surface at O. Then O-O is the axis perpendicular to the plane of the surface.

Let $\bar{y}$ = distance of the C.G. of the inclined surface from O-O

y^* = distance of the centre of pressure from O-O.

Consider a small strip of area dA at a depth 'h' from free surface and at a distance y from the axis O-O as shown in Fig. 3.18.

Pressure intensity on the strip, $p = \rho gh$

$\therefore$ Pressure force, dF, on the strip, $dF = p \times \text{Area of strip} = \rho gh \times dA$

Total pressure force on the whole area, $F = \int dF = \int \rho ghdA$

But from Fig. 3.18, $\dfrac{h}{y} = \dfrac{\bar{h}}{\bar{y}} = \dfrac{h^*}{y^*} = \sin\theta$

$\therefore$ $h = y\sin\theta$

$\therefore$ $F = \int \rho g \times y \times \sin\theta \times dA = \rho g \sin\theta \int ydA$

But $\int ydA = A\bar{y}$

where $\bar{y}$ = Distance of C.G. from axis O-O

$\therefore$ $F = \rho g \sin\theta\, \bar{y} \times A$

$= \rho gA\bar{h}$ $\qquad (\because \bar{h} = \bar{y}\sin\theta)$ ...(3.6)

Centre of Pressure (h*)

Pressure force on the strip, $dF = \rho ghdA$

$= \rho gy \sin\theta\, dA$ $\qquad [h = y\sin\theta]$

Moment of the force, dF, about axis O-O

$= dF \times y = \rho gy\sin\theta\, dA \times y = \rho g \sin\theta\, y^2 dA$

Sum of moments of all such forces about O-O

$= \int \rho g \sin\theta\, y^2\, dA = \rho g \sin\theta \int y^2\, dA$

But $\int y^2\, dA$ = M.O.I. of the surface about O-$O = I_0$

$\therefore$ Sum of moments of all forces about O-$O = \rho g \sin\theta\, I_0$

Moment of the total force, F, about O-O is also given by

$= F \times y^*$

where y^* = Distance of centre of pressure from O-O.

Equating the two values given by equations (3.7) and (3.8)

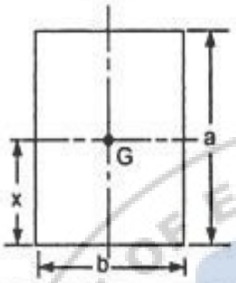
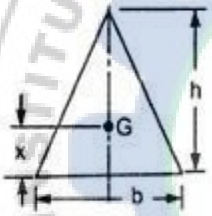
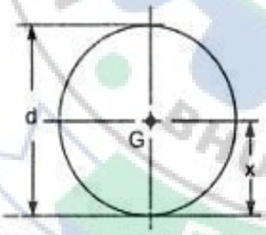
$F \times y^* = \rho g \sin\theta\, I_0$

or $y^* = \dfrac{\rho g \sin\theta\, I_0}{F}$

Now $y^* = \dfrac{h^*}{\sin\theta}$, $F = \rho gA\bar{h}$

and I_0 by the theorem of parallel axis $= I_G + A\bar{y}^2$.

Table: M.I and Geometric Properties of some plane surfaces

Plane surface	*C.G. from the base*	*Area*	*Moment of inertia about an axis passing through C.G. and parallel to base (I_G)*
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$

Pascal's law

The basic property of a static fluid is pressure.
Pressure is the surface force exerted by a fluid against the walls of its container.
Pressure also exists at every point within a volume of fluid.
For a static fluid, as shown by the following analysis, pressure turns to be independent direction.

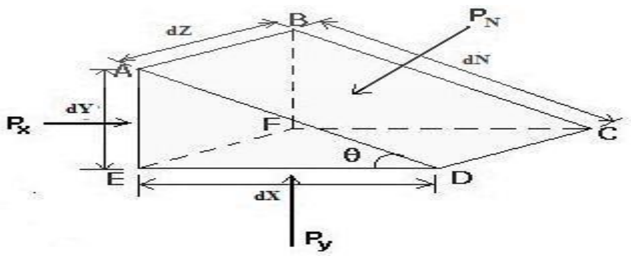


Fig:11. Pascal Law

Consider a triangular prism of small fluid element ABCDEF in equilibrium. Let P_x is the intensity of pressure in the X direction acting at right angle on the face ABFE, Py is the intensity of pressure in the Y direction acting at right angle on the face CDEF, and Ps is the intensity of pressure normal to inclined plane at an angle θ as shown in figure at right angle to ABC ..

For a fluid at rest there will be no shear stress, there will be no accelerating forces, and therefore the sum of the forces in any direction must be zero.

Thus the forces acting on the fluid element are the pressures on the surrounding and the gravity force.

Force due to $p_x = p_x$ x Area ABFE $= p_x\,dydz$

Horizontal component of force due to $p_N = -(p_N$ x Area ABC) $\sin(\theta) = -p_N dNdz\,dy/ds = -P_N dydz$

As Py has no component in the x direction, the element will be in equilibrium, if

$p_x\,dydz + (-p_N dydz) = 0$

i.e. $p_x = p_N$

Similarly in the y direction, force due to $p_y = p_y dxdz$

Component of force due to $p_N = -(p_N$ x Area ABC) $\cos(\theta) = -p_N dsdz\,dx/ds = -$

$p_N dxdz$ Force due to weight of element is negligible and the equation reduces to,

$p_y = p_N$

Therefore, $p_x = p_y = p_N$

Thus, Pressure at a point in a fluid at rest is same in all directions.

Manometers:

Manometer is an instrument for measuring the pressure of a fluid, consisting of a tube filled with a heavier gauging liquid, the level of the liquid being determined by the fluid pressure and the height of the liquid being indicated on a scale. A U-tube manometer consists of a glass tube bent in U-Shape, one end of which is connected to gauge point and the other end is exposed to atmosphere.

Manometric liquids:

1. Manometric liquids should neither mix nor have any chemical reaction with the liquid whose pressure intensity is to be measured.
2. It should not undergo any thermal variation.
3. Manometric liquid should have very low vapour pressure.
4. Manometric liquid should have pressure sensitivity depending upon the magnitude of pressure to be measured and accuracy requirement.

Convert all vertical columns of liquids to meters of water by multiplying them by corresponding specify gravity.

To write the manometric equation:

1. Convert all given pressure to meters of water and assume unknown pressure in meters of waters.
2. Proceeding from one end towards the other the following points must be considered.

- Any horizontal movement inside the same liquid will not cause change in pressure.
- Vertically downward movement causes increase in pressure and upward motion cause decrease in pressure.
- Convert all vertical columns of liquids to meters of water by multiplying them by corresponding specify gravity.
- Take atmospheric pressure as zero (gauge pressure computation).

Simple U-Tube Manometer: It consist of glass tube in U shape one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in fig. The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

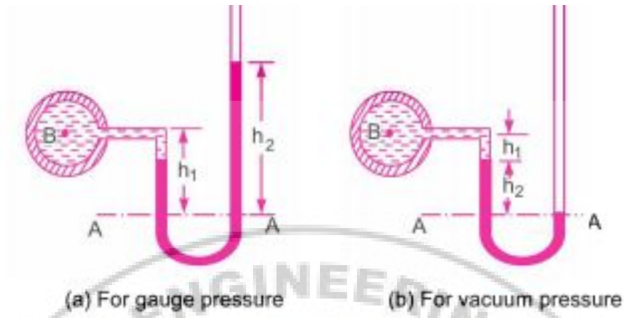


Fig: 12. Simple U tube Manometer

For Gauge Pressure. Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A Let, H_1 = Height of light liquid above the datum line

H_2 = Height of heavier liquid above the datum line

S_1 = Specific gravity of light liquid

ρ_1 = Density of light liquid = 1000 x S_1

S_2 = Specific gravity of heavy liquid

ρ_2 = Density of heavy liquid = 1000 x S_2

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line *A-A* in the left column and in the right column of U-tube manometer should be same.

Pressure above *A-A* in the left column $= p + \rho_1 \times g \times h_1$

Pressure above *A-A* in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1).$

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above *A-A* in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above *A-A* $= 0$

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$

Differential U-tube manometer

Let, A and B are the two pipes carrying liquids of specific gravity s_1 and s_3 & s_2 = specific gravity of manometer liquid.

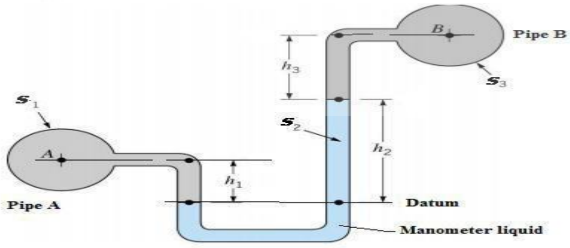


Fig:13. Differential U-tube Manometer

Let two point A & B are at different level and also contains liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B

Let h = Difference of mercury level in the U-tube.

y = Distance of the centre of B, from the mercury level in the right limb.

x = Distance of the centre of A, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X.

Pressure above X-X in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at A.

Pressure above X-X in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at B.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$$

$\therefore$ Difference of pressure at A and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

In Fig. 2.18 (*b*), the two points A and B are at the same level and contains the same liquid of density ρ_1. Then

Pressure above X-X in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above X-X in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1).$$

Buoyant force: The upward force exerted by a liquid on a body when the body is immersed in the liquid is known as buoyancy or buoyant force.

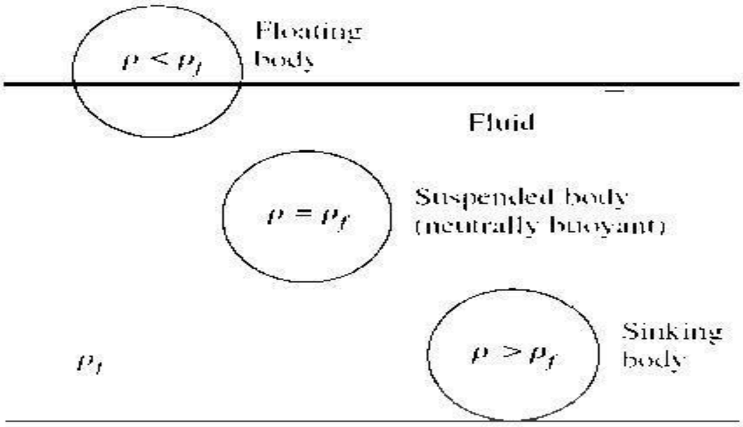
The point through which force of buoyancy is supposed to act is called centre of buoyancy.

The buoyant force acting on a body is equal to the weight of the liquid displaced by the body.

For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body.

Archimedes principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

For **floating bodies**, the weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body.



ρ - density of body; ρ_f – density of fluid

A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.

Fig:14. Floating Body

Stability of Immersed and Floating Bodies

A floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.

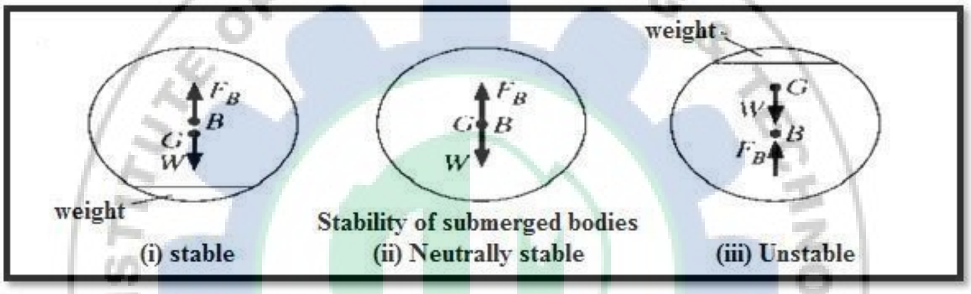


Fig:15. An immersed neutrally buoyant body is (a) stable if the center of gravity G is directly below the center of buoyancy B of the body, (b) neutrally stable if G and B are coincident, and (c) unstable if G is directly above B.

Stability of floating bodies: A floating body is stable if the body is bottom-heavy and thus the center of gravity G is below the centroid B of the body, or if the metacentre M is above point G. However, the body is unstable if point M is below point G.

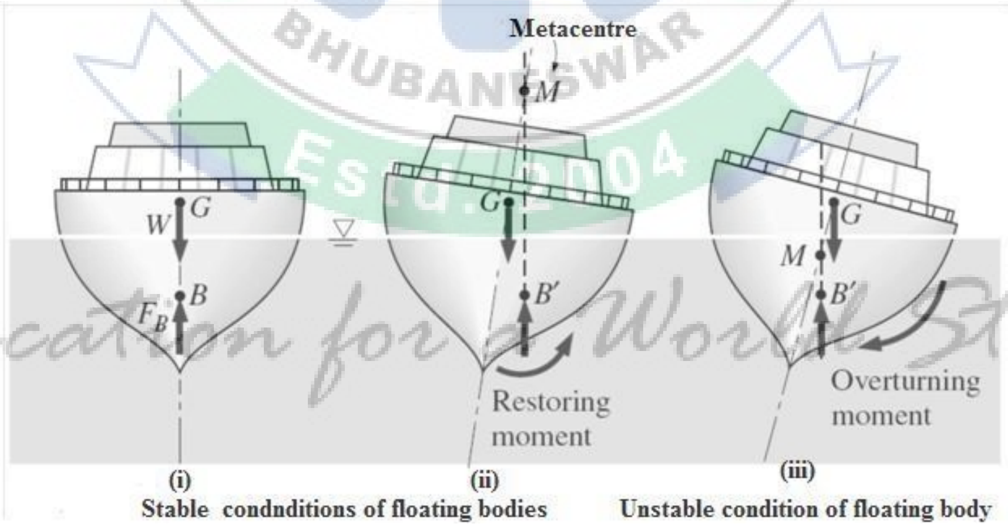


Fig.16.Stability of Floating Bodies

Metacentre: The point about which a body starts oscillating when the body is tilted is known meta-centre.

Metacentric height GM: The distance between the center of gravity G and the metacenter M is known as Meta centric height. It is the point of intersection of line of action of buoyant force with the line passing through centre of gravity, when the body is slightly tilted.

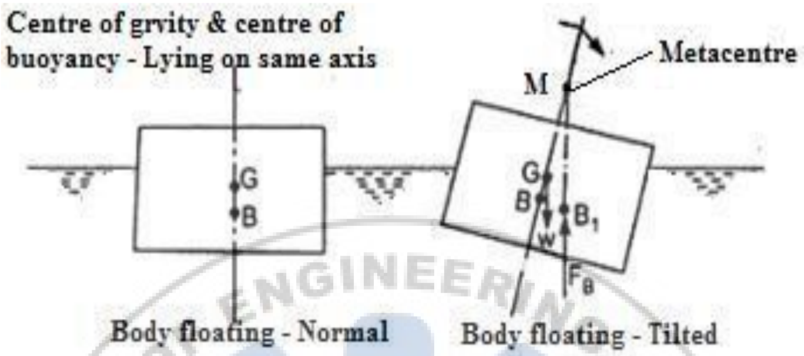


Fig.17. Metacentric Height

The length of the metacentric height GM above G is a measure of the stability: If the metacentric height increases, then the floating body will be more.. The meta-centric height (GM) is.given by, GM = V - BG

Where, I = Moment of Inertia of the floating body (in plan) at water surface about the axis Y-Y V = Volume of ihe body sub merged in water

BG = Distance between centre of gravity and centre of buoyancy.

Conditions of equilibrium of a floating and submerged body are :

Table.2. Condition of Equilibrium of a Floating bodies

Equilibrium	Floating Body	Sub-merged Body
(i) Stable Equilibrium (a) Unstable Equilibrium (Hi) Neutral Equilibrium	M is above G M is below G Af and G coincide	B is above G B is below G B and G coincide

Problems:

1. Calculate the sp.weight, density and sp.gravity of one litre of liquid which weights 7N.

Sol:

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because \ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3\right)$$

$$\text{Weight} = 7 \text{ N}$$

$$(i) \text{ Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000}\right) \text{m}^3} = \mathbf{7000 \text{ N/m}^3. \text{ Ans.}}$$

$$(ii) \text{ Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = \mathbf{713.5 \text{ kg/m}^3. \text{ Ans.}}$$

$$(iii) \text{ Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{\because \ \text{Density of water} = 1000 \text{ kg/m}^3\}$$

$$= \mathbf{0.7135. \text{ Ans.}}$$

2. Calculate the density, sp.weight and weight of one litre of petrol of specific gravity = 0.7

Solution. Given : Volume = 1 litre = $1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

Sp. gravity $S = 0.7$

(*i*) *Density* (ρ)

Using equation (1.1*A*),

Density (ρ) $= S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 =$ **700 kg/m³. Ans.**

(*ii*) *Specific weight* (*w*)

Using equation (1.1), $w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 =$ **6867 N/m³. Ans.**

(*iii*) *Weight* (*W*)

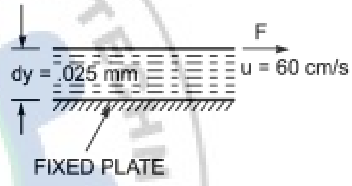
We know that specific weight $= \frac{\text{Weight}}{\text{Volume}}$

or $w = \frac{W}{0.001}$ or $6867 = \frac{W}{0.001}$

∴ $W = 6867 \times 0.001 =$ **6.867 N. Ans.**

3. A plate 0.023 mm distant from a fixed plate moves at 60 cm/s and requires a force of 2N per unit area i.e 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.

Solution. Given :



Distance between plates, $dy = .025 \text{ mm}$
$= .025 \times 10^{-3} \text{ m}$

Velocity of upper plate, $u = 60 \text{ cm/s} = 0.6 \text{ m/s}$

Force on upper plate, $F = 2.0 \frac{\text{N}}{\text{m}^2}$.

This is the value of shear stress *i.e.*, τ

Let the fluid viscosity between the plates is μ.

Using the equation (1.2), we have $\tau = \mu \frac{du}{dy}$.

where du = Change of velocity $= u - 0 = u = 0.60$ m/s

dy = Change of distance $= .025 \times 10^{-3}$ m

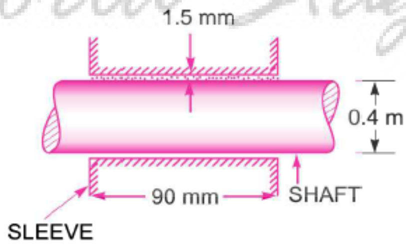
τ = Force per unit area $= 2.0 \frac{\text{N}}{\text{m}^2}$

∴ $2.0 = \mu \frac{0.60}{.025 \times 10^{-3}}$ ∴ $\mu = \frac{2.0 \times .025 \times 10^{-3}}{0.60} = 8.33 \times 10^{-5} \frac{\text{Ns}}{\text{m}^2}$

$= 8.33 \times 10^{-5} \times 10$ poise $=$ **8.33 × 10⁻⁴ poise. Ans.**

4. The dynamic viscosity of oil used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90mm. The thickness of the oil film is 1.5mm.

Solution. Given :



Viscosity $\mu = 6$ poise
$= \frac{6}{10} \frac{\text{N s}}{\text{m}^2} = 0.6 \frac{\text{N s}}{\text{m}^2}$

Dia. of shaft, $D = 0.4$ m

Speed of shaft, $N = 190$ **r.p.m**

Sleeve length, $L = 90 \text{ mm} = 90 \times 10^{-3} \text{ m}$

Thickness of oil film, $t = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

Tangential velocity of shaft, $u = \frac{\pi D N}{60} = \frac{\pi \times 0.4 \times 190}{60} = 3.98$ m/s

Using the relation $\tau = \mu \frac{du}{dy}$

where du = Change of velocity = $u - 0 = u = 3.98$ m/s

dy = Change of distance = $t = 1.5 \times 10^{-3}$ m

$$\tau = 10 \times \frac{3.98}{1.5 \times 10^{-3}} = 1592 \text{ N/m}^2$$

This is shear stress on shaft

∴ Shear force on the shaft, F = Shear stress × Area

$$= 1592 \times \pi D \times L = 1592 \times \pi \times .4 \times 90 \times 10^{-3} = 180.05 \text{ N}$$

Torque on the shaft, $T = \text{Force} \times \frac{D}{2} = 180.05 \times \frac{0.4}{2} = 36.01$ Nm

∴ *Power lost $= \frac{2\pi NT}{60} = \frac{2\pi \times 190 \times 36.01}{60} =$ **716.48 W. Ans.**

5. The surface tension of water in contact with air at 20°C is 0.0725N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater then the outside pressure. Calculate the diameter of the droplet of water.

Solution. Given :

Surface tension, $\sigma = 0.0725$ N/m

Pressure intensity, p in excess of outside pressure is

$$p = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \frac{\text{N}}{\text{m}^2}$$

Let d = dia. of the droplet

we get $p = \frac{4\sigma}{d}$ or $0.02 \times 10^4 = \frac{4 \times 0.0725}{d}$

$$d = \frac{4 \times 0.0725}{0.02 \times (10)^4} = .00145 \text{ m} = .00145 \times 1000 = \textbf{1.45 mm. Ans.}$$

6. Calculate the capillary rise in a glass tube of 2.5mm diameter when immersed vertically in a) water b) Mercury. Take surface tension of 2.5 mm diameter when immersed vertically in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°

Solution. Given :

Dia. of tube, $d = 2.5 \text{ mm} = 2.5 \times 10^{-3}$ m

Surface tension, σ for water $= 0.0725$ N/m

σ for mercury $= 0.52$ N/m

Sp. gr. of mercury $= 13.6$

∴ Density $= 13.6 \times 1000$ kg/m³.

(a) Capillary rise for water ($\theta = 0°$)

Using equation (1.20), we get $h = \dfrac{4\sigma}{\rho \times g \times d} = \dfrac{4 \times 0.0725}{1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= .0118$ m $=$ **1.18 cm. Ans.**

(b) For mercury

Angle of contact between mercury and glass tube, $\theta = 130°$

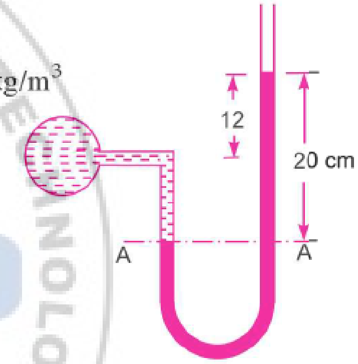
Using equation (1.21), we get $h = \dfrac{4\sigma \cos\theta}{\rho \times g \times d} = \dfrac{4 \times 0.52 \times \cos 130°}{13.6 \times 1000 \times 9.81 \times 2.5 \times 10^{-3}}$

$= -.004$ m $= -$ **0.4 cm. Ans.**

The negative sign indicates the capillary depression.

7. The right limb of a single U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp.gravity is 0.9 is flowing. The centre of the pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury in the two limbs is 20cm.

Solution. Given :



Sp. gr. of fluid, $S_1 = 0.9$

∴ Density of fluid, $\rho_1 = S_1 \times 1000 = 0.9 \times 1000 = 900$ kg/m³

Sp. gr. of mercury, $S_2 = 13.6$

∴ Density of mercury, $\rho_2 = 13.6 \times 1000$ kg/m³

Difference of mercury level, $h_2 = 20$ cm $= 0.2$ m

Height of fluid from A-A, $h_1 = 20 - 12 = 8$ cm $= 0.08$ m

Let p = Pressure of fluid in pipe

Equating the pressure above A-A, we get

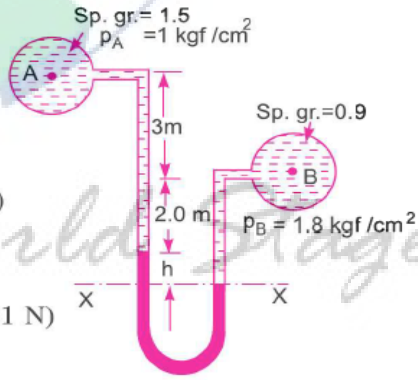
$$p + \rho_1 g h_1 = \rho_2 g h_2$$

$$p + 900 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times .2$$

$$p = 13.6 \times 1000 \times 9.81 \times .2 - 900 \times 9.81 \times 0.08$$

$= 26683 - 706 = 25977$ N/m² $=$ **2.597 N/cm². Ans.**

8. A differential manometer is connected at the two points A and B of two pipes as shown in fig. The pipe A contains a liquid of Sp.gravity = 1.5 while pipe B contains a liquid of sp.gravity = 0.9. The pressure at A and B are 1 Kgf/cm² and 1.80 Kgf/cm² respectively. Find the difference in mercury level in the differential manometer.



Solution. Given :

Sp. gr. of liquid at A, $S_1 = 1.5$ ∴ $\rho_1 = 1500$

Sp. gr. of liquid at B, $S_2 = 0.9$ ∴ $\rho_2 = 900$

Pressure at A, $p_A = 1$ kgf/cm² $= 1 \times 10^4$ kgf/m²

$= 10^4 \times 9.81$ N/m² (∵ 1 kgf = 9.81 N)

Pressure at B, $p_B = 1.8$ kgf/cm²

$= 1.8 \times 10^4$ kgf/m²

$= 1.8 \times 10^4 \times 9.81$ N/m² (∵ 1 kgf = 9.81 N)

Density of mercury $= 13.6 \times 1000$ kg/m³

Taking X-X as datum line.

Pressure above X-X in the left limb

$$= 13.6 \times 1000 \times 9.81 \times h + 1500 \times 9.81 \times (2 + 3) + p_A$$
$$= 13.6 \times 1000 \times 9.81 \times h + 7500 \times 9.81 + 9.81 \times 10^4$$

Pressure above X-X in the right limb $= 900 \times 9.81 \times (h + 2) + p_B$

$$= 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Equating the two pressure, we get

$$13.6 \times 1000 \times 9.81h + 7500 \times 9.81 + 9.81 \times 10^4 = 900 \times 9.81 \times (h + 2) + 1.8 \times 10^4 \times 9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h + 2.0) \times .9 + 18$$
$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$
$$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.7h = 2.3$$

$$\therefore \quad h = \frac{2.3}{12.7} = 0.181 \text{ m} = \mathbf{18.1 \text{ cm. Ans.}}$$

9. A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and a) coincide with water surface b) 2.5 m below the free water surface.

Solution. Given :

Width of plane surface, $b = 2$ m

Depth of plane surface, $d = 3$ m

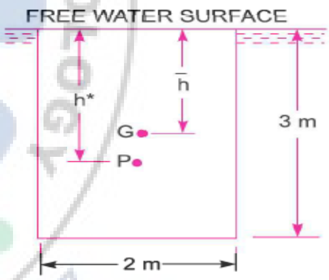
(*a*) Upper edge coincides with water surface

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3, $g = 9.81$ m/s^2

$$A = 3 \times 2 = 6 \text{ m}^2, \ \bar{h} = \frac{1}{2}(3) = 1.5 \text{ m.}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 1.5 = \mathbf{88290 \text{ N. Ans.}}$$



Depth of centre of pressure is given by equation (3.5) as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

where I_G = M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5}{6 \times 1.5} + 1.5 = 0.5 + 1.5 = \mathbf{2.0 \text{ m. Ans.}}$$

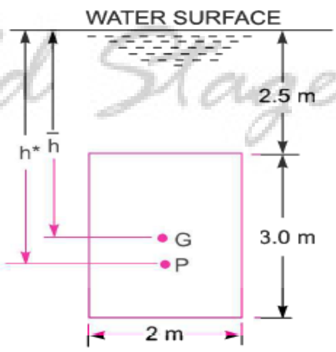
(*b*) Upper edge is 2.5 m below water surface

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G. from free surface of water

$$= 2.5 + \frac{3}{2} = 4.0 \text{ m}$$

$$\therefore \quad F = 1000 \times 9.81 \times 6 \times 4.0 = \mathbf{235440 \text{ N. Ans.}}$$



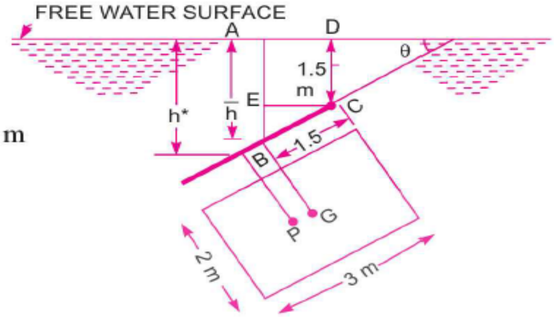
Centre of pressure is given by $h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

where $I_G = 4.5$, $A = 6.0$, $\bar{h} = 4.0$

$$\therefore \quad h^* = \frac{4.5}{6.0 \times 4.0} + 4.0$$

$$= 0.1875 + 4.0 = 4.1875 = \mathbf{4.1875 \text{ m. Ans.}}$$

10. A rectangular plane surface 2m wide and 3m deep lies in water in such a way that its plane makes an angle of 30° with the free surface of water. Determine the total surface and position of centre of pressure when the upper edge is 1.5m below the free water surface.



Solution. Given :

Width of plane surface, $b = 2$ m

Depth, $d = 3$ m

Angle, $\theta = 30°$

Distance of upper edge from free water surface = 1.5 m

(*i*) **Total pressure** force is given by equation

$$F = \rho g A \bar{h}$$

where $\rho = 1000$ kg/m^3

$A = b \times d = 3 \times 2 = 6$ m^2

∴ $\bar{h}$ = Depth of C.G. from free water surface

$= 1.5 + 1.5 \sin 30°$

$\{\because \bar{h} = AE + EB = 1.5 + BC \sin 30° = 1.5 + 1.5 \sin 30°\}$

$= 1.5 + 1.5 \times \frac{1}{2} = 2.25$ m

∴ $F = 1000 \times 9.81 \times 6 \times 2.25 =$ **132435 N. Ans.**

(*ii*) **Centre of pressure (h*)**

Using equation (3.10), we have

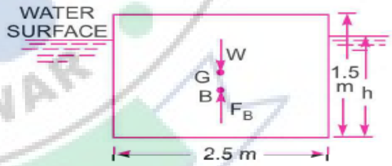
$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}, \quad \text{where } I_G = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = 4.5 \text{ m}^4$$

$$\therefore \quad h^* = \frac{4.5 \times \sin^2 30°}{6 \times 2.25} + 2.25 = \frac{4.5 \times \frac{1}{4}}{6 \times 2.25} + 2.25$$

$= 0.0833 + 2.25 =$ **2.3333 m. Ans.**

11. Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5m and depth 1.5m. When it floats horizontally in water. The density of wooden block is 650 kg/m^3 and its length 6m.

Solution. Given :



Width = 2.5 m

Depth = 1.5 m

Length = 6.0 m

Volume of the block $= 2.5 \times 1.5 \times 6.0 = 22.50$ m^3

Density of wood, $\rho = 650$ kg/m^3

∴ Weight of block $= \rho \times g \times$ Volume

$= 650 \times 9.81 \times 22.50$ N $= 143471$ N

For equilibrium the weight of water displaced = Weight of wooden block

= 143471 N

∴ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625 \text{ m}^3. \text{ Ans.}}$$

($\because$ Weight density of water $= 1000 \times 9.81$ N/m^3)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

$2.5 \times h \times 6.0 = 14.625$ m^3, where h is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875 \text{ m from base. Ans.}}$$

12. A rectangular pontoon is 5m long, 3m wide and 1.20m high. The depth of immersion of the position is 0.80 m in sea water. If the centre of gravity is 0.6m above the bottom of the position, determine the meta centric height. The density for sea water is 1025 kg/m^3.

Solution. Given :

Dimension of pontoon $= 5 \text{ m} \times 3 \text{ m} \times 1.20 \text{ m}$

Depth of immersion $= 0.8 \text{ m}$

Distance $AG = 0.6$ m

Distance $AB = \frac{1}{2} \times$ Depth of immersion

$= \frac{1}{2} \times .8 = 0.4$ m

Density for sea water $= 1025$ kg/m^3

Meta-centre height GM, given by equation

$$GM = \frac{I}{\forall} - BG$$

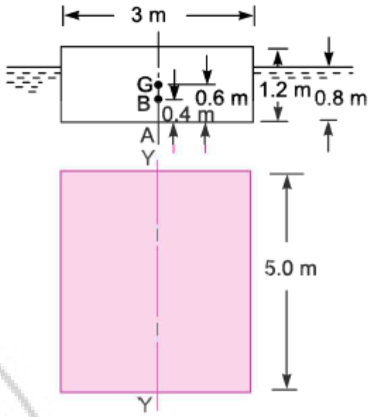
where I = M.O. Inertia of the plan of the pontoon about Y-Y axis

$$= \frac{1}{12} \times 5 \times 3^3 \text{ m}^4 = \frac{45}{4} \text{ m}^4$$

$\forall$ = Volume of the body sub-merged in water

$= 3 \times 0.8 \times 5.0 = 12.0$ m^3

$BG = AG - AB = 0.6 - 0.4 = 0.2$ m



PLAN AT WATER SURFACE

$$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = \mathbf{0.7375 \text{ m.}} \quad \textbf{Ans.}$$

Questions for practice:

PART - A

1. Define fluid and fluid mechanics.
2. Define real and ideal fluids.
3. Define mass density and specific weight.
4. Distinguish between fluid statics and kinematics.
5. Define viscosity.
6. Define specific volume.
7. Define specific gravity.
8. Distinct b/w capillarity and surface tension.
9. Calculate the specific weight, density and specific gravity of 1 liter liquid which weighs 7N.
10. State Newton's law of viscosity.
11. Name the types of fluids.
12. Define compressibility.
13. Define kinematic viscosity.
14. Find the kinematic viscosity of oil having density 981 kg/m^3. The shear stress at a point

in oil is 0.2452N/m and velocity gradient at that point is 0.2/sec.

15. Determine the specific gravity of a fluid having 0.05 poise and kinematic viscosity 0.035 stokes.
16. Find out the minimum size of glass tube that can be used to measure water level if the capillary rise is restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.
17. Write down the expression for capillary fall.
18. Explain vapour pressure .
19. Two horizontal plates are placed 1.25 cm apart. The space between them is being filled with oil of viscosity 14 poises. Calculate the shear stress in oil if upper plate is moved with a velocity of 2.5 m/s.
20. State Pascal's law.
21. What is mean by absolute and gauge pressure and vacuum pressure?
22. Define Manometer and list out its types.
23. Define centre of pressure and total pressure.
24. Define buoyancy and centre of buoyancy.
25. Define Meta centre.
26. Define Hydro static Pressure.
27. What is stable equilibrium of floating bodies?
28. What is stable equilibrium of submerged bodies?

PART – B

1. Calculate the capillary effect in a glass tube of 4.5 mm diameter, when immersed in (a) water (a) mercury. The temperature of the liquid is 20^o C and the values of the surface tension of water and mercury at 20^o C in contact with air are 0.073575 N/m and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 130^o. Take specific weight of water as

9800 N/m .

2. If the velocity profile of a liquid over a plate is a parabolic with the vertex 202 cm from the plate, where the velocity is 120 cm/sec. calculate the velocity gradients and shear stress at a distance of 0, 10 and 20 cm from the plate, if the viscosity of the fluid is 8.5 poise.

3. The dynamic viscosity of oil, used for lubrication between a shaft and sleeve is 6 poise. The shaft is of diameter 0.4 m and rotates at 190 rpm. Calculate the power lost in the bearing for a sleeve length of 90mm. the thickness of the oil film is 1.5 mm.

4. If the velocity distribution over a plate is given by $u=2/3\ y - y^2$ in which u is the velocity in m/s at a distance y meter above the plate, determine the shear stress at y = 0 and y = 0.15 m.

5. The velocity distribution of flow is given by $u = ly^2 + my + c$ with vertex 30 cm from the plate, where velocity is 1.8 m/s. If $\mu = 0.9\ Ns/m^2$, find the velocity gradients and shear stresses at y = 0, 15 and 30 cm from the plate.

6. Derive Pascal's law.

7. Derive expression for capillary rise and fall.

8. Two large plane surfaces are 2.4 cm apart. The space between the gap is filled with glycerin. What force is required to drag a thin plate of size 0.5 m between two large plane surfaces at a speed of 0.6 m/sec. if the thin plate is (i) in the middle gap (ii) thin plate is 0.8 cm from one of the plane surfaces? Take dynamic viscosity of fluid is 8.1 poise.

9. Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water

   (b) mercury. Take surface tension = 0.0725 N/m for water and = 0.52 N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact of mercury with glass = 130 degree.

10. A U - Tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains water and mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U tube is 10 cm and the free surface of mercury is in level with over the centre of the pipe. If the pressure of water in pipe line is reduced to $9810\ N/m^2$, Calculate the new difference in the level of mercury. Sketch the arrangement in both cases.

11. Calculate the total hydrostatic force and location of centre of pressure for a circular plate of 2.5 m diameter when immersed vertically in an oil of specific gravity 0.8 with its top edge 1.5 m below the oil.

13. A rectangular plate 2.5m x 3.5 m is submerged in water and makes an angle of 60° with the horizontal, the 2.5m sides being horizontal. Calculate the total force on the plate and the location of the point of application of the force, when the top edge of the plate is 1.6m below the water surface.

14. A rectangular plate 1.5 m x 3 m is immersed in an oil of specific gravity 0.82 such that its upper and lower edge is at depths 1.5 m and 3 m respectively. Determine the total pressure acting on the plate and its location.
15. In an open container water is filled to a height of 2.5m and above that an oil of Specific gravity 0.85 is filled for a depth of 1.4 m. Find the intensity of pressure at the interface of two liquids and at the bottom of the tank.
16. The pressure Intensity at a point is 40kPa. Find corresponding pressure head in (a) water (b) Mercury (c) oil of specific gravity 0.9.

UNIT – 2 Equations of Motion

Fluid flow is described by two methods: Lagrangian method & Eulerian method In the Lagrangian method a single particle is followed over the flow field with the co ordinate system following the particle. The flow description is particle based and not space based. A moving coordinate system has to be used. In the Eulerian method, the description of flow is based on fixed coordinate system and the description of the velocity is with reference to location and time. Hence, Eulerian approach is easily adoptable to describe fluid motion mathematically.

Control Volume:
A fixed volume in space whose size and shape is entirely arbitrary, through which a fluid is continuously flowing is known as *control volume.* The boundary of a control volume is termed as the *control surface*. The size and shape is arbitrary and normally chosen such that it encloses part of the flow of particular interest.

Types of Fluid Flow:
1)Steady flow: The flow in which the fluid characteristics like velocity, pressure, density etc. at a point do not change with time is defined as steady flow.

Mathematically, for steady flow,

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0, \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

Unsteady Flow: The flow, in which the velocity, pressure and density at a point changes with respect to time is defined as unsteady flow.
Mathematically, for unsteady flow

$$\left(\frac{\partial V}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0 \text{ etc.}$$

2. Uniform flows: The flow in which the velocity at any given time does not change with respect to distance is defined as Uniform flow.
Mathematically, for uniform flow

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} = 0$$

Non-uniform flow: The flow in which the velocity at any given time changes with respect to distance is defined as non uniform flow.
Mathematically, for non-uniform flow,

$$\left(\frac{\partial V}{\partial s}\right)_{t = \text{constant}} \neq 0.$$

3. Laminar flow: The flow in which the fluid particles move along well-defined paths which are straight and parallel is defined as laminar flow. Thus the particles move in layers and do not cross each other.
Turbulent flow: The flow in which the fluid particles do not move in a zig-zag way and the adjacent layers cross each other is defined as turbulent flow.

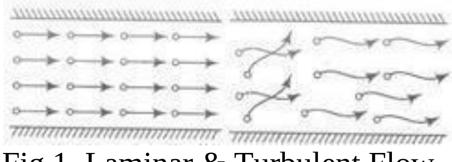


Fig.1. Laminar & Turbulent Flow

4. Compressible flow: The flow in which the density of fluid changes from point to point ie., ρ is not constant for the fluid, is defined as compressible flow.
Mathematically, for compressible flow, $\rho \neq$ constant.

Incompressible flow: The flow in which the density of fluid is constant is defined as incompressible flow.
Liquids are generally incompressible while gases are compressible.

Mathematically, for incompressible flow, ρ = Constant

5.*Rotational flow*: The flow in which the fluid particles while flowing along stream-lines, also rotate about their own axes, that type of flow is known as rotational flow.

Irrotational flow: The flow in which the fluid particles while flowing along stream-lines, do not rotate about their own axes, that type of flow is called irrotational flow.

6. One Dimensional Flow:

One dimensional flow is that type of flow in which the fluid velocity is a function of one- space-co-ordinate only. The variation of velocities in other two mutually perpendicular directions is assumed negligible.

Mathematically, for one-dimensional flow

$$u = f(x),\ v = 0 \quad \text{and } w = 0$$

where u, v and w are velocity components in x, y and z directions respectively.

Two-dimensional flow:

It is that type of flow in which the velocity is a function of two space co-ordinates only. Thus, mathematically for two dimensional flow

$$u = f_1(x, y), \quad v = f_2(x, y) \text{ and } w = 0$$

Three-dimensional flow:

It is the type of flow in which the velocity is a function of three space co-ordinates (x, y and z). Mathematically for three dimensional flow,

$$u = f_1(x, y, z), \quad v = f_2(x, y, z), \quad w = f_3(x, y, z).$$

Path Line:

A path line is the trajectory of an individual element of fluid.

Streamline:

A streamline is an imaginary continuous line within a moving fluid such that the tangent at each point is the direction of the flow velocity vector at that point.

Stream Tube:

An imaginary tube (need not be circular) formed by collection of neighboring streamlines through which the fluid flows is known as stream tube.

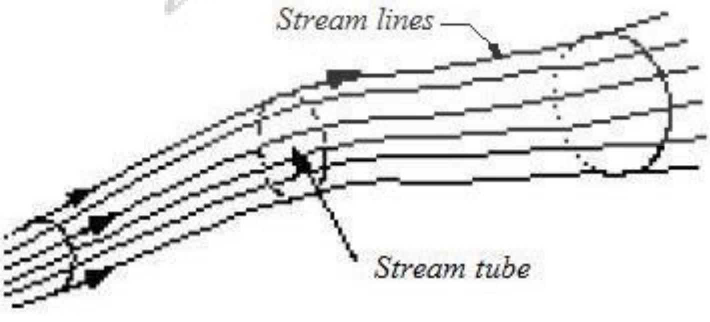


Fig.2.Conservation of mass: Integral Form

Let us consider a control volume V bounded by the control surface S. The efflux of mass across the control surface S is given by

$$\iint_S \rho \vec{V} . d\vec{A}$$

where $\vec{V}$ is the velocity vector at an elemental area(which is treated as a vector by considering its positive direction along the normal drawn outward from the surface).

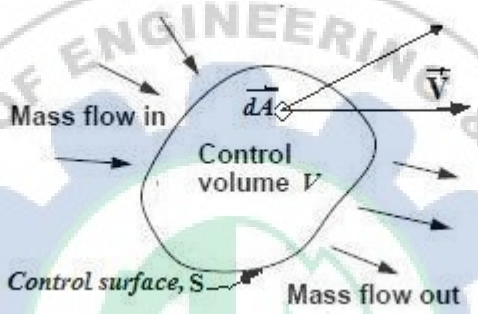


Fig.3.A Control Volume for integral form of derivation

The rate of mass accumulation within the control volume becomes

$$\frac{\partial}{\partial t} \iiint_V \rho . dV$$

where *dV* is an elemental volume, ρ is the density and *V* is the total volume bounded by the control surface S. Hence, the continuity equation becomes

$$\frac{\partial}{\partial t} \iiint_V \rho . dV + \iint_S \rho \vec{V} . d\vec{A} = 0$$ ....(1)

The second term of the Equation can be converted into a volume integral by the use of the Gauss divergence theorem as

$$\iint_S \rho \vec{V} \, d\vec{A} = \iiint_V \nabla . (\rho \vec{V}) dV$$

Since the volume *V* does not change with time, the sequence of differentiation and integration in the first term of Equation (1) can be interchanged and it can be written as

$$\iiint_V \left[\frac{\partial \rho}{\partial t} + \nabla . (\rho \vec{V}) \right] dV = 0$$ ---------------------(2)

Equation (2) is valid for any arbitrary control volume irrespective of its shape and size. So we can write

$$\frac{\partial \rho}{\partial t} + \nabla . (\rho \vec{V}) = 0$$

Conservation of Energy:

The law of conservation of energy says "energy cannot be created or be destroyed; One form of energy can be changed into another form only".

Consider the Control Volume shown in Figure as a thermodynamic system. Let amount of heat ***δq*** be added to the system from the surrounding. Also let ***δw*** be the work done on the system by the surroundings. Both heat and work are the forms of energy. Addition of any form of the energy to the system, changes the amount of internal energy of the system. Lets denote this change of internal energy by ***de***. As per the principle of energy conservation,

$$\delta q + \delta w = de$$

Therefore in terms of rate of change, the above equation changes to

$$\frac{\delta q}{\delta t} + \frac{\delta w}{\delta t} = \frac{de}{dt} \text{ ----------------- (1)}$$

For an open system there will be a change in all the forms of energies possessed by the system, like internal energy and kinetic energy. The right hand side of the equation (1) is representing change in the content of energy of the system.

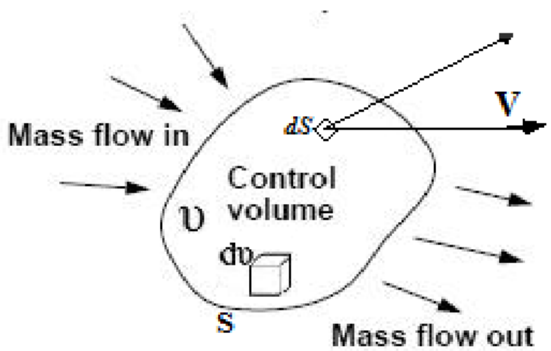
If q is the amount of heat added per unit mass, then the rate of heat addition for any elemental volume will be $q(\rho dv)$. The total external volumetric heat addition on the entire control volume and heat got added by viscous effects like conduction can be,

$$\frac{\delta q}{\delta t} = \iiint_V q\,\rho dv + Q_{viscous} \text{ ------------------(2)}$$

The main source of work transfer is due to the surface forces like pressure, body force etc. Consider an elemental area **ds** of the control surface. The pressure force on this elemental area is -P**ds** and the rate of work done on the fluid passing through **ds** with velocity **V** is *(-P**ds**).V*. Integrating over the complete control surface, rate of work done due to pressure force is,

$$-\oiint_S (P ds).V \text{ ----------------(3)}$$

In addition, consider an elemental volume **dυ** inside the control volume, as shown in Figure.



volume due to body force is $(\rho F_b du).V$. Here F_b is the body force per unit mass. Summing over the complete control volume, we obtain, rate of work done on fluid inside **υ** due to body forces is

$$\iiint_V (\rho F_b dv).V \text{ ----------------- (4)}$$

The rate of work done on the elemental

If the flow is viscous, the shear stress on the control surface will also do work on the fluid as it passes across the surface. Let *Wviscous* denote the work done due to the shear stress. Therefore, the total work done on the fluid inside the control volume is the sum of terms given by (3) and (4) and *Wviscous*, that is

$$\frac{\delta w}{\delta t} = -\oiint_s PV.ds + \iiint_v \rho(F_b.V)dv + \dot{W}_{viscous} \quad \text{------------(5)}$$

For the open system considered, the changes in internal energy as well as kinetic energy need to be accounted. Therefore, right hand side of equation (1) should deal with total energy (sum of internal and kinetic energies) of the system. Let, ***e*** be the internal energy per unit mass of the system and kinetic energy per unit mass due to local velocity **V** be $\boldsymbol{V^2/2}$.

Total energy in the control volume might also change due to influx and outflux of the fluid. The elemental mass flow across **ds** is (ρ**V**.**ds**). Therefore the elemental flow of total energy across the ds is (ρ**V**.**ds**)(e+V^2/2).

Hence the net energy change of the control volume is,

$$\frac{de}{dt} = \frac{\partial}{\partial t}\iiint_v \rho\left(e + \frac{V^2}{2}\right)dv + \oiint_s (\rho V.ds)\left(e + \frac{V^2}{2}\right)) \quad \text{------------------(6)}$$

Thus, substituting Equations (2), (5) and (6) in equation (1), we have

This is the energy equation in the integral form. It is essentially the first law thermodynamics applied to fluid flow or open system.

One dimensional form of Conservation of Energy

Consider the control volume shown in Figure for steady inviscid flow without body force, Then the equation (9) reduces to,

$$\iiint_v q\rho dv - \oiint_s PV.ds = \oiint_s \rho\left(e + \frac{V^2}{2}\right)V.ds$$

Let us denote the first term on left hand side of above equation by $\dot{Q}$ to represent the total external heat addition in the system. Thus, above equation becomes

Evaluating the surface integrals over the control volume in Figure, we obtain

$$\dot{Q} - (-P_1u_1A + P_2u_2A) = -\rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1A + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2A$$

or

$$\frac{\dot{Q}}{A} + P_1 u_1 + \rho_1\left(e_1 + \frac{u_1^2}{2}\right)u_1 = P_2 u_2 + \rho_2\left(e_2 + \frac{u_2^2}{2}\right)u_2$$

or

$$\frac{\dot{Q}}{\rho_1 u_1 A} + \frac{P_1}{\rho_1} + e_1 + \frac{u_1^2}{2} = \frac{P_2}{\rho_2} + e_2 + \frac{u_2^2}{2}$$

or

$$\frac{\dot{Q}}{\rho_1 u_1 A} + P_1 \upsilon_1 + e_1 + \frac{u_1^2}{2} = P_2 \upsilon_2 + e_2 + \frac{u_2^2}{2}$$

Here, $\dot{Q}/\rho_1 u_1 A$ is the external heat added per unit mass, *q*. Also, we know e + Pu = h. Hence, above equation can be re-written as,

$$h_1 + \frac{u_1^2}{2} + q = h_2 + \frac{u_2^2}{2}$$

This is the energy equation for steady one-dimensional flow for inviscid flow

Conservation of Momentum (Integral Form)

Momentum is defined as the product of mass and velocity, and represents the energy of motion stored in the system. It is a vector quantity and can only be defined by specifying its direction as well as magnitude.

The conservation of momentum is defined by Newton’s second law of motion.

Newton's Second Law of Motion

"The rate of change of momentum is proportional to the net force acting, and takes place in the direction of that force".

This can be expressed as

$$\frac{d}{dt}(mV) = F \qquad \text{...(1)}$$

Consider the same Control Volume shown in Figure for deriving the momentum conservation equation. Right hand side of equation (1) is the summation of all forces like surface forces and body forces. Let $\mathbf{F_b}$ and **P** be the net body force per unit mass and pressure exerted on control surface respectively. The body force on the elemental volume **dυ** is therefore ***ρ F*** $\rho F_b\, dv$ and the total body force exerted on the fluid in the control volume is

$$\iiint_v \rho F_b\, d\upsilon \qquad \text{...(2)}$$

The surface force due to pressure acting on the element of area **ds** is –Pds, where the negative sign indicates that the force is in the opposite direction of **ds**.

The total pressure force over the entire control surface

is expressed as $-\oiint_s P\, ds$ ....(3)

Let $\mathbf{F_{viscous}}$ be the total viscous force exerted on the control surface. Hence, the resultant

force
experienced by the fluid is given by

$$F = -\oiint_s P ds + \iiint_v \rho F_b d\upsilon + F_{viscous} \quad \text{.......(4)}$$

The left hand side term of the Equation (1) gives the time rate of change of momentum following a fixed fluid element or substantial derivative of the momentum. It can be evaluated using equation (2) by evaluating the sum of net flow of momentum leaving the control volume through the control surface **S** and time rate of change of momentum due to fluctuations of flow properties inside the control volume.

The mass flow across the elemental area **ds** is *(ρV.ds)*.Therefore, the flow of momentum per second across **ds** is *(ρV.ds)V*

The net flow of momentum out of the control volume through **s** is ,

$$\oiint_s (\rho V.ds)V \quad \text{.......(5)}$$

The momentum of the fluid in the elemental volume **dυ** is *(ρ du)V*. The momentum contained at any
instant inside the control volume is

$$\iiint_v \rho V d\upsilon$$

and its time rate of change due to unsteady flow fluctuation is

$$\frac{\partial}{\partial t}\iiint_v \rho V d\upsilon \quad \text{.......(6)}$$

Combining Equations (5) and (6) to obtain the left hand side of equation (1), we get

$$\frac{d}{dt}(mV) = \frac{\partial}{\partial t}\iiint_v \rho V d\upsilon + \oiint_s (\rho V.ds)V \quad \text{.......(7)}$$

Thus, substituting Equations (4) and (7) into (1), we have

$$\frac{\partial}{\partial t}\iiint_v \rho V d\upsilon + \oiint_s (\rho V.ds)V = -\oiint_s P ds + \iiint_v \rho F_b d\upsilon + F_{viscous} \quad \text{.......(8)}$$

This is the momentum equation in integral form.
It is a general equation, applies to the unsteady, three-dimensional flow of any fluid, compressible or incompressible, viscous or non viscous.

One dimensional form of Momentum conservation equation

For the steady and non viscous flow with no body forces, the Equation (8) reduces to

$$\oiint_s (\rho V.ds)V = -\oiint_s P ds$$

Above equation is a vector equation. However, since we are dealing with the one-dimensional flow, we need to consider only the scalar x component of equation.

$$\oiint_s (\rho V.ds)u = -\oiint_s (P ds)_x$$

Considering the control volume shown in Figure, above equation

transforms to, $\rho_1(-u_1A)u_1 + \rho_2(-u_2A)u_2 = -(-P_1A + P_2A)$
or

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 \quad \text{-------------------------------------} \quad (9)$$

This is the momentum equation for steady,non viscous one-dimensional

One Dimensional flow – forces of fluid in a curved pipe

In the case where fluid flows in a curved pipe as shown in figure, let ABCD be the control volume, A_1, A_2 the areas, v_1, v_2 the velocities, and p_1,p_2 the pressures of sections AB and CD respectively. Let F be the force of fluid acting on the pipe; the force of the pipe acting on the fluid is -F. This force and the pressures acting on sections AB and CD act on the fluid, increasing the fluid momentum by such a combined force (Increase in momentum = momentum going out - momentum coming in).

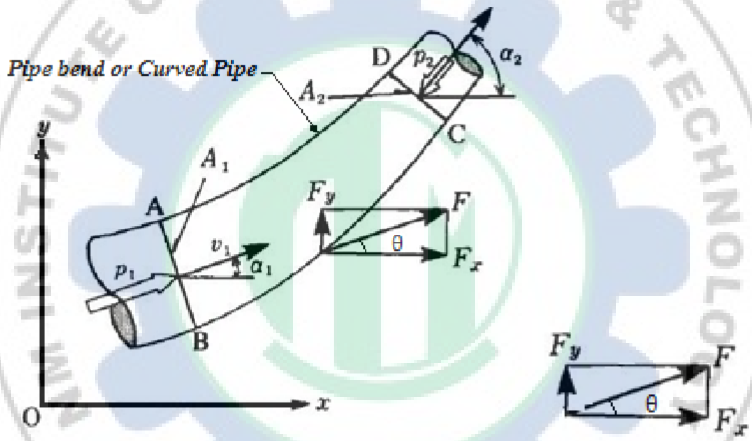


Fig.4. One dimensional flow

If F_x and F_y are the component forces in the x and y directions of F respectively, then from the equation of momentum,

$F_x + A_1p_1 \cos \alpha_1 - A_2p_2 \cos \alpha_2 = m (v_2 \cos$

$\alpha_2 - v_1 \cos \alpha_1)$ $F_y + A_2p_2 \sin\alpha_2 - A_2p_2 \sin\alpha_2$

$= m(v_2 \sin\alpha_2 – v_1 \sin\alpha_1)$

From the above equations , F_x and F_y are given by
In these equations, m is the mass flow rate. If Q is the volumetric flow rate, then the following relation exists:
$m = \rho A_1 v_1 = \rho A_2 v_2 = \rho Q$

If the curved pipe is a pipe bend in a horizontal plane, then $\alpha_1 = 0$. Therefore

Rate of Flow (or) Discharge Q

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible flow of liquid, the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section. Thus

(i) For liquids the units of Q are ***m*** ^{3} ***/s*** or ***litres/s***
(ii)For gases the units of Q are kgf/s or Newton/s Consider a fluid flowing through a pipe in which A= Cross-sectional area of pipe.

V= Average area of fluid across the section Then, discharge $Q = A \times V$ m^3/s

Continuity Equation:

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant. Consider two cross-sections of a pipe as shown in figure

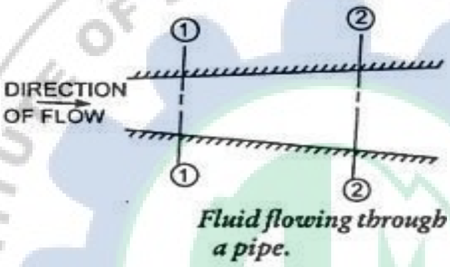


Fig.5. Continuity Equation

Let V_1=Average velocity at cross-section at 1-1
ρ_1 =Density at section 1-1 A_1=Area of pipe at section 1-1 and
V_2, ρ_2, A_2 are corresponding values at section 2-2
Then rate of flow at section 1-1 = $\rho_1 A_1 V_1$ Rate of flow at section 2-2 = $\rho_2 A_2 V_2$
According to law of conservation of mass,

Rate of flow at section 1-1 = Rate of flow at section 2-2

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \quad \text{..............................(1)}$$

The above equation applicable to the compressible as well as incompressible fluids is called Continuity Equation. If the fluid is incompressible, then $\rho_1 = \rho_2$ and continuity equation (1) reduces to

$$A_1 V_1 = A_2 V_2$$

Energy Equations: This is equation of motion in which the forces due to gravity and pressure are taken into consideration. The common fluid mechanics equations used in fluid dynamics are given below
Let, Gravity force F_g, Pressure force F_p, Viscous force F_v , Compressibility force F_c , and Turbulent force F_t.

$$F_{net} = F_g + F_p + F_v + F_c + F_t$$

1. If fluid is incompressible, then $F_c = 0$
   This is known as Reynolds equation of motion.
2. If fluid is incompressible and turbulence is negligible, then
   This equation is called as Navier-Stokes equation.
3. If fluid flow is considered ideal then, viscous effect will also be negligible. Then

$$F_{net} = F_g + F_p$$

This equation is known as Euler's equation.

Euler`s Equation:

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line. Consider a stream-line in which flow is taking place in S-direction as shown in figure. Consider a cylindrical element of cross-section dA and length dS. The forces acting on the cylindrical element are:

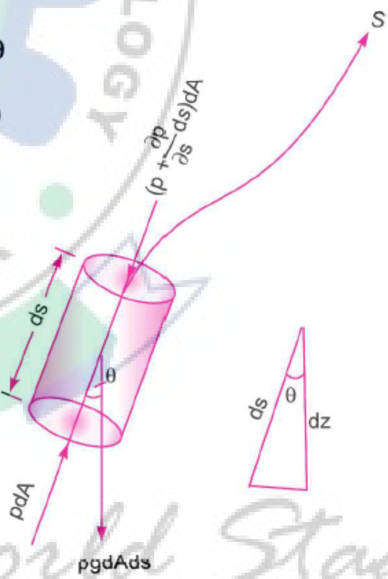
1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta = \rho dA ds \times a_s \quad \text{...(6.2)}$$

where a_s is the acceleration in the direction of s.



Now $\quad a_s = \frac{dv}{dt}$, where v is a function of s and t.

$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{v\partial v}{\partial s}$$

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho\partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (*b*), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$

or $$\frac{dp}{\rho} + gdz + vdv = 0$$

is known as Euler's equation of motion.

Bernoulli`s Equation: is obtained by integrating the above Euler's equation of motion. If the flow is incompressible, ρ is a constant and

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

$$\text{or} \qquad \frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

Assumptions made in deriving Bernoulli`s Equation:

The following are the assumptions made in the derivation of Bernoulli's equation:

(i) The fluid is ideal,
(ii) The flow is steady
(iii) The flow is frictionless
(iv) The flow is incompressible
(v) The flow is irrotational

Statement of Bernoulli`s Theorem:

In a steady, frictionless, incompressible and irrotational flow of an ideal fluid, the total energy at any point of the fluid is constant".
The total energy consists of pressure energy, kinetic energy and potential energy or datum energy.
Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant}$$

Application of Bernoulli`s Equation:

1. Venturimeter 2, Orificemeter 3. Pitot Tube

Flow Measurement Devices:

Venturimeter and Orifice meter are the devices used for measurement of flow rate or actual discharge through pipes.
Pitot tube is used to measure the velocity of flow in open canals, pipes as well as measurement of speed of ships, Aircrafts.

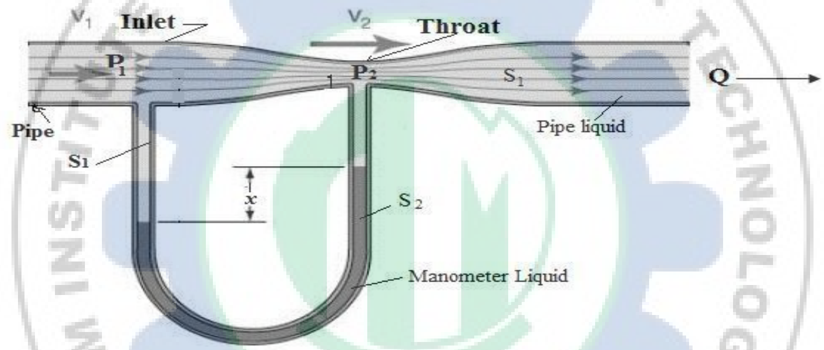
Venturimeter:

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It is based on the principle of Bernoulli's equation. The Venturimeter has a converging conical section from the initial pipe diameter, followed by a throat, ended with a diverging conical section back to the original pipe diameter.
As the inlet area of the venturimeter is larger than the throat area, the velocity at the throat increases resulting in decrease of pressure. By this, a pressure difference is created between the inlet and the throat of the venturimeter. The pressure difference is measured by using a differential U-tube manometer. This pressure difference helps in the determination of rate of flow of fluid or discharge through the pipe line.

Let D1 and D2 – Diameter at inlet and throat

P1 and P2 – Pressure at inlet and throat

V1 and V2 – Velocity at inlet and throat



$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g}+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+\frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1-p_2}{\rho g}=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

But $\frac{p_1-p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1-p_2}{\rho g}=h$

Substituting this value of $\frac{p_1-p_2}{\rho g}$ in the above equation, we get

$$h=\frac{v_2^2}{2g}-\frac{v_1^2}{2g}$$

Now applying continuity equation at sections 1 and 2

$$a_1v_1=a_2v_2 \quad \text{or} \quad v_1=\frac{a_2v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h=\frac{v_2^2}{2g}-\frac{\left(\frac{a_2v_2}{a_1}\right)^2}{2g}=\frac{v_2^2}{2g}\left[1-\frac{a_2^2}{a_1^2}\right]=\frac{v_2^2}{2g}\left[\frac{a_1^2-a_2^2}{a_1^2}\right]$$

or $$v_2^2=2gh\,\frac{a_1^2}{a_1^2-a_2^2}$$

$\therefore \qquad v_2 = \sqrt{2gh\frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gh}$

$\therefore$ Discharge, $\qquad Q = a_2 v_2$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Equation gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$$\therefore \qquad Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid

S_o = Sp. gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Then $\qquad h = x\left[\frac{S_h}{S_o} - 1\right]$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where S_l = Sp. gr. of lighter liquid in U-tube

S_o = Sp. gr. of fluid flowing through pipe

x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right]$$

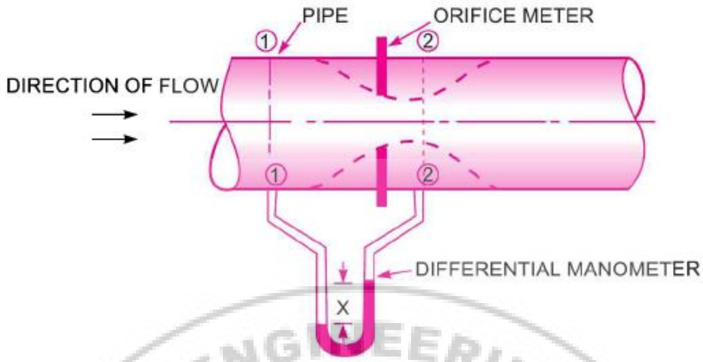
Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right]$$

Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and



p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$\therefore$
$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = {v_2}^2 - {v_1}^2$$

or
$$v_2 = \sqrt{2gh + v_1^2}$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$\therefore$
$$a_2 = a_0 \times C_c$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0 \, C_c}{a_1} v_2$$

Substituting the value of v_1 in equation (i), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or
$$ {v_2}^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 {C_c}^2 {v_2}^2 \text{ or } {v_2}^2 \left[1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$\therefore$
$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0 \, C_c$ $\qquad \because a_2 = a_0 C_c$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1 - \left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$\therefore \qquad C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (iv), we get

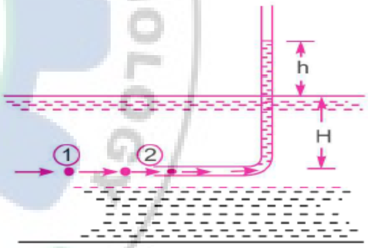
$$Q = a_0 \times C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}.$$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

Pitot-tube. It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig.



The lower end, which is bent through 90° is directed in the up-stream direction as shown in Fig. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just as the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equation at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \qquad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

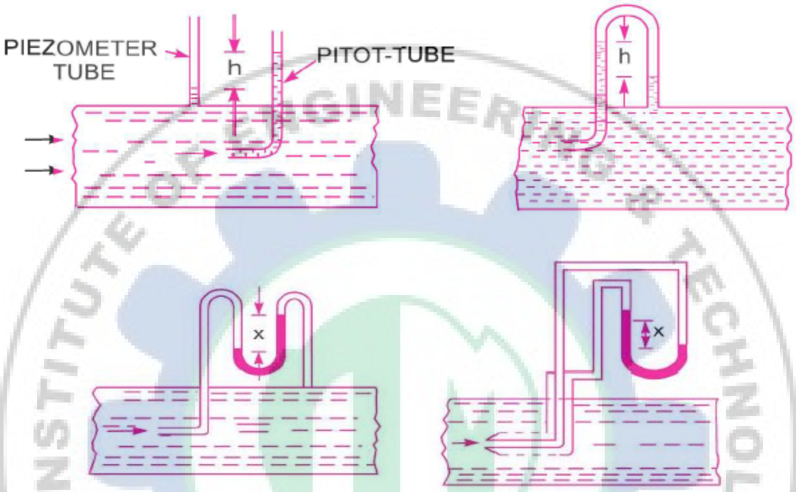
$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

$\therefore$ Velocity at any point $\quad v = C_v \sqrt{2gh}$

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig.
2. Pitot-tube connected with piezometer tube as shown in Fig.
3. Pitot-tube and vertical piezometer tube connected with a differential U-tube manometer as shown in Fig.



4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. . The outlet of these two tubes are connected to the differential manometer where the difference of pressure head 'h' is measured by knowing the difference of the levels of the manometer liquid say x. Then $h = x\left[\frac{S_g}{S_o} - 1\right]$.

Solved Problems:

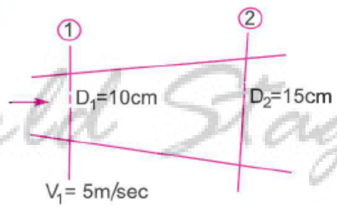
1. The diameter of a pipe at the section 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section1 is 5m/s. Determine the velocity at section 2.

Solution. Given :

At section 1, $\quad D_1 = 10 \text{ cm} = 0.1 \text{ m}$

$$A_1 = \frac{\pi}{4}(D_1^2) = \frac{\pi}{4}(.1)^2 = 0.007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$



At section 2, $\quad D_2 = 15 \text{ cm} = 0.15 \text{ m}$

$$A_2 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$$

(i) Discharge through pipe is given by equation

or $\quad Q = A_1 \times V_1$

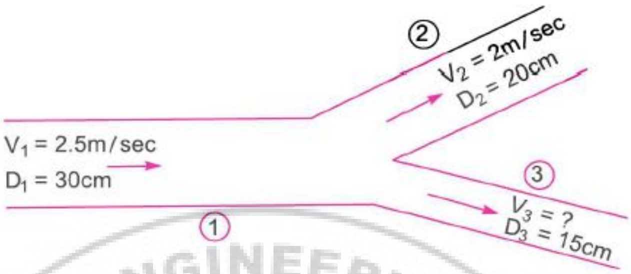
$$= 0.007854 \times 5 = \mathbf{0.03927 \text{ m}^3\text{/s. Ans.}}$$

Using equation , we have $A_1V_1 = A_2V_2$

(ii) $\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{0.007854}{0.01767} \times 5.0 = \mathbf{2.22 \text{ m/s. Ans.}}$

2. A 30cm diameter pipe conveying water branches in to two pipes of diameters 20cm and 15cm respectively. If the average velocity in the 30cm diameter pipe is 2.5m/s. find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s.

Solution. Given :



$D_1 = 30 \text{ cm} = 0.30 \text{ m}$

$\therefore \quad A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .3^2 = 0.07068 \text{ m}^2$

$V_1 = 2.5 \text{ m/s}$

$D_2 = 20 \text{ cm} = 0.20 \text{ m}$

$\therefore \quad A_2 = \frac{\pi}{4} (.2)^2 = \frac{\pi}{4} \times .4 = 0.0314 \text{ m}^2,$

$V_2 = 2 \text{ m/s}$

$D_3 = 15 \text{ cm} = 0.15 \text{ m}$

$\therefore \quad A_3 = \frac{\pi}{4} (.15)^2 = \frac{\pi}{4} \times 0.225 = 0.01767 \text{ m}^2$

Find (*i*) Discharge in pipe 1 or Q_1

(*ii*) Velocity in pipe of dia. 15 cm or V_3

Let Q_1, Q_2 and Q_3 are discharges in pipe 1, 2 and 3 respectively.

Then according to continuity equation

$$Q_1 = Q_2 + Q_3$$

(*i*) **The discharge Q_1** in pipe 1 is given by

$$Q_1 = A_1 V_1 = 0.07068 \times 2.5 \text{ m}^3/\text{s} = \mathbf{0.1767 \text{ m}^3/\text{s}}. \text{ Ans.}$$

(*ii*) **Value of V_3**

$$Q_2 = A_2 V_2 = 0.0314 \times 2.0 = 0.0628 \text{ m}^3/\text{s}$$

Substituting the values of Q_1 and Q_2 in equation (1)

$$0.1767 = 0.0628 + Q_3$$

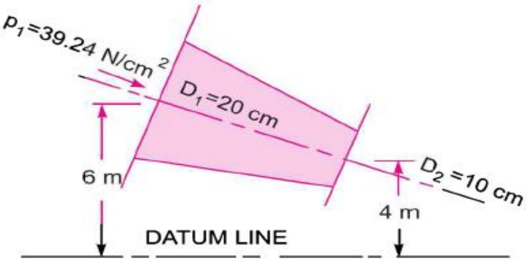
$\therefore \quad Q_3 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3/\text{s}$

But $\quad Q_3 = A_3 \times V_3 = 0.01767 \times V_3 \quad \text{or} \quad 0.1139 = 0.01767 \times V_3$

$\therefore \quad V_3 = \frac{0.1139}{0.01767} = \mathbf{6.44 \text{ m/s}}. \text{ Ans.}$

3. The water is flowing through a pipe having diameters 20cm and 10cm at section 1 and 2 respectively. The rate of flow through pipe is 35 litres/sec. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24N/cm^2. Find the intensity of pressure at section 2.

Solution:



At section 1, $D_1 = 20 \text{ cm} = 0.2 \text{ m}$

$$A_1 = \frac{\pi}{4} (.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2, $D_2 = 0.10 \text{ m}$

$$A_2 = \frac{\pi}{4} (0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow, $Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$

Now $Q = A_1V_1 = A_2V_2$

$\therefore$ $$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and $$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or $$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or $$46.063 = \frac{p_2}{9810} + 5.012$$

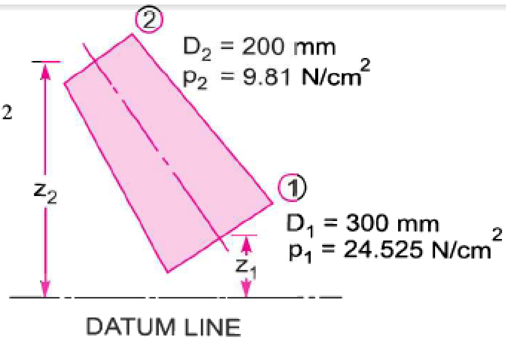
$\therefore$ $$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$ $$p_2 = 41.051 \times 9810 \text{ N/m}^2$$

$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}\text{. Ans.}$$

4.Water is flowing through a pipe having diameter 300mm and 200mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm^2 and the pressure at the upper end is 9.81 N/cm^2. Determine the difference in datum head if the rate of flow through pipe is 40 lit/sec.

Solution. Given :



Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

$$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

Now $A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$

$$\therefore \quad V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

$$25 + .32 + z_1 = 10 + 1.623 + z_2$$

$$25.32 + z_1 = 11.623 + z_2$$

$$\therefore \quad z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

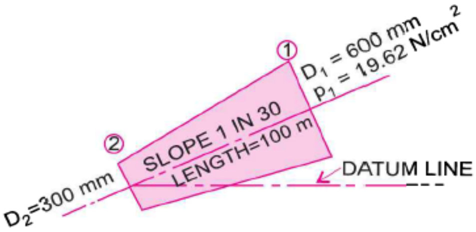
$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

5. The water is flowing through a taper pipe of length 100m having diameters 600mm at the upper end and 300mm at the lower end at the rate of 50lit/sec. The pipe has a slope of sec 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm^2.

Solution. Given :

Length of pipe, $L = 100$ m

Dia. at the upper end, $D_1 = 600$ mm $= 0.6$ m



$\therefore$ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$

$= 0.2827 \text{ m}^2$

p_1 = pressure at upper end

$= 19.62 \text{ N/cm}^2$

$= 19.62 \times 10^4 \text{ N/m}^2$

Dia. at lower end, $D_2 = 300$ mm $= 0.3$ m

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.3)^2 = 0.07068$ m

Q = rate of flow = 50 litres/s $= \frac{50}{1000} = 0.05 \text{ m}^3\text{/s}$

Let the datum line passes through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3}$ m

Also we know $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768$ m/sec $= 0.177$ m/s

and $V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074$ m/sec $= 0.707$ m/s

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

$$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

$$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

$p_2 = 23.3 \times 9810 \text{ N/m}^2 = 228573 \text{ N/m}^2 =$ **22.857 N/cm²**. **Ans.**

6.A pipe of diameter 400mm carries water at a velocity of 25m/s. The pressure at the points A and B are given as 29.43 N/cm^2 and 22.563 N/cm^2 respectively while the datum head at A and B are 28m and 30m. Find the loss of head between A and B.

Solution. Given :

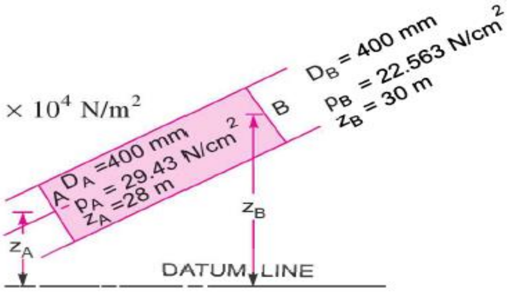
Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

Velocity, $V = 25 \text{ m/s}$

At point *A*, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$



$\therefore$ Total energy at *A*, $E_A = \dfrac{p_A}{\rho g} + \dfrac{v_A^2}{2g} + z_A$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

At point *B*, $p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$

$z_B = 30 \text{ m}$

$v_B = v = v_A = 25 \text{ m/s}$

$\therefore$ Total energy at *B*, $E_B = \dfrac{p_B}{\rho g} + \dfrac{v_B^2}{2g} + z_B$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

$\therefore$ Loss of energy $= E_A - E_B = 89.85 - 84.85 = \mathbf{5.0 \text{ m. Ans.}}$

7.A horizontal venturimeter with inlet and throat diameters 30cm and 15cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20cm of mercury. Determine the rate of flow take $C_d = 0.98$.

Solution. Given :

Dia. at inlet, $d_1 = 30 \text{ cm}$

$\therefore$ Area at inlet, $a_1 = \dfrac{\pi}{4} d_1^{\,2} = \dfrac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15 \text{ cm}$

$\therefore$ $a_2 = \dfrac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20$ cm of mercury.

$\therefore$ Difference of pressure head is given by

or $$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20 \left[\frac{13.6}{1} - 1 \right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn.

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s. Ans.}}$$

8.A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of oil of sp.gravity 0.8. The discharge of oil through venturimeter is 60 lit/sec. Find the reading of the oil – mercury differential manometer take $C_d = 0.98$.

Solution. Given : $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$

Using the equation $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$

or $\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$

$\therefore$ $h = (17.029)^2 = 289.98$ cm of oil

But $h = x\left[\frac{S_h}{S_o} - 1\right]$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$\therefore$ $289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$

$\therefore$ $x = \frac{289.98}{16} = 18.12$ cm.

$\therefore$ Reading of oil-mercury differential manometer = **18.12 cm. Ans.**

9.A horizontal venturimeter with inlet diameter 20cm and throat diameter 10cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm2 and the vacuum pressure at the throat is 30cm of mercury. Find the discharge of water through veturimeter take C_d = 0.98.

Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$

$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$

ρ for water $= 1000 \frac{\text{kg}}{\text{m}^3}$ and $\therefore \frac{p_1}{\rho g} = \frac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$\frac{p_2}{\rho g} = -30$ cm of mercury

$= -0.30$ m of mercury $= -0.30 \times 13.6 = -4.08$ m of water

$\therefore$ Differential head $= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$

$= 18 + 4.08 = 22.08$ m of water $= 2208$ cm of water

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3/\text{s} = \mathbf{165.555 \text{ lit/s. Ans.}}$$

10.An orifice meter with orifice diameter 10cm is inserted in a pipe of 20cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm2 and 9.81 N/cm2 respectively. Coefficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

∴ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. of pipe, $d_1 = 20$ cm

∴ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

$p_1 = 19.62 \text{ N/cm}^2 = 19.62 \times 10^4 \text{ N/m}^2$

∴ $\frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20$ m of water

Similarly $\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10$ m of water

∴ $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10$ m of water $= 1000$ cm of water

$C_d = 0.6$

The discharge, Q is given by equation

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3\text{/s} = \mathbf{68.21 \text{ litres/s. Ans.}}$$

11.An orifice meter with orifice diameter 15cm is inserted in a pipe of 30cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50cm of mercury. Find the rate of flow of sp.gravity 0.9 when the co-efficient of discharge of the orifice meter is 0.64.

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

∴ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Dia. of pipe, $d_1 = 30$ cm

∴ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Sp. gr. of oil, $S_o = 0.9$

Reading of diff. manometer, $x = 50$ **cm of mercury**

∴ **Differential head,** $h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right]$ **cm of oil**

$$= 50 \times 14.11 = 705.5 \text{ cm of oil}$$

$$C_d = 0.64$$

The rate of the flow, Q is given by equation

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3\text{/s} = \mathbf{137.414 \text{ litres/s. Ans.}}$$

12.A pitot tube placed in the centre of a 300mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifice is 60mm of water. Take the coefficient of pitot tube as $C_v = 0.98$.

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Diff. of pressure head, $h = 60 \text{ mm of water} = .06 \text{ m of water}$

$C_v = 0.98$

Mean velocity, $\overline{V} = 0.80 \times \text{Central velocity}$

Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore$ $\overline{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$

Discharge, $Q = \text{Area of pipe} \times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06 \text{ m}^3\text{/s. Ans.}}$$

Questions for practice

1. Define Control Volume and control surface continuity equation, Rate of Flow
2. List the types of fluid flow.
3. Define Steady and Unsteady flow.
4. Define Uniform and Non-uniform flow.
5. Compare Laminar and Turbulent flow.
6. What is the variation of viscosity with temperature for fluids?
7. Define Compressible and incompressible flow
8. Define Rotational and Irrotational flow.
9. Define One, Two and Three dimensional flow.
10. State the Bernoulli's equation and its applications.
11. State the assumptions used in deriving Bernoulli's equation.
12. State Momentum Equation.
13. What is the use of an orifice meter?
14. What is the use of a Venturimeter?
15. State the difference between Venturimeter and Orificemeter.
16. What is the use of Pitot tube?

Part B

1. Derive Euler's equation of motion along the stream line for an ideal fluid and thereby deduce Bernoulli's equation stating clearly the assumptions
2. What is venturimeter? Derive an expression for the discharge through a venturimeter.
3. Derive differential form of continuity equation.
4. Differentiate between Venturimeter and Orificemeter.

5.Water is flowing through a pipe having diameters 20 cm and 15 cm at sections 1 and 2 respectively. The rate of flow through pipe is 40 liters/sec. The section 1 is 6m above datum line and section 2 is 3m above the datum. If pressure at section 1is 29.43 N/cm2. Find the intensity pressure at section 2.

6.An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take C_v=0.98.

7.A 300mm diameter pipe carries water under a head of 20m with a velocity of 3.5 m/s. If the axis of the pipe turns through 45^0. Find the magnitude and direction of resultant force at the bend of the pipe turns through 45^0 , find the magnitude and direction of resultant force at the bend.

8.An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted on upstream and downstream of the orifice meter give readings of 19.62 N/cm^2 and 9.81 N/cm^2 respectively. Co-efficient of discharge for the meter is 0.6. Find the discharge of water through the pipe.

9.A horizontal venturimeter with inlet and throat diameter 300mm and 100mm respectively is used to measure the flow of water. The pressure intensity at inlet is 130KN/m while the vacuum pressure head at throat is 350mm of mercury. Assuming 3% head lost between inlet and the throat find the value of co-efficient of discharge for the venturimeter and also determine the rate of flow.

10.A pipe of 300 mm diameter inclined at 30° to the horizontal is carrying gasoline (specific gravity =0.82). A Venturimeter is fitted in the pipe to find out the flow rate whose throat diameter is 150 mm. The throat is 1.2 m from the entrance along its length. The pressure gauges fitted to the Venturimeter read 140 kN/m² and 80 kN/m² respectively. Find out the coefficient of discharge of Venturimeter if the flow is 0.20 m³/s.

11. Find the velocity of flow of an oil through a pipe when the difference of mercury level in a differential U tube manometer connected to the two tappings of pitot tube is 10cm.Take the co- efficient of of pitot tube as 0.98and Specific gravity of oil is 0.8.Find the discharge through the pipe if the diameter is 30 cm.

12. Water is flowing through a pipe having diameters 20 cm and 15 cm at sections 1 and 2 respectively. The rate of flow through pipe is 40 liters/sec. The section 1 is 6m above datum

line and section2 is 3m above the datum. If pressure at section 2 is 3m above the datum. If pressure at section 1 is 29.43 N/cm^2. Find the intensity of pressure at section 2.

13.Water flows upwards in a vertical pipe line of gradually varying section from point 1 to point 2, which is 1.5m above point 1, at the rate of 0.9 m^3/s. At section 1 the pipe dia is 0.5m and pressure is 300 kPa. If pressure at section 2 is 600 kPa, determine the pipe diameter at that location. Neglect losses.

14.Water flows up a conical pipe 450 mm diameter at the lower end and 250 mm diameter at 2.3 m above the lower end. If the pressure and velocity at the lower end are 63 kN/m^2 (gauge) and 4.1 m/s, assuming a head loss in the pipe to be 10% of the pressure head at the lower end, calculate the discharge through the pipe. Also calculate the pressure and velocity at the upper end.

15.Water is flowing through a tapering pipe of length 200 m having diameters 500 mm at the upper end and 250 mm at the lower end, the pipe has a slope of 1 in 40. The rate of flow through the pipe is 250 lit/ sec. the pressure at the lower end and the upper end are 20 N/cm^2 and 10 N/cm^2 respectively. Find the loss of head and direction of flow.

16.Water at 36 m above sea level has a velocity of 18 m/s and a pressure of 350 kN/m^2. Determine the potential, kinetic and pressure energy of the water in metres of head. Also determine the total head.

17.A liquid with specific gravity 0.8 flows at the rate of 3 *l/s* through a venturimeter of diameters 6 cm and 4 cm. If the manometer fluid is mercury, determine the value of manometer reading.

UNIT 3 Flow through Orifice, Notches and Weir and Pipes

Orifice

Orifice is a small opening on the side or at the bottom of a tank, through which a fluid is flowing. The orifices are classified according to the size, shape, nature of discharge and shape of the edge.

1. According to the size of orifice and head of liquid from the centre of the orifice: Small orifice and Large orifice.
   Small Orifice: If the head of liquid from the centre of orifice is more than five times the depth of orifice, the orifice is called small orifice.
   Large Orifice: If the head of liquid is less than five times the depth of orifice, it is known as large orifice.
2. According to shape of orifice: (i) Circular orifice, (ii) Triangular orifice,(iii) Rectangular orifice and (iv) Square orifice
3. According to their cross-sectional area or edge: (i) Sharp-edged orifice and (ii) Bell mouthed orifice

According to the discharge condition: (i) Free discharging orifices (ii) Fully drowned or submerged orifices and (iii) Partially submerged orifices.

Flow through a Small Orifice

Flow from a tank through a hole in the side.

Fig.1. Flow through a small Orifice

The edges of the hole are sharp to minimize frictional losses by minimizing the contact between the hole and the liquid. The streamlines at the orifice contract reducing the area of flow. This contraction is called the vena contracta.

The amount of contraction must be known to calculate the flow.

Applying Bernoulli's equation along the streamline joining point 1 on the surface to point 2 at the centre of the orifice.

At the surface velocity is negligible ($v_1 = 0$) and the pressure atmospheric ($p_1 = 0$). At the orifice the jet is open to the atmosphere so again the pressure is atmospheric ($p_2 = 0$).

If we take the datum line through the orifice then Z1 = H and Z2 = 0 leaving h = 2Z2 = h=√2

This theoretical value of velocity is an overestimate as friction losses have not been taken into account. A coefficient of velocity is used to correct the theoretical velocity,

Each orifice has its own coefficient of velocity, they usually lie in the range 0.97 - 0.99

The discharge through the orifice = jet area X jet velocity

The area of the jet is the area of the vena contracta and not the area of the orifice. We use a Coefficient of contraction to get the area of the

jet, A_a.
A_a = Cc x area of orifice

Discharge through the Orifice Q = Area x Velocity

Actual Discharge Qa = Cd x Qth

Qth = Area of Orifice x Vth

Hydraulic Coefficient

The following three coefficients are known as hydraulic coefficients or orifice coefficient

Coefficient of Contraction

Coefficient of Velocity

Coefficient of Discharge

Coefficient of Contraction:

The ratio of the area of the jet, at vena-contracta, to the area of the orifice is known as *coefficient of contraction*. Mathematically coefficient of contraction,
The value of Coefficient of contraction varies slightly with the available head of the

$$C_c = \frac{Area\ of\ jet\ at\ vena\ contracta}{Area\ of\ the\ orifice}$$

liquid, size and shape of the orifice. The average value of C_c is 0.64.

Coefficient of Velocity:

The ratio of actual velocity of the jet, at vena-contracta, to the theoretical velocity is known as *coefficient of velocity*.

The theoretical velocity of jet at vena-contracta is given by the relation, $h = \sqrt{2}$, where *H* is the head of water at vena-contracta. Mathematically coefficient of velocity.

$$C_v = \frac{Actual\ velocity\ of\ the\ jet\ at\ vena\ contracta}{Theoretical\ velocity\ of\ the\ jet}$$

The difference between the velocities is due to friction of the orifice. The value of Coefficient of velocity varies slightly with the different shapes of the edges of the orifice. This value is very small for sharp-edged orifices. For a sharp edged orifice, the value of increases with the head of water.

Coefficient of Discharge:

The ratio of a actual discharge through an orifice to the theoretical discharge is known as *coefficient of discharge*. Mathematically coefficient of discharge,

$$C_d = \frac{Actual\ discharge}{Theoretical\ discharge}$$

$$= \frac{Actual\ velocity \times Actual\ area}{Theoretical\ velocity \times Theoretical\ area}$$
$$= C_v \times C_c$$

Thus the value of coefficient of discharge varies with the values of and . An average of coefficient of discharge varies from 0.60 to 0.64.

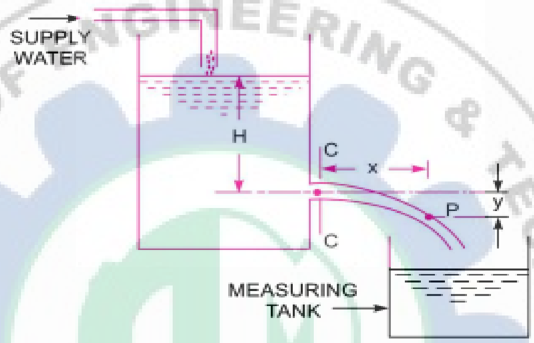
Determination of Coefficient of Discharge (C_d):

The water is allowed to flow through an orifice provided in a tank under a constant head H. The water is collected in a collecting tank for a known height. The time of collection of water in the collecting tank is noted down.

Then

$$Q = \frac{\text{Area of measuring tank} \times \text{Height of water in measuring tank}}{\text{Time } (t)}$$

and theoretical discharge = area of orifice $\times \sqrt{2gH}$



$$\therefore \quad C_d = \frac{Q}{a \times \sqrt{2gH}}$$

Determination of Coefficient of Velocity (Cv): Let C-C represents the vena – contracta of a jet water coming out from an orifice under constant head H as shown in fig. Consider a liquid particle which is at vena contracta at any time and takes the position at P along the jet time t.

Let x = horizontal distance travelled by the particle in time 't'
y = vertical distance between P and C-C
V = actual velocity of jet at vena-contracta.

Then horizontal distance, $x = V \times t$

and vertical distance, $y = \frac{1}{2} gt^2$

From equation (i), $t = \frac{x}{V}$

Substituting this value of 't' in (ii), we get

$$y = \frac{1}{2} g \times \frac{x^2}{V^2}$$

$$V^2 = \frac{gx^2}{2y}$$

$$\therefore \quad V = \sqrt{\frac{gx^2}{2y}}$$

But theoretical velocity,

$$V_{th} = \sqrt{2gH}$$

$$\therefore \text{ Co-efficient of velocity, } C_v = \frac{V}{V_{th}} = \sqrt{\frac{gx^2}{2y}} \times \frac{1}{\sqrt{2gH}} = \sqrt{\frac{x^2}{4yH}}$$

$$= \frac{x}{\sqrt{4yH}}.$$

Determination of Coefficient of Contraction (C_C):

The coefficient of contraction is determined from the equation $C_d = C_V \times C_C$

$C_C = C_d / Cv$

Flow through Large Orifices:

If the head of liquid is less than 5 times the depth of the orifice, the orifice is called large orifice. In case of small orifice, the velocity in the entire cross-section of the jet is considered to be constant and discharge can be calculated by $Q = C_d \times a \times \sqrt{2gh}$. But in case of a large orifice, the velocity is not constant over the entire cross-section of the jet and hence Q cannot be calculated by $Q = C_d \times a \times \sqrt{2gh}$.

Discharge through Large Rectangular Orifice:

Consider a large rectangular orifice in one side of the tank discharging freely in to atmosphere under a constant head H as shown in fig.

Let H_1 = height of liquid above top edge of orifice

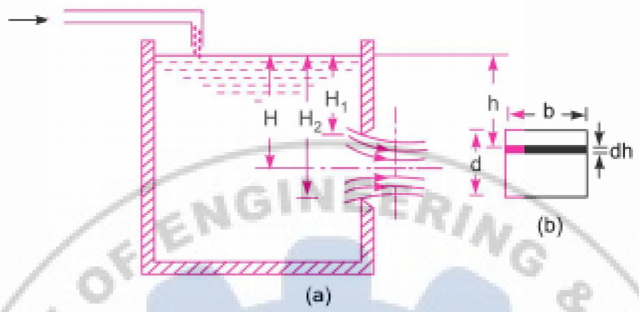
H_2 = height of liquid above bottom edge of orifice

b = breadth of orifice

d = depth of orifice = $H_2 - H_1$

C_d = co-efficient of discharge.

Consider an elementary horizontal strip of depth 'dh' at a depth of 'h' below the free surface of the liquid in the tank as shown in Fig.



Large rectangular orifice.

$\therefore$ Area of strip = $b \times dh$

and theoretical velocity of water through strip = $\sqrt{2gh}$.

$\therefore$ Discharge through elementary strip is given

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity}$$
$$= C_d \times b \times dh \times \sqrt{2gh} = C_d\, b \times \sqrt{2gh}\ dh$$

By integrating the above equation between the limits H_1 and H_2, the total discharge through the whole orifice is obtained

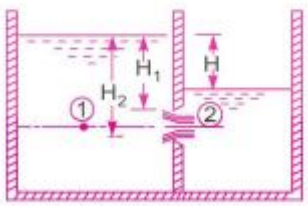
$$\therefore \quad Q = \int_{H_1}^{H_2} C_d \times b \times \sqrt{2gh}\, dh$$

$$= C_d \times b \times \sqrt{2g} \int_{H_1}^{H_2} \sqrt{h}\, dh = C_d \times b \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2}\right]_{H_1}^{H_2}$$

$$= \frac{2}{3} C_d \times b\, \sqrt{2g}\, \left[H_2^{3/2} - H_1^{3/2}\right].$$

Discharge through Fully Sub-Merged Orifice:

Fully sub-merged orifice is one which has its whole of the outlet side sub merged under liquid so that it discharges a jet of liquid in to the liquid of the same kind. It is also called totally drowned orifice as shown in Fig. Consider two points (1) & (2). Point 1 being in the reservoir on the upstream side of the orifice and point 2 being at vena contracta.



Fully sub-merged orifice.

Fig.4.Fully Sub-merged Orifice

Let H_1 = Height of water above the top of the orifice on the upstream side,

H_2 = Height of water above the bottom of the orifice,

H = Difference in water level,

b = Width of orifice,

C_d = Co-efficient of discharge.

Height of water above the centre of orifice on upstream side

$$= H_1 + \frac{H_2 - H_1}{2} = \frac{H_1 + H_2}{2}$$

Height of water above the centre of orifice on downstream side

$$= \frac{H_1 + H_2}{2} - H$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g}$$

Now $\frac{p_1}{\rho g} = \frac{H_1 + H_2}{2}$, $\frac{p_2}{\rho g} = \frac{H_1 + H_2}{2} - H$ and V_1 is negligible

$$\therefore \quad \frac{H_1 + H_2}{2} + 0 = \frac{H_1 + H_2}{2} - H + \frac{V_2^2}{2g}$$

$$\therefore \quad \frac{V_2^2}{2g} = H$$

$$\therefore \quad V_2 = \sqrt{2gH}$$

$$\therefore \quad V_2 = \sqrt{2gH}$$

Area of orifice $= b \times (H_2 - H_1)$

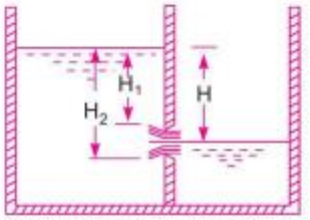
$\therefore$ Discharge through orifice $= C_d \times$ Area $\times$ Velocity

$= C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}$

$$\therefore \quad Q = C_d \times b\,(H_2 - H_1) \times \sqrt{2gH}\,.$$

Discharge through Partially Sub-Merged Orifice:

Partially sub-merged orifice is one which has its outlet side partially sub-merged under liquid as shown in Fig. It is also known as partially drowned orifice. Thus the partially sub-merged orifice has two portions. The upper portion behaves as an orifice discharging free while the lower portion behaves as a sub-merged orifice. Only a large orifice can behave as a partially sub-merged orifice. The total discharge Q through partially sub-merged orifice is equal to the discharges through free and the sub-merged portions.



Partially sub-merged orifice.

Fig.5. Partially sub-merged orifice

Discharge through the sub-merged portion is given by equation.

$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

Discharge through the free portion is given by equation (7.8) as

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

∴ Total discharge

$$Q = Q_1 + Q_2$$

$$= C_d \times b \times (H_2 - H) \times \sqrt{2gH} + \frac{2}{3} C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H^{3/2}]. \ ..$$

Time of Emptying a Tank through an Orifice at its Bottom:

Consider a tank containing some liquid up to a height of H_1. Let an orifice is fitted at the bottom of the tank. It is required to find the time for the liquid surface to fall from the height H_1 to a height H_2.

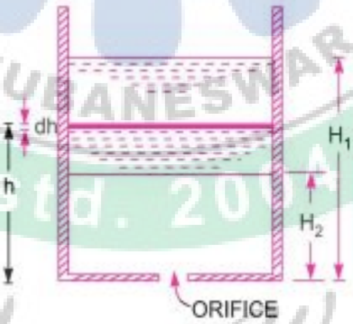


Fig.6.Time of Emptying a Tank

Let A = Area of the tank

a = Area of the orifice

H_1 = Initial height of the liquid

H_2 = Final height of the liquid

T = Time in seconds for the liquid to fall from H_1 to H_2.

Let at any time, the height of liquid from orifice is h and let the liquid surface fall by a small height dh in time dT. Then

Volume of liquid leaving the tank in time, $dT = A \times dh$

Also the theoretical velocity through orifice, $V = \sqrt{2gh}$

∴ Discharge through orifice/sec,

$$dQ = C_d \times \text{Area of orifice} \times \text{Theoretical velocity} = C_d \,.\, a \,.\, \sqrt{2gh}$$

∴ Discharge through orifice in time interval

$$dT = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT$$

As the volume of liquid leaving the tank is equal to the volume of liquid flowing through orifice in time dT, we have

$$A(-dh) = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT$$

– ve sign is inserted because with the increase of time, head on orifice decreases.

$$\therefore \quad -A dh = C_d \,.\, a \,.\, \sqrt{2gh} \,.\, dT \text{ or } dT = \frac{-A\, dh}{C_d \,.\, a \,.\, \sqrt{2gh}} = \frac{-A(h)^{-1/2}}{C_d \,.\, a \,.\, \sqrt{2g}} dh$$

By integrating the above equation between the limits H_1 and H_2, the total time, T is obtained as

$$\int_0^T dT = \int_{H_1}^{H_2} \frac{-Ah^{-1/2}\, dh}{C_d \,.\, a \,.\, \sqrt{2g}} = \frac{-A}{C_d \,.\, a \,.\, \sqrt{2g}} \int_{H_1}^{H_2} h^{-1/2}\, dh$$

or

$$T = \frac{-A}{C_d \,.\, a \,.\, \sqrt{2g}} \left[\frac{h^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \right]_{H_1}^{H_2} = \frac{-A}{C_d \,.\, a \,.\, \sqrt{2g}} \left[\frac{\sqrt{h}}{\frac{1}{2}} \right]_{H_1}^{H_2}$$

$$= \frac{-2A}{C_d \,.\, a \,.\, \sqrt{2g}} \left[\sqrt{H_2} - \sqrt{H_1} \right] = \frac{2A\left[\sqrt{H_1} - \sqrt{H_2} \right]}{C_d \,.\, a \,.\, \sqrt{2g}}$$

For emptying the tank completely, $H_2 = 0$ and hence

$$T = \frac{2A\sqrt{H_1}}{C_d \,.\, a \,.\, \sqrt{2g}}.$$

Time of Emptying a Hemispherical Tank

Consider a hemispherical tank of radius R fitted with an orifice of area “a” at its bottom as shown in Fig. The tank contains some liquid whose initial height is H_1 and in time T, the height of liquid falls to H2. It is required to find the time T.

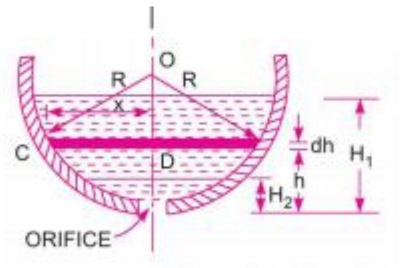


Fig.7. Hemispherical Tank

Let at any instant of time, the head of liquid over the orifice is h and at this instant let x be the radius of the liquid surface. Then

Area of liquid surface, $A = \pi x^2$

and theoretical velocity of liquid $= \sqrt{2gh}$.

Let the liquid level falls down by an amount of dh in time dT.

$\therefore$ Volume of liquid leaving tank in time $dT = A \times dh$

$$= \pi x^2 \times dh$$

Also volume of liquid flowing through orifice

$$= C_d \times \text{area of orifice} \times \text{velocity} = C_d.a.\ \sqrt{2gh}\ \text{second}$$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d.a.\ \sqrt{2gh} \times dT$$

From equations (*i*) and (*ii*), we get

$$\pi x^2 (-dh) = C_d.a.\ \sqrt{2gh}\ .\ dT$$

–ve sign is introduced, because with the increase of T, h will decrease

$$\therefore \quad -\pi x^2\, dh = C_d.a.\ \sqrt{2gh}\ .\ dT$$

But from Fig. for ΔOCD, we have $OC = R$

$$DO = R - h$$

$$\therefore \quad CD = x = \sqrt{OC^2 - OD^2} = \sqrt{R^2 - (R-h)^2}$$

$$\therefore \quad x^2 = R^2 - (R-h)^2 = R^2 - (R^2 + h^2 - 2Rh) = 2Rh - h^2$$

Substituting x^2 in equation (*iii*), we get

$$-\pi(2Rh - h^2)dh = C_d.a.\ \sqrt{2gh}\ .\ dT$$

or

$$dT = \frac{-\pi\left(2Rh - h^2\right)dh}{C_d\ .\ a\ .\sqrt{2gh}} = \frac{-\pi}{C_d\ .\ a\ .\sqrt{2g}}\left(2Rh - h^2\right)h^{-1/2}\ dh$$

$$= \frac{-\pi}{C_d\ .\ a\ .\sqrt{2g}}\,(2Rh^{1/2} - h^{3/2})dh$$

The total time T required to bring the liquid level from H_1 to H_2 is obtained by integrating the above equation between the limits H_1 and H_2.

$$\therefore \quad T = \int_{H_1}^{H_2} \frac{-\pi}{C_d\ .\ a\ .\sqrt{2g}}\,(2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d\ .\ a\ .\sqrt{2g}} \int_{H_1}^{H_2} (2Rh^{1/2} - h^{3/2})dh$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[2R \frac{h^{1/2+1}}{\frac{1}{2}+1} - \frac{h^{3/2}+1}{\frac{3}{2}+1} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[2 \times \frac{2}{3} Rh^{3/2} - \frac{2}{5} h^{5/2} \right]_{H_1}^{H_2}$$

$$= \frac{-\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R\left(H_2^{3/2} - H_1^{3/2}\right) - \frac{2}{5}\left(H_2^{5/2} - H_1^{5/2}\right) \right]$$

$$= \frac{\pi}{C_d \times a \times \sqrt{2g}} \left[\frac{4}{3} R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right) \right]$$

For completely emptying the tank, $H_2 = 0$ and hence

$$T = \frac{\pi}{C_d \, . \, a \, . \sqrt{2g}} \left[\frac{4}{3} RH_1^{3/2} - \frac{2}{5} H_1^{5/2} \right].$$

Time of Emptying a Circular Horizontal Tank:

Consider a circular horizontal tank of length L and radius R, containing liquid upto a height of H_1. Let an orifice of area 'a' is fitted at the bottom of the tank. Then the time required to bring the liquid level from H_1 to H_2 is obtained as :

Let at any time, the height of liquid over orifice is 'h' and in time dT, let the height falls by an height of 'dh'. Let at this time, the width of liquid surface = AC as shown in Fig.

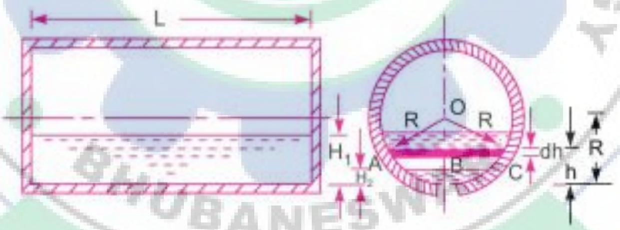


Fig.8. Time of Emptying a Circular Horizontal Tank

$\therefore$ Surface area of liquid = $L \times AC$

But $AC = 2 \times AB = 2\left[\sqrt{AO^2 - OB^2}\right] = 2\left[\sqrt{R^2 - (R-h)^2}\right]$

$$= 2\sqrt{R^2 - \left(R^2 + h^2 - 2Rh\right)} = 2\sqrt{2Rh - h^2}$$

$\therefore$ Surface area, $A = L \times 2\sqrt{2Rh - h^2}$

$\therefore$ Volume of liquid leaving tank in time dT

$$= A \times dh = 2L\sqrt{2Rh - h^2} \times dh \quad \ldots(i)$$

Also the volume of liquid flowing through orifice in time dT

$$= C_d \times \text{Area of orifice} \times \text{Velocity} \times dT$$

But the velocity of liquid at the time considered = $\sqrt{2gh}$

$\therefore$ Volume of liquid flowing through orifice in time dT

$$= C_d \times a \times \sqrt{2gh} \times dT \quad \ldots(ii)$$

Equating (*i*) and (*ii*), we get

$$2L\sqrt{2Rh - h^2} \times (-dh) = C_d \times a \times \sqrt{2gh \times dT}$$

– ve sign is introduced as with the increase of T, the height h decreases,

$$\therefore \quad dT = \frac{-2L\sqrt{2Rh - h^2}\, dh}{C_d \times a \times \sqrt{2gh}} = \frac{-2L\sqrt{(2R - h)}\, dh}{C_d \times a \times \sqrt{2g}}$$

[Taking $\sqrt{h}$ common]

$$\therefore \quad \text{Total time, } T = \int_{H_1}^{H_2} \frac{-2L(2R-h)^{1/2}\, dh}{C_d \times a \times \sqrt{2g}}$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \int_{H_1}^{H_2} [2R - h]^{1/2}\, dh$$

$$= \frac{-2L}{C_d \times a \times \sqrt{2g}} \left[\frac{(2R-h)^{1/2+1}}{\frac{1}{2}+1} \times (-1) \right]_{H_1}^{H_2}$$

$$= \frac{2L}{C_d \times a \times \sqrt{2g}} \times \frac{2}{3} \times \left[(2R-h)^{3/2} \right]_{H_1}^{H_2}$$

$$= \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R - H_2)^{3/2} - (2R - H_1)^{3/2} \right]$$

For completely emptying the tank, $H_2 = 0$ and hence

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} \left[(2R)^{3/2} - (2R - H_1)^{3/2} \right].$$

Classification of Mouthpieces:

1. The mouthpieces are classified as (*i*) External mouthpiece or (*ii*) Internal mouthpiece depending upon their position with respect to the tank or vessel to which they are fitted.

2. The mouthpiece are classified as (*i*) Cylindrical mouthpiece or (*ii*) Convergent mouthpiece or (*iii*) Convergent-divergent mouthpiece depending upon their shapes.

3. The mouthpieces are classified as (*i*) Mouthpieces running full or (*ii*) Mouthpieces running free, depending upon the nature of discharge at the outlet of the mouthpiece. This classification is only for internal mouthpieces which are known Borda's or Re-entrant mouthpieces. A mouthpiece is said to be running free if the jet of liquid after contraction does not touch the sides of the mouthpiece. But if the jet after contraction expands and fills the whole mouthpiece it is known as running full.

Flow through an External Cylindrical Mouthpiece:

A mouthpiece is a short length of a pipe which is two or three times its diameter in length. If this pipe is fitted externally to the orifice, the mouthpiece is called external cylindrical mouthpiece and the discharge through orifice increases.

Consider a tank having an external cylindrical mouthpiece of cross-sectional area a_1, attached to one of its sides as shown in Fig. 7.13. The jet of liquid entering the mouthpiece contracts to form a vena-contracta at a section *C-C*. Beyond this section, the jet again expands and fill the mouthpiece completely.

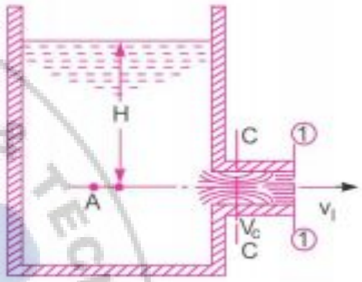


Fig.9. External Cylindrical Mouthpiece

Let H = Height of liquid above the centre of mouthpiece

v_c = Velocity of liquid at *C-C* section

a_c = Area of flow at vena-contracta

v_1 = Velocity of liquid at outlet

a_1 = Area of mouthpiece at outlet

C_c = Co-efficient of contraction.

Applying continuity equation at *C-C* and (1)-(1), we get

$$a_c \times v_c = a_1 v_1$$

$$\therefore \quad v_c = \frac{a_1 v_1}{a_c} = \frac{v_1}{a_c / a_1}$$

But $$\frac{a_c}{a_1} = C_c = \text{Co-efficient of contraction}$$

Taking $C_c = 0.62$, we get $\frac{a_c}{a_1} = 0.62$

$$\therefore \quad v_c = \frac{v_1}{0.62}$$

The jet of liquid from section *C-C* suddenly enlarges at section (1)-(1). Due to sudden enlargement, there will be a loss of head, h_L* which is given as $h_L = \frac{(v_c - v_1)^2}{2g}$

But $v_c = \frac{v_1}{0.62}$ hence $h_L = \frac{\left(\frac{v_1}{0.62} - v_1\right)^2}{2g} = \frac{v_1^2}{2g}\left[\frac{1}{0.62} - 1\right]^2 = \frac{0.375\, v_1^2}{2g}$

Applying Bernoulli's equation to point A and (1)-(1)

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 + h_L$$

where $z_A = z_1$, v_A is negligible,

$\frac{p_1}{\rho g}$ = atmospheric pressure = 0

$$\therefore \quad H + 0 = 0 + \frac{v_1^2}{2g} + .375\,\frac{v_1^2}{2g}$$

$$\therefore \quad H = 1.375\,\frac{v_1^2}{2g}$$

$$\therefore \quad v_1 = \sqrt{\frac{2gH}{1.375}} = 0.855\,\sqrt{2gH}$$

Theoretical velocity of liquid at outlet is $v_{th} = \sqrt{2gH}$

$\therefore$ Co-efficient of velocity for mouthpiece

$$C_v = \frac{\text{Actual velocity}}{\text{Theoretical velocity}} = \frac{0.855\sqrt{2gH}}{\sqrt{2gH}} = 0.855.$$

C_c for mouthpiece = 1 as the area of jet of liquid at outlet is equal to the area of mouthpiece at outlet.

Thus $\quad C_d = C_c \times C_v = 1.0 \times .855 = 0.855$

Thus the value of C_d for mouthpiece is more than the value of C_d for orifice, and so discharge through mouthpiece will be more.

Flow through a Convergent – Divergent Mouthpiece:

If a mouthpiece converges upto vena-contracta and then diverges as shown in Fig. then that type of mouthpiece is called Convergent-Divergent Mouthpiece. As in this mouthpiece there is no sudden enlargement of the jet, the loss of energy due to sudden enlargement is eliminated. The co-efficient of discharge for this mouthpiece is unity. Let H is the head of liquid over the mouthpiece.

Applying Bernoulli's equation to the free surface of water in tank and section C-C, we have

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c$$

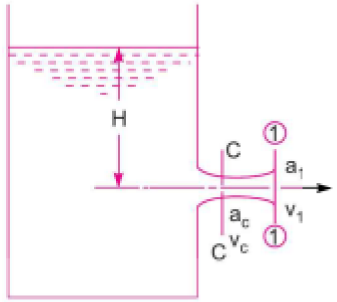


Fig.10.Convergent – Divergent Mouthpiece

Taking datum passing through the centre of orifice, we get

$$\frac{p}{\rho g} = H_a,\ v = 0,\ z = H,\ \frac{p_c}{\rho g} = H_c,\ z_c = 0$$

$$\therefore \quad H_a + 0 + H = H_c + \frac{v_c^2}{2g} + 0 \quad \ldots(i)$$

$$\therefore \quad \frac{v_c^2}{2g} = H_a + H - H_c \quad \ldots(ii)$$

or

$$v_c = \sqrt{2g(H_a + H - H_c)}$$

Now applying Bernoulli's equation at sections C-C and (1)-(1)

$$\frac{p_c}{\rho g} + \frac{v_c^2}{2g} + z_c = \frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1$$

But $z_c = z_1$ and $\frac{p_1}{\rho g} = H_a$

$$\therefore \quad H_c + \frac{v_c^2}{2g} = H_a + \frac{v_1^2}{2g}$$

Also from (*i*), $H_c + v_c^2/2g = H + H_a$

$$\therefore \quad H_a + v_1^2/2g = H + H_a$$

$$\therefore \quad v_1 = \sqrt{2gH}$$

Now by continuity equation, $a_c\, v_c = v_1 \times a_1$

$$\therefore \quad \frac{a_1}{a_c} = \frac{v_c}{v_1} = \frac{\sqrt{2g(H_a + H - H_c)}}{\sqrt{2gH}} = \sqrt{\frac{H_a}{H} + 1 - \frac{H_c}{H}}$$

$$= \sqrt{1 + \frac{H_a - H_c}{H}}$$

The discharge, Q is given as $Q = a_c \times \sqrt{2gH}$

where a_c = area at vena-contracta.

NOTCHES:

A **notch** is a device used for measuring the rate of flow of a liquid through a small channel or a tank. It may be defined as an opening in the side of a tank or a small channel in such a way that the liquid surface in the tank or channel is below the top edge of the opening.

A **weir** is a concrete or masonary structure, placed in an open channel over which the flow occurs. It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel. The notch is of small size while the weir is of a bigger size. The notch is generally made of metallic plate while weir is made of concrete or masonary structure.

1. Nappe or Vein. The sheet of water flowing through a notch or over a weir is called Nappe or Vein.

2. Crest or Sill. The bottom edge of a notch or a top of a weir over which the water flows, is known as the sill or crest.

Classification of Notches and Weirs:

The notches are classified as :

1. According to the shape of the opening :
   (*a*) Rectangular notch,
   (*b*) Triangular notch,
   (*c*) Trapezoidal notch, and
   (*d*) Stepped notch.
2. According to the effect of the sides on the nappe :
   (*a*) Notch with end contraction.
   (*b*) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening, the shape of the crest, the effect of the sides on the nappe and nature of discharge. The following are important classifications.

(*a*) According to the shape of the opening :
   (*i*) Rectangular weir, (*ii*) Triangular weir, and
   (*iii*) Trapezoidal weir (Cipolletti weir)

(*b*) According to the shape of the crest :
   (*i*) Sharp-crested weir, (*ii*) Broad-crested weir,
   (*iii*) Narrow-crested weir, and (*iv*) Ogee-shaped weir.

(*c*) According to the effect of sides on the emerging nappe :
   (*i*) Weir with end contraction, and (*ii*) Weir without end contraction.

Discharge over a Rectangular Notch or Weir:

The expression for discharge over a rectangular notch or weir is the same.

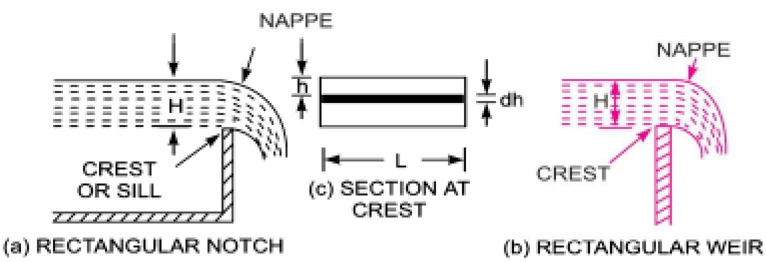


Fig.11.Rectangulat Notch and Weir

Consider a rectangular notch or weir provided in a channel carrying water as shown in Fig.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water as shown in Fig.

The area of strip $= L \times dh$

and theoretical velocity of water flowing through strip $= \sqrt{2gh}$

The discharge dQ, through strip is

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity}$$

$$= C_d \times L \times dh \times \sqrt{2gh} \quad \ldots(i)$$

where C_d = Co-efficient of discharge.

The total discharge, Q, for the whole notch or weir is determined by integrating equation (i) between the limits 0 and H.

$$\therefore \quad Q = \int_0^H C_d \, . \, L \, . \, \sqrt{2gh} \, . \, dh = C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} \, dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{\frac{1}{2}+1} \right]_0^H = C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= \frac{2}{3} C_d \times L \times \sqrt{2g} \, [H]^{3/2}.$$

Discharge over a Triangular Notch or Weir:

The expression for the discharge over a triangular notch or weir is the same. It is derived as :

Let H = head of water above the V- notch

θ = angle of notch

Consider a horizontal strip of water of thickness 'dh' at a depth of h from the free surface of water as shown in Fig.

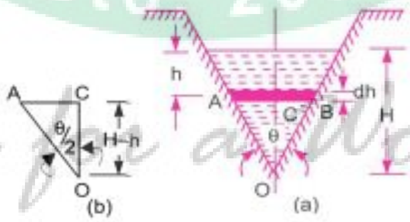


Fig.12.Triangular Notch or Weir

From Fig.   we have

$$\tan\frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$\therefore$ $$AC = (H-h)\tan\frac{\theta}{2}$$

Width of strip $$= AB = 2AC = 2\,(H-h)\tan\frac{\theta}{2}$$

$\therefore$ Area of strip $$= 2\,(H-h)\tan\frac{\theta}{2}\times dh$$

The theoretical velocity of water through strip = $\sqrt{2gh}$

$\therefore$ Discharge, through the strip,

$$dQ = C_d \times \text{Area of strip} \times \text{Velocity (theoretical)}$$

$$= C_d \times 2\,(H-h)\tan\frac{\theta}{2}\times dh \times \sqrt{2gh}$$

$$= 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times dh$$

$\therefore$ Total discharge, $$Q = \int_0^H 2C_d\,(H-h)\tan\frac{\theta}{2}\times\sqrt{2gh}\times dh$$

$$= 2C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (H-h)h^{1/2}\,dh$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\int_0^H (Hh^{1/2}-h^{3/2})\,dh$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{Hh^{3/2}}{3/2}-\frac{h^{5/2}}{5/2}\right]_0^H$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H.H^{3/2}-\frac{2}{5}H^{5/2}\right]$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{2}{3}H^{5/2}-\frac{2}{5}H^{5/2}\right]$$

$$= 2\times C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\left[\frac{4}{15}H^{5/2}\right]$$

$$= \frac{8}{15}C_d\times\tan\frac{\theta}{2}\times\sqrt{2g}\times H^{5/2}$$

For a right-angled V-notch, if $C_d = 0.6$

$$\theta = 90°,\quad\therefore\quad \tan\frac{\theta}{2} = 1$$

Discharge, $$Q = \frac{8}{15}\times 0.6\times 1\times\sqrt{2\times 9.81}\times H^{5/2}$$

$$= 1.417\ H^{5/2}.$$

Discharge over a Trapezoidal Notch or Weir:

As shown in Fig. a trapezoidal notch or weir is a combination of a rectangular and triangular notch or weir. Thus the total discharge will be equal to the sum of discharge through a rectangular weir or notch and discharge through a triangular notch or weir.

Let H = Height of water over the notch

L = Length of the crest of the notch

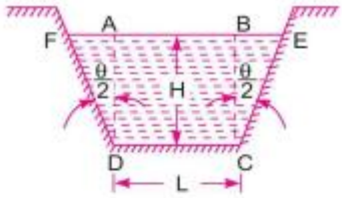


Fig.13. Trapezoidal Notch

C_{d_1} = Co-efficient of discharge for rectangular portion $ABCD$ of Fig.

C_{d_2} = Co-efficient of discharge for triangular portion [FAD and BCE]

The discharge through rectangular portion $ABCD$ is given by

or $$Q_1 = \frac{2}{3} \times C_{d_1} \times L \times \sqrt{2g} \times H^{3/2}$$

The discharge through two triangular notches FDA and BCE is equal to the discharge through a single triangular notch of angle θ and it is given by equation

$$Q_2 = \frac{8}{15} \times C_{d_2} \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

∴ Discharge through trapezoidal notch or weir $FDCEF = Q_1 + Q_2$

$$= \frac{2}{3} C_{d_1} L \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan\theta/2 \times \sqrt{2g} \times H^{5/2}.$$

Discharge over a Broad – Crested Weir:

A weir having a wide crest is known as broad-crested weir.

Let H = height of water above the crest

L = length of the crest

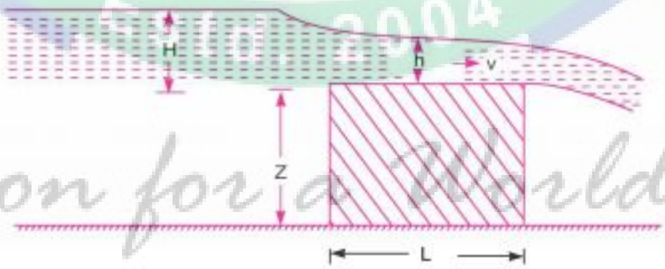


Fig.14. Broad – Crested Weir

If $2L > H$, the weir is called broad-crested weir

If $2L < H$, the weir is called a narrow-crested weir

Fig. 8.10 shows a broad-crested weir.

Let h = head of water at the middle of weir which is constant

v = velocity of flow over the weir

Applying Bernoulli's equation to the still water surface on the upstream side and running water at the end of weir,

$$0 + 0 + H = 0 + \frac{v^2}{2g} + h$$

$$\therefore \quad \frac{v^2}{2g} = H - h$$

$$\therefore \quad v = \sqrt{2g\,(H - h)}$$

$\therefore$ The discharge over weir $Q = C_d \times \text{Area of flow} \times \text{Velocity}$

$$= C_d \times L \times h \times \sqrt{2g\,(H - h)}$$

$$= C_d \times L \times \sqrt{2g\left(Hh^2 - h^3\right)}$$

The discharge will be maximum, if $(Hh^2 - h^3)$ is maximum

$$\text{or} \quad \frac{d}{dh}(Hh^2 - h^3) = 0 \text{ or } 2h \times H - 3h^2 = 0 \text{ or } 2H = 3h$$

$$\therefore \quad h = \frac{2}{3}H$$

Q_{max} will be obtained by substituting this value of h in equation

$$Q_{max} = C_d \times L \times \sqrt{2g\left[H \times \left(\frac{2}{3}H\right)^2 - \left(\frac{2}{3}H\right)^3\right]}$$

$$= C_d \times L \times \sqrt{2g}\,\sqrt{H \times \frac{4}{9} \times H^2 - \frac{8}{27}H^3}$$

$$= C_d \times L \times \sqrt{2g}\,\sqrt{\frac{4}{9}H^3 - \frac{8}{27}H^3} = C_d \times L \times \sqrt{2g}\,\sqrt{\frac{(12-8)H^3}{27}}$$

$$= C_d \times L \times \sqrt{2g}\,\sqrt{\frac{4}{27}H^3} = C_d \times L \times \sqrt{2g} \times 0.3849 \times H^{3/2}$$

$$= .3849 \times \sqrt{2 \times 9.81} \times C_d \times L \times H^{3/2} = 1.7047 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times C_d \times L \times H^{3/2}.$$

Discharge over a Narrow – Crested Weir:

For a narrow-crested weir, $2L < H$. It is similar to a rectangular weir or notch hence, Q is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

Discharge over an OGEE weir:

Fig. shows an Ogee weir, in which the crest of the weir rises upto maximum height of $0.115 \times H$ (where H is the height of water above inlet of the weir) and then falls as shown in Fig.
The discharge for an Ogee weir is the same as that of a rectangular weir, and it is given by

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

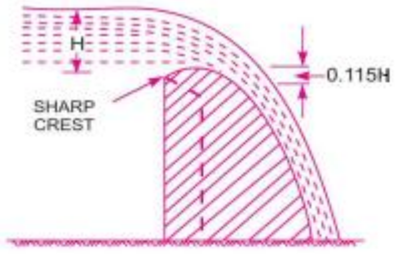


Fig.15. OGEE Weir

Viscous Flow:

This chapter deals with the flow of fluids which are viscous and flowing at very low velocity. At low velocity the fluid moves in layers. Each layer of fluid slides over the adjacent layer. Due to relative velocity between two layers the velocity gradient $\frac{du}{dy}$ exists and hence a shear stress $\tau = \mu \frac{du}{dy}$ acts on the layers.

The following cases will be considered in this chapter :

1. Flow of viscous fluid through circular pipe.
2. Flow of viscous fluid between two parallel plates.
3. Kinetic energy correction and momentum correction factors.
4. Power absorbed in viscous flow through
   (*a*) Journal bearings, (*b*) Foot-step bearings, and (*c*) Collar bearings.

Flow of Viscous Fluid through Circular Pipe:

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through the circular pipe will be viscous or laminar, if the Reynolds number (R_e*) is less than 2000. The expression for Reynold number is given by

$$R_e = \frac{\rho V D}{\mu}$$

where ρ = Density of fluid flowing through pipe
V = Average velocity of fluid
D = Diameter of pipe and
μ = Viscosity of fluid.

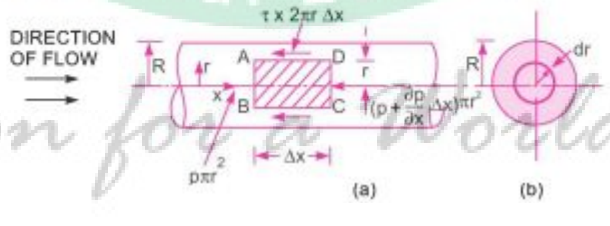


Fig.16.Viscous flow through a pipe

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in Fig. 9.1 (*a*). Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius $(r + dr)$. Let the length of fluid element be Δx. If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Then the forces acting on the fluid element are :

1. The pressure force, $p \times \pi r^2$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r \Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero *i.e.*,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x} \cdot r - 2\tau = 0$$

$\therefore$
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear as shown in Fig.

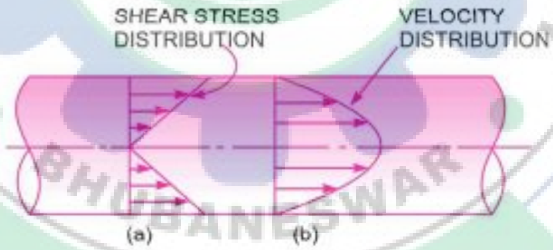


Fig.17.Shear stress and velocity distribution across a section

(*i*) **Velocity Distribution.** To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in equation (9.1).

But in the relation $\tau = \mu \frac{du}{dy}$, y is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$\therefore$
$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting this value in , we get

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{or} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

Integrating this above equation w.r.t. 'r', we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$, $u = 0$.

$$\therefore \quad 0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

$$\therefore \quad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Substituting this value of C in equation (9.2), we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2]$$

In equation values of μ, $\frac{\partial p}{\partial x}$ and R are constant, which means the velocity, u varies with the square of r. Thus equation is a equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in Fig.

(*ii*) **Ratio of Maximum Velocity to Average Velocity.** The velocity is maximum, when $r = 0$ in equation . Thus maximum velocity, U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge of the fluid across the section by the area of the pipe (πR^2). The discharge (Q) across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr as shown in Fig. (*b*). The fluid flowing per second through this elementary ring

$$dQ = \text{velocity at a radius } r \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\, dr$$

$$\therefore \quad Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\, r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

$$\therefore \quad \text{Average velocity,} \qquad \bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^4}{\pi R^2}$$

or
$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2$$

Dividing equation (9.4) by equation (9.5),

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x}R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right)R^2} = 2.0$$

$\therefore$ Ratio of maximum velocity to average velocity = 2.0.

(*iii*) **Drop of Pressure for a given Length (L) of a pipe**

From equation (9.5), we have

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right)R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

Integrating the above equation w.r.t. x, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\,dx$$

$$\therefore \qquad -[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or } (p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2}L$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2} \qquad \left\{\because \; R = \frac{D}{2}\right\}$$

or
$$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2}, \quad \text{where } p_1 - p_2 \text{ is the drop of pressure.}$$

$$\therefore \quad \text{Loss of pressure head} = \frac{p_1 - p_2}{\rho g}$$

$$\therefore \qquad \frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$$

Equation (9.6) is called **Hagen Poiseuille Formula.**

Flow in Pipes:

In this chapter, however, a method of expressing the loss using an average flow velocity is stated. Studies will be made on how to express losses caused by a change in the cross sectional area of a pipe, a pipe bend and a valve, in addition to the frictional loss of a pipe. Consider a case where fluid runs from a tank into a pipe whose entrance section is fully rounded. At the entrance, the velocity distribution is roughly uniform while the pressure head is lower by V2/2g . The section from the entrance to just where the boundary layer develops to the tube centre is called the inlet or entrance region, whose length is called the inlet or entrance length. For steady flow at a known flow rate, these regions exhibit the following: **Laminar flow**: A local velocity constant with time, but which varies spatially due to viscous shear and geometry. **Turbulent**

flow: A local velocity which has a constant mean value but also has a statistically random fluctuating component due to turbulence in the flow. Typical plots of velocity time histories for laminar flow, turbulent flow, and the region of transition between the two are shown below.

Principal parameter used to specify the type of flow regime is the Reynolds number :

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

V- Flow velocity

D – Flow dimension

μ - Dynamic Viscosity

Ʊ – Kinematic Viscosity

Frictional Loss in Pipe flow

When a liquid is flowing through a pipe, the velocity of the liquid layer adjacent to the pipe wall is zero. The velocity of liquid goes on increasing from the wall and thus velocity gradient and hence shear stresses are produced in the whole liquid due to viscosity. This viscous action causes loss of energy which is usually known as frictional loss.

On the basis of his experiments, William Froude gave the following laws of fluid fraction for turbulent flow.

The frictional resistance for turbulent flow is :

(*i*) proportional to V^n, where n varies from 1.5 to 2.0,

(*ii*) proportional to the density of fluid,

(*iii*) proportional to the area of surface in contact,

(*iv*) independent of pressure,

(*v*) dependent on the nature of the surface in contact.

Expression for Loss of Head due to friction in pipes:

Consider a uniform horizontal pipe having steady flow as shown in fig 18. Let 1-1 and 2-2 are two sections of pipe.

Let P_1 = pressure intensity at section 1-1

V_1 = Velocity of flow at section 1-1

L = length of the pipe between sections 1-1 and 2-2,

d = diameter of pipe,

f' = frictional resistance per unit wetted area per unit velocity,

h_f = loss of head due to friction,

and p_2, V_2 = are values of pressure intensity and velocity at section 2-2.

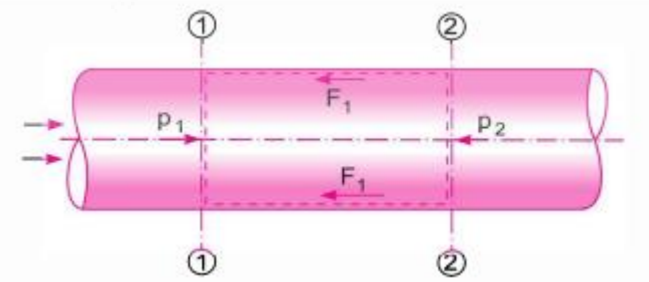


Fig.18.Uniform Horizontal Pipe

Applying Bernoulli's equations between sections 1-1 and 2-2,

Total head at 1-1 = Total head at 2-2 + loss of head due to friction between 1-1 and 2-2

or $$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

But $z_1 = z_2$ as pipe is horizontal

$V_1 = V_2$ as dia. of pipe is same at 1-1 and 2-2

$$\therefore \quad \frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f \text{ or } h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} \quad \ldots(i)$$

But h_f is the head lost due to friction and hence intensity of pressure will be reduced in the direction of flow by frictional resistance.

Now frictional resistance = frictional resistance per unit wetted area per unit velocity × wetted area × velocity^2

or $$F_1 = f' \times \pi dL \times V^2 \quad [\because \text{ wetted area} = \pi d \times L, \text{ velocity} = V = V_1 = V_2]$$

$$= f' \times P \times L \times V^2 \quad [\because \pi d = \text{Perimeter} = P] \ldots(ii)$$

The forces acting on the fluid between sections 1-1 and 2-2 are :

1. pressure force at section 1-1 = $p_1 \times A$

where A = Area of pipe

2. pressure force at section 2-2 = $p_2 \times A$
3. frictional force F_1 as shown in Fig. 10.3.

Resolving all forces in the horizontal direction, we have

$$p_1 A - p_2 A - F_1 = 0 \quad \ldots(10.1)$$

or $$(p_1 - p_2)A = F_1 = f' \times P \times L \times V^2 \quad [\because \text{ From } (ii), F_1 = f'PLV^2]$$

or $$p_1 - p_2 = \frac{f' \times P \times L \times V^2}{A}$$

But from equation (i), $p_1 - p_2 = \rho g h_f$

Equating the value of $(p_1 - p_2)$, we get

$$\rho g h_f = \frac{f' \times P \times L \times V^2}{A}$$

or $$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \quad \ldots(iii)$$

In equation (iii), $\frac{P}{A} = \frac{\text{Wetted perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$

$$\therefore \quad h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L \times V^2 = \frac{f'}{\rho g} \times \frac{4LV^2}{d} \quad \ldots(iv)$$

Putting $\frac{f'}{\rho} = \frac{f}{2}$, where f is known as co-efficient of friction.

Equation (*iv*), becomes as $h_f = \frac{4 \cdot f}{2g} \cdot \frac{LV^2}{d} = \frac{4f \cdot L \cdot V^2}{d \times 2g}$

is known as Darcy-Weisbach equation. This equation is commonly used for finding loss of head due to friction in pipes.

Sometimes equation (10.2) is written as

$$h_f = \frac{f \cdot L \cdot V^2}{d \times 2g}$$

Then f is known as friction factor.

Loss of Energy in Pipes:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

   This is due to friction and it is calculated by the following formulae :
   (*a*) Darcy-Weisbach Formula
   (*b*) Chezy's Formula

2. Minor Energy Losses

   This is due to
   (*a*) Sudden expansion of pipe
   (*b*) Sudden contraction of pipe
   (*c*) Bend in pipe
   (*d*) Pipe fittings etc.
   (*e*) An obstruction in pipe.

Loss of Energy due to friction:

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter and is given by

$$h_f = \frac{4 \cdot f \cdot L \cdot V^2}{d \times 2g}$$

where h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$= \frac{16}{R_e}$ for $R_e < 2000$ (viscous flow)

$= \frac{0.079}{R_e^{1/4}}$ for R_e varying from 4000 to 10^6

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

(*b*) **Chezy's Formula for loss of head due to friction in pipes.** Refer to chapter article in which expression for loss of head due to friction in pipes is derived. Equation (*iii*) of article

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,
A = area of cross-section of pipe, L = length of pipe,
and V = mean velocity of flow.

Now the ratio of $\frac{A}{P}\left(= \frac{\text{Area of flow}}{\text{Perimeter (wetted)}}\right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation , we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m} \text{ or } V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore \quad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}}\sqrt{m\frac{h_f}{L}}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation (11.3), we get

$$V = C\sqrt{mi}$$

Equation is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

Minor Energy Losses

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

Loss of Head Due to Sudden Enlargement. Consider a liquid flowing through a pipe which has sudden enlargement as shown in Fig. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

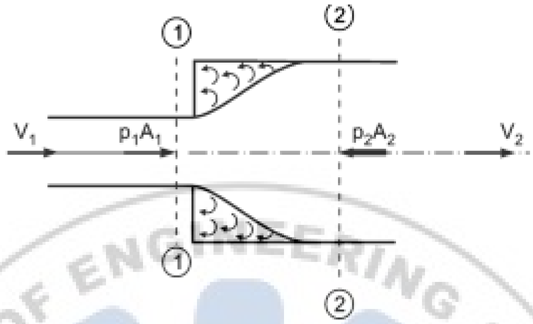


Fig.19. Sudden Enlargement

$$h_e = \frac{(V_1 - V_2)^2}{2g}.$$

Loss of Head due to Sudden Contraction. Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in Fig. Consider two sections 1-1 and 2-2 before and after contraction. As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section *C-C* as shown in Fig. This section *C-C* is called Vena-contracta. After section *C-C*, a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

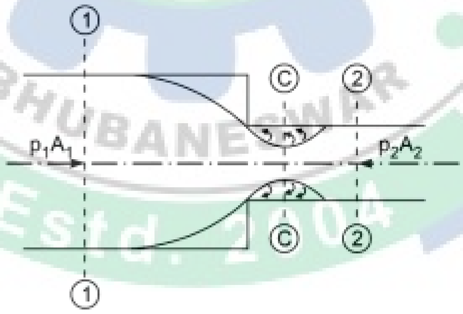


Fig.20.Sudden Contraction

$$h_c = 0.5\,\frac{V_2^2}{2g}.$$

Loss of Head at the Entrance of a Pipe. This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken = $0.5 \frac{V^2}{2g}$, where V = velocity of liquid in pipe.

This loss is denoted by h_i

$$\therefore \qquad h_i = 0.5 \frac{V^2}{2g}$$

Loss of Head at the Exit of Pipe. This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir). This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the outlet of pipe. This loss is denoted h_o.

$$\therefore \qquad h_o = \frac{V^2}{2g}$$

where V = velocity at outlet of pipe.

Loss of Head Due to an Obstruction in a Pipe. Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in Fig.

Consider a pipe of area of cross-section A having an obstruction as shown in Fig.

Let a = Maximum area of obstruction

A = Area of pipe

V = Velocity of liquid in pipe

Then $(A - a)$ = Area of flow of liquid at section 1-1.

As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity, V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

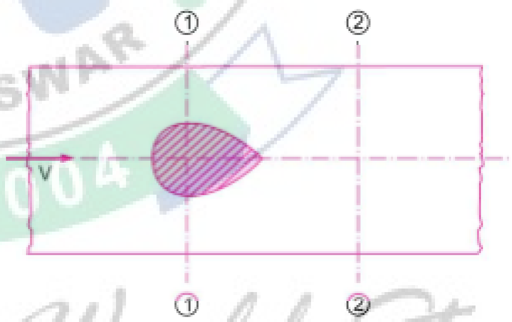


Fig.21. Obstruction in a pipe

$$\text{Head loss due to obstruction} = \frac{V^2}{2g}\left(\frac{A}{C_c(A-a)} - 1\right)^2$$

Loss of Head due to Bend in Pipe. When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g}$$

where h_b = loss of head due to bend, V = velocity of flow, k = co-efficient of bend

The value of k depends on

(*i*) Angle of bend, (*ii*) Radius of curvature of bend, (*iii*) Diameter of pipe.

Loss of Head in Various Pipe Fittings. The loss of head in the various pipe fittings such as valves, couplings etc., is expressed as

$$= \frac{kV^2}{2g}$$

where V = velocity of flow, k = co-efficient of pipe fitting.

HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

FLOW THROUGH SYPHON

Syphon is a long bent pipe which is used to transfer liquid from a reservoir at a higher elevation to another reservoir at a lower level when the two reservoirs are separated by a hill or high level ground as shown in Fig.

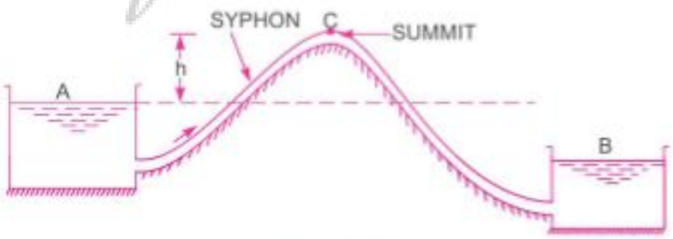


Fig.22.Flow through Syphon

The point C which is at the highest of the syphon is called the summit. As the point C is above the free surface of the water in the tank A, the pressure at C will be less than atmospheric pressure. Theoretically, the pressure at C may be reduced to – 10.3 m of water but in actual practice this pressure is only – 7.6 m of water or 10.3 – 7.6 = 2.7 m of water absolute. If the pressure at C becomes less than 2.7 m of water absolute, the dissolved air and other gases would come out from water and collect at the summit. The flow of water will be obstructed. Syphon is used in the following cases :

1. To carry water from one reservoir to another reservoir separated by a hill or ridge.
2. To take out the liquid from a tank which is not having any outlet.
3. To empty a channel not provided with any outlet sluice.

FLOW THROUGH PIPES IN SERIES OR FLOW THROUGH COMPOUND PIPES

Pipes in series or compound pipes are defined as the pipes of different lengths and different diameters connected end to end (in series) to form a pipe line as shown in Fig.

Let, L_1, L_2, L_3 = length of pipes 1, 2 and 3 respectively
d_1, d_2, d_3 = diameter of pipes 1, 2, 3 respectively
V_1, V_2, V_3 = velocity of flow through pipes 1, 2, 3
f_1, f_2, f_3 = co-efficient of frictions for pipes 1, 2, 3
H = difference of water level in the two tanks.

Fig.23.Pipes in series

The discharge passing through each pipe is same.

$\therefore$ $$Q = A_1V_1 = A_2V_2 = A_3V_3$$

The difference in liquid surface levels is equal to the sum of the total head loss in the pipes.

$\therefore$ $$H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}.$$

If minor losses are neglected, then above equation becomes as

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}.$$

If the co-efficient of friction is same for all pipes

i.e., $f_1 = f_2 = f_3 = f$, then equation becomes as

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right].$$

Equivalent Pipe

This is defined as the pipe of uniform diameter having loss of head and discharge equal to the loss of head and discharge of a compound pipe consisting of several pipes of different lengths and diameters. The uniform diameter of the equivalent pipe is called equivalent size of the pipe. The length of equivalent pipe is equal to sum of lengths of the compound pipe consisting of different pipes.

Let L_1 = length of pipe 1 and d_1 = diameter of pipe 1

L_2 = length of pipe 2 and d_2 = diameter of pipe 2

L_3 = length of pipe 3 and d_3 = diameter of pipe 3

H = total head loss

L = length of equivalent pipe

d = diameter of the equivalent pipe

Then $L = L_1 + L_2 + L_3$

Total head loss in the compound pipe, neglecting minor losses

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

Assuming $f_1 = f_2 = f_3 = f$

Discharge, $Q = A_1V_1 = A_2V_2 = A_3V_3 = \frac{\pi}{4} d_1^2V_1 = \frac{\pi}{4} d_2^2V_2 = \frac{\pi}{4} d_3^2V_3$

$\therefore$ $V_1 = \frac{4Q}{\pi d_1^2}, V_2 = \frac{4Q}{\pi d_2^2}$ and $V_3 = \frac{4Q}{\pi d_3^2}$

Substituting these values in equation

$$H = \frac{4fL_1 \times \left(\frac{4Q}{\pi d_1^2}\right)^2}{d_1 \times 2g} + \frac{4fL_2\left(\frac{4Q}{\pi d_2^2}\right)^2}{d_2 \times 2g} + \frac{4fL_3\left(\frac{4Q}{\pi d_3^2}\right)^2}{d_3 \times 2g}$$

$$= \frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right]$$

Head loss in the equivalent pipe, $H = \frac{4f \, . \, L \, . \, V^2}{d \times 2g}$ [Taking same value of f as in compound pipe]

where $V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{4Q}{\pi d^2}$

$$\therefore \quad H = \frac{4\, f \, . \, L \, . \left(\frac{4Q}{\pi d^2}\right)^2}{d \times 2g} = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

Head loss in compound pipe and in equivalent pipe is same hence equating equations

$$\frac{4 \times 16 fQ^2}{\pi^2 \times 2g}\left[\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}\right] = \frac{4 \times 16 Q^2 f}{\pi^2 \times 2g}\left[\frac{L}{d^5}\right]$$

or $$\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} = \frac{L}{d^5} \quad \text{or} \quad \frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

Equation is known as Dupuit's equation. In this equation $L = L_1 + L_2 + L_3$ and d_1, d_2 and d_3 are known. Hence the equivalent size of the pipe, *i.e.*, value of d can be obtained.

Flow through Parallel Pipes:

Consider a main pipe which divides into two or more branches as shown in Fig. and again join together downstream to form a single pipe, then the branch pipes are said to be connected in parallel. The discharge through the main is increased by connecting pipes in parallel.

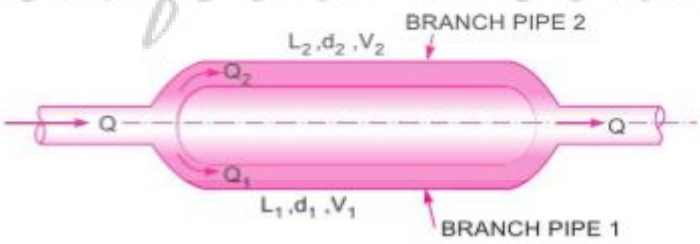


Fig.24.Parallel Pipes

The rate of flow in the main pipe is equal to the sum of rate of flow through branch pipes. Hence from Fig.

$$Q = Q_1 + Q_2$$

In this, arrangement, the loss of head for each branch pipe is same.

$\therefore$ Loss of head for branch pipe 1 = Loss of head for branch pipe 2

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$f_1 = f_2, \text{ then } \frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}$$

Flow through Branched Pipes:

When three or more reservoirs are connected by means of pipes, having one or more junctions, the system is called a branching pipe system. Fig. shows three reservoirs at different levels connected to a single junction, by means of pipes which are called branched pipes. The lengths, diameters and co-efficient of friction of each pipes is given. It is required to find the discharge and direction of flow in each pipe. The basic equations used for solving such problems are :

1. **Continuity equation** which means the inflow of fluid at the junction should be equal to the outflow of fluid.
2. **Bernoulli's equation, and**
3. **Darcy-Weisbach equation**

Also it is assumed that reservoirs are very large and the water surface levels in the reservoirs are constant so that steady conditions exist in the pipes. Also minor losses are assumed very small. The flow from reservoir A takes place to junction D. The flow from junction D is towards reservoirs C. Now the flow from junction D towards reservoir B will take place only when piezometric head at D $\left(\text{which is equal to } \frac{p_D}{\rho g} + Z_D\right)$ is more than the piezometric head at B (*i.e.*, Z_B). Let us consider that flow is from D to reservoir B.

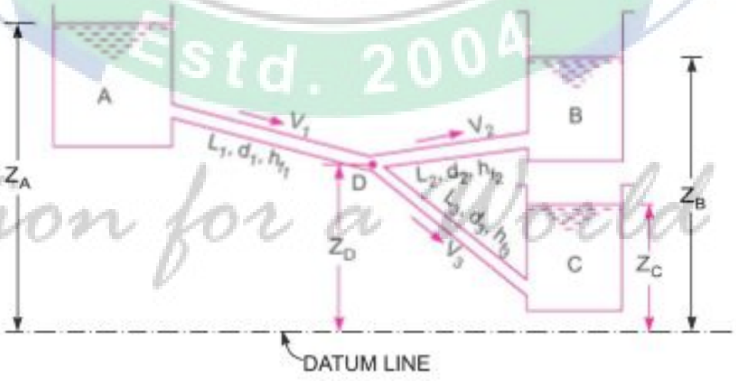


Fig.25.Branched Pipes

For flow from A to D from Bernoulli's equation

$$Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1} \quad \text{...}(i)$$

For flow from D to B from Bernoulli's equation

$$Z_D + \frac{p_D}{\rho g} = Z_B + h_{f_2} \quad \text{...}(ii)$$

For flow from D to C from Bernoulli's equation

$$Z_D + \frac{p_D}{\rho g} = Z_C + h_{f_3} \quad \text{...}(iii)$$

From continuity equation,
Discharge through AD = Discharge through DB + Discharge through DC

$$\therefore \quad \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 \times V_2 + \frac{\pi}{4} d_3^2 V_3$$

or

$$d_1^2 V_1 = d_2^2 V_2 + d_3^2 V_3 \quad \text{...}(iv)$$

There are four unknowns *i.e.*, V_1, V_2, V_3 and $\frac{p_D}{\rho g}$ and there are four equations (*i*), (*ii*), (*iii*) and (*iv*).

Hence unknown can be calculated.

Power Transmission through Pipes

Power is transmitted through pipes by flowing water or other liquids flowing through them. The power transmitted depends upon (*i*) the weight of liquid flowing through the pipe and (*ii*) the total head available at the end of the pipe. Consider a pipe AB connected to a tank as shown in Fig. The power available at the end B of the pipe and the condition for maximum transmission of power will be obtained as mentioned below :

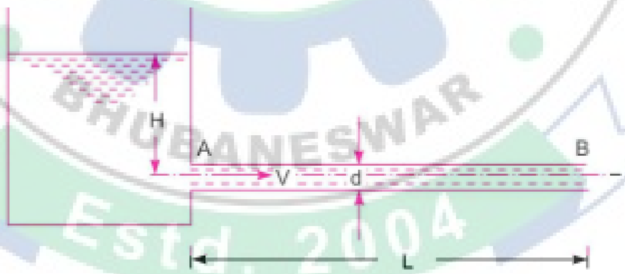


Fig.26.Power transmission through pipes

Let L = length of the pipe,
d = diameter of the pipe,
H = total head available at the inlet of pipe,
V = velocity of flow in pipe,
h_f = loss of head due to friction, and f = co-efficient of friction.

The head available at the outlet of the pipe, if minor losses are neglected

= Total head at inlet − loss of head due to friction

$$= H - h_f = H - \frac{4f \times L \times V^2}{d \times 2g} \qquad \left\{\because\ h_f = \frac{4f \times L \times V^2}{d \times 2g}\right\}$$

Weight of water flowing through pipe per sec,

$$W = \rho g \times \text{volume of water per sec} = \rho g \times \text{Area} \times \text{Velocity}$$

$$= \rho g \times \frac{\pi}{4} d^2 \times V$$

∴ The power transmitted at the outlet of the pipe

= weight of water per sec × head at outlet

$$= \left(\rho g \times \frac{\pi}{4} d^2 \times V\right) \times \left(H - \frac{4f \times L \times V^2}{d \times 2g}\right) \text{ Watts}$$

∴ Power transmitted at outlet of the pipe,

$$P = \frac{\rho g}{1000} \times \frac{\pi}{4} d^2 \times V \left(H - \frac{4fLV^2}{d \times 2g}\right) \text{ kW}$$

Efficiency of power transmission,

$$\eta = \frac{\text{Power available at outlet of the pipe}}{\text{Power supplied at the inlet of the pipe}}$$

$$= \frac{\text{Weight of water per sec} \times \text{Head available at outlet}}{\text{Weight of water per sec} \times \text{Head at inlet}}$$

$$= \frac{W \times (H - h_f)}{W \times H} = \frac{H - h_f}{H}.$$

Flow through Nozzle:

Fig. shows a nozzle fitted at the end of a long pipe. The total energy at the end of the pipe consists of pressure energy and kinetic energy. By fitting the nozzle at the end of the pipe, the total energy is converted into kinetic energy. Thus nozzles are used, where higher velocities of flow are required. The examples are :

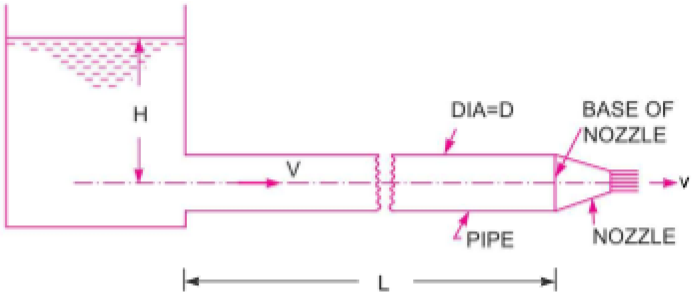


Fig.27.Flow through Nozzle

1. In case of Pelton turbine, the nozzle is fitted at the end of the pipe (called penstock) to increase velocity.
2. In case of the extinguishing fire, a nozzle is fitted at the end of the hose pipe to increase velocity.

Let D = diameter of the pipe, L = length of the pipe,

A = area of the pipe = $\frac{\pi}{4} D^2$,

V = velocity of flow in pipe,

H = total head at the inlet of the pipe,

d = diameter of nozzle at outlet,

v = velocity of flow at outlet of nozzle,

a = area of the nozzle at outlet = $\frac{\pi}{4} d^2$,

f = co-efficient of friction for pipe.

Loss of head due to friction in pipe, $h_f = \frac{4fLV^2}{2g \times D}$

$\therefore$ Head available at the end of the pipe or at the base of nozzle

$$= \text{Head at inlet of pipe} - \text{head lost due to friction}$$

$$= H - h_f = \left(H - \frac{4fLV^2}{2g \times D}\right)$$

Neglecting minor losses and also assuming losses in the nozzle negligible, we have

Total head at inlet of pipe = total head (energy) at the outlet of nozzle + losses

But total head at outlet of nozzle = kinetic head = $\frac{v^2}{2g}$

$$\therefore \quad H = \frac{v^2}{2g} + h_f = \frac{v^2}{2g} + \frac{4fLV^2}{2gD} \quad \left(\because h_f = \frac{4fLV^2}{2gD}\right) \quad ...(i)$$

From continuity equation in the pipe and outlet of nozzle,

$$AV = av$$

$$\therefore \quad V = \frac{av}{A}$$

Substituting this value in equation (*i*), we get

$$H = \frac{v^2}{2g} + \frac{4fL}{2gD} \times \left(\frac{av}{A}\right)^2 = \frac{v^2}{2g} + \frac{4fLa^2v^2}{2g \times D \times A^2} = \frac{v^2}{2g}\left(1 + \frac{4fLa^2}{DA^2}\right)$$

$$\therefore \quad v = \sqrt{\frac{2gH}{\left(1 + \frac{4fL}{D} \times \frac{a^2}{A^2}\right)}}$$

$\therefore$ Discharge through nozzle = $a \times v$.

Water Hammer in Pipes:

Consider a long pipe *AB* as shown in Fig. connected at one end to a tank containing water at a height of *H* from the centre of the pipe. At the other end of the pipe, a valve to regulate the flow of water is provided. When the valve is completely open, the water is flowing with a velocity, *V* in the pipe. If now the valve is suddenly closed, the momentum of the flowing water will be destroyed and consequently a wave of high pressure will be set up. This wave of high pressure will be transmitted along the pipe with a velocity equal to the velocity of sound wave and may create noise called knocking. Also this wave of high pressure has the effect of hammering action on the walls of the pipe and hence it is also known as water hammer.

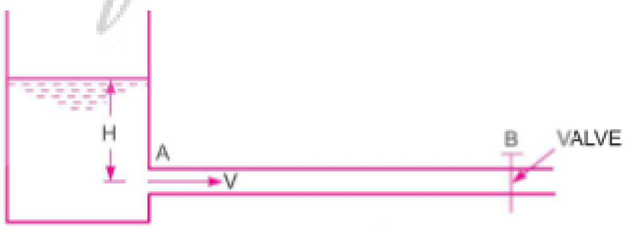


Fig.28.Water Hammer

The pressure rise due to water hammer depends upon : (*i*) the velocity of flow of water in pipe, (*ii*) the length of pipe, (*iii*) time taken to close the valve, (*iv*) elastic properties of the material of the pipe. The following cases of water hammer in pipes will be considered :

1. Gradual closure of valve,
2. Sudden closure of valve and considering pipe rigid, and

Practice Problems:

Problem .1 *The head of water over an orifice of diameter 40 mm is 10 m. Find the actual discharge and actual velocity of the jet at vena-contracta. Take $C_d = 0.6$ and $C_v = 0.98$.*

Solution. Given :

Head, $H = 10$ cm

Dia. of orifice, $d = 40$ mm $= 0.04$ m

∴ Area, $a = \frac{\pi}{4}(.04)^2 = .001256\ \text{m}^2$

$C_d = 0.6$

$C_v = 0.98$

(*i*) $\dfrac{\text{Actual discharge}}{\text{Theoretical discharge}} = 0.6$

But Theoretical discharge $= V_{th} \times$ Area of orifice

V_{th} = Theoretical velocity, where $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10}$ =14 m/s

∴ Theoretical discharge $= 14 \times .001256 = 0.01758\ \frac{\text{m}^2}{\text{s}}$

∴ Actual discharge $= 0.6 \times$ Theoretical discharge

$= 0.6 \times .01758 = \mathbf{0.01054\ m^3/s.}$ **Ans.**

(*ii*) $\dfrac{\text{Actual velocity}}{\text{Theoretical velocity}} = C_v = 0.98$

∴ Actual velocity $= 0.98 \times$ Theoretical velocity

$= 0.98 \times 14 = \mathbf{13.72\ m/s.}$ **Ans.**

Problem .2 *The head of water over the centre of an orifice of diameter 20 mm is 1 m. The actual discharge through the orifice is 0.85 litre/s. Find the co-efficient of discharge.*

Solution. Given :

Dia. of orifice, $d = 20$ mm $= 0.02$ m

∴ Area, $a = \frac{\pi}{4}(0.02)^2 = 0.000314\ \text{m}^2$

Head, $H = 1$ m

Actual discharge, $Q = 0.85$ litre/s $= 0.00085\ \text{m}^3/\text{s}$

Theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 1} = 4.429$ m/s

∴ Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice

$= 4.429 \times 0.000314 = 0.00139\ \text{m}^3/\text{s}$

∴ Co-efficient of discharge $= \dfrac{\text{Actual discharge}}{\text{Theoretical discharge}} = \dfrac{0.00085}{0.00139} = \mathbf{0.61.}$ **Ans.**

Problem .3 *A jet of water, issuing from a sharp-edged vertical orifice under a constant head of 10.0 cm, at a certain point, has the horizontal and vertical co-ordinates measured from the vena-contracta as 20.0 cm and 10.5 cm respectively. Find the value of C_v. Also find the value of C_c if $C_d = 0.60$.*

Solution. Given :

Head, $H = 10.0$ cm

Horizontal distance, $x = 20.0$ cm

Vertical distance, $y = 10.5$ cm

$C_d = 0.6$

The value of C_v is given by equation (7.6) as

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{20.0}{\sqrt{4 \times 10.5 \times 10.0}} = \frac{20}{20.493} = 0.9759 = \mathbf{0.976.\ Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.6}{0.976} = 0.6147 = \mathbf{0.615.\ Ans.}$$

Problem .4 *The head of water over an orifice of diameter 100 mm is 10 m. The water coming out from orifice is collected in a circular tank of diameter 1.5 m. The rise of water level in this tank is 1.0 m in 25 seconds. Also the co-ordinates of a point on the jet, measured from vena-contracta are 4.3 m horizontal and 0.5 m vertical. Find the co-efficients, C_d, C_v and C_c.*

Solution. Given :

Head, $H = 10$ m

Dia. of orifice, $d = 100$ mm $= 0.1$ m

$\therefore$ Area of orifice, $a = \frac{\pi}{4}(.1)^2 = 0.007853\ m^2$

Dia. of measuring tank, $D = 1.5$ m

$\therefore$ Area, $A = \frac{\pi}{4}(1.5)^2 = 1.767\ m^2$

Rise of water, $h = 1$ m

Time, $t = 25$ seconds

Horizontal distance, $x = 4.3$ m

Vertical distance, $y = 0.5$ m

Now theoretical velocity, $V_{th} = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 10} = 14.0$ m/s

$\therefore$ Theoretical discharge, $Q_{th} = V_{th} \times$ Area of orifice $= 14.0 \times 0.007854 = 0.1099\ m^3/s$

Actual discharge, $Q = \frac{A \times h}{t} = \frac{1.767 \times 1.0}{25} = 0.07068$

$\therefore$ $C_d = \frac{Q}{Q_{th}} = \frac{0.07068}{0.1099} = \mathbf{0.643.\ Ans.}$

The value of C_v is given by equation (7.6) as

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.3}{\sqrt{4 \times 0.5 \times 10}} = \frac{4.3}{4.472} = \mathbf{0.96.\ Ans.}$$

C_c is given by equation (7.7) as $C_c = \frac{C_d}{C_v} = \frac{0.643}{0.96} = \mathbf{0.669.\ Ans.}$

Problem .5 *Water discharge at the rate of 98.2 litres/s through a 120 mm diameter vertical sharp-edged orifice placed under a constant head of 10 metres. A point, on the jet, measured from the vena-contracta of the jet has co-ordinates 4.5 metres horizontal and 0.54 metres vertical. Find the co-efficient C_v, C_c and C_d of the orifice.*

Solution. Given :

Discharge, $Q = 98.2 \text{ lit/s} = 0.0982 \text{ m}^3\text{/s}$

Dia. of orifice, $d = 120 \text{ mm} = 0.12 \text{ m}$

$\therefore$ Area of orifice, $a = \frac{\pi}{4}(0.12)^2 = 0.01131 \text{ m}^2$

Head, $H = 10 \text{ m}$

Horizontal distance of a point on the jet from vena-contracta, $x = 4.5$ m
and vertical distance, $y = 0.54$ m

Now theoretical velocity, $V_{th} = \sqrt{2g \times H} = \sqrt{2 \times 9.81 \times 10} = 14.0 \text{ m/s}$

Theoretical discharge, $Q_{th} = V_{th} \times \text{Area of orifice}$
$= 14.0 \times 0.01131 = 0.1583 \text{ m}^3\text{/s}$

The value of C_d is given by, $C_d = \frac{\text{Actual discharge}}{\text{Theoretical discharge}} = \frac{Q}{Q_{th}} = \frac{0.0982}{0.1583} = $ **0.62. Ans.**

The value of C_c is given by equation (7.6),

$$C_v = \frac{x}{\sqrt{4yH}} = \frac{4.5}{\sqrt{4 \times 0.54 \times 10}} = \mathbf{0.968.}\ \textbf{Ans.}$$

The value of C_c is given by equation (7.7) as

$$C_c = \frac{C_d}{C_v} = \frac{0.62}{0.968} = \mathbf{0.64.}\ \textbf{Ans.}$$

Problem *Find the discharge through a rectangular orifice 2.0 m wide and 1.5 m deep fitted to a water tank. The water level in the tank is 3.0 m above the top edge of the orifice. Take $C_d = 0.62$.*

Solution. Given :

Width of orifice, $b = 2.0$ m

Depth of orifice, $d = 1.5$ m

Height of water above top edge of the orifice, $H_1 = 3$ m

Height of water above bottom edge of the orifice,

$$H_2 = H_1 + d = 3 + 1.5 = 4.5 \text{ m}$$

$$C_d = 0.62$$

Discharge Q is given by equation (7.8) as

$$Q = \frac{2}{3} C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.62 \times 2.0 \times \sqrt{2 + 9.81}\,[4.5^{1.5} - 3^{1.5}] \text{ m}^3\text{/s}$$

$$= 3.66[9.545 - 5.196] \text{ m}^3\text{/s} = \mathbf{15.917\ m^3/s.}\ \textbf{Ans.}$$

Problem *A rectangular orifice, 1.5 m wide and 1.0 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take the co-efficient of discharging for the orifice = 0.6.*

Solution. Given :

Width of orifice, $b = 1.5$ m

Depth of orifice, $d = 1.0$ m

$H_1 = 3.0$ m

$H_2 = H_1 + d = 3.0 + 1.0 = 4.0$ m

$C_d = 0.6$

Discharge, Q is given by the equation (7.8) as

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 1.5 \times \sqrt{2 + 9.81}\ [4.0^{1.5} - 3.0^{1.5}] \text{ m}^3\text{/s}$$

$$= 2.657\ [8.0 - 5.196] \text{ m}^3\text{/s} = \mathbf{7.45\ m^3/s.\ Ans.}$$

Problem *A rectangular orifice, 1.5 m wide and 1.0 m deep is discharging water from a tank. If the water level in the tank is 3.0 m above the top edge of the orifice, find the discharge through the orifice. Take the co-efficient of discharging for the orifice = 0.6.*

Solution. Given :

Width of orifice, $b = 1.5$ m

Depth of orifice, $d = 1.0$ m

$H_1 = 3.0$ m

$H_2 = H_1 + d = 3.0 + 1.0 = 4.0$ m

$C_d = 0.6$

Discharge, Q is given by the equation (7.8) as

$$Q = \frac{2}{3} \times C_d \times b \times \sqrt{2g}\ [H_2^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.6 \times 1.5 \times \sqrt{2+9.81}\ [4.0^{1.5} - 3.0^{1.5}] \text{ m}^3\text{/s}$$

$$= 2.657\ [8.0 - 5.196] \text{ m}^3\text{/s} = \mathbf{7.45\ m^3/s.\ Ans.}$$

Problem *Find the discharge through a fully sub-merged orifice of width 2 m if the difference of water levels on both sides of the orifice be 50 cm. The height of water from top and bottom of the orifice are 2.5 m and 2.75 m respectively. Take $C_d = 0.6$.*

Solution. Given :

Width of orifice, $b = 2$ m

Difference of water level, $H = 50$ cm $= 0.5$ m

Height of water from top of orifice, $H_1 = 2.5$ m

Height of water from bottom of orifice, $H_2 = 2.5$ m

$C_d = 0.6$

Discharge through fully sub-merged orifice is given by equation (7.9)

or $$Q = C_d \times b \times (H_2 - H_1) \times \sqrt{2gH}$$

$$= 0.6 \times 2.0 \times (2.75 - 2.5) \times \sqrt{2 \times 9.81 \times 0.5} \text{ m}^3\text{/s}$$

$$= \mathbf{0.9396\ m^3/s.\ Ans.}$$

Problem *Find the discharge through a totally drowned orifice 2.0 m wide and 1 m deep, if the difference of water levels on both the sides of the orifice be 3 m. Take $C_d = 0.62$.*

Solution. Given :

Width of orifice, $b = 2.0$ m

Depth of orifice, $d = 1$ m.

Difference of water level on both the sides

$H = 3$ m

$C_d = 0.62$

Discharge through orifice is $Q = C_d \times \text{Area} \times \sqrt{2gH}$

$$= 0.62 \times b \times d \times \sqrt{2gH}$$

$$= 0.62 \times 2.0 \times 1.0 \times \sqrt{2 \times 9.81 \times 3} \text{ m}^3\text{/s} = \mathbf{9.513\ m^3/s.\ Ans.}$$

Problem *A rectangular orifice of 2 m width and 1.2 m deep is fitted in one side of a large tank. The water level on one side of the orifice is 3 m above the top edge of the orifice, while on the other side of the orifice, the water level is 0.5 m below its top edge. Calculate the discharge through the orifice if $C_d = 0.64$.*

Solution. Given : Width of orifice, $b = 2$ m

Depth of orifice, $d = 1.2$ m

Height of water from top edge of orifice, $H_1 = 3$ m

Difference of water level on both sides, $H = 3 + 0.5 = 3.5$ m

Height of water from the bottom edge of orifice, $H_2 = H_1 + d = 3 + 1.2 = 4.2$ m

The orifice is partially sub-merged. The discharge through sub-merged portion,

$$Q_1 = C_d \times b \times (H_2 - H) \times \sqrt{2gH}$$

$$= 0.64 \times 2.0 \times (4.2 - 3.5) \times \sqrt{2 \times 9.81 \times 3.5} = 7.4249 \text{ m}^3/\text{s}$$

The discharge through free portion is

$$Q_2 = \frac{2}{3} C_d \times b \times \sqrt{2g}\,[H^{3/2} - H_1^{3/2}]$$

$$= \frac{2}{3} \times 0.64 \times 2.0 \times \sqrt{2 \times 9.81}\ [3.5^{3/2} - 3.0^{3/2}]$$

$$= 3.779\,[6.5479 - 5.1961] = 5.108 \text{ m}^3/\text{s}$$

∴ Total discharge through the orifice is

$$Q = Q_1 + Q_2 = 7.4249 + 5.108 = \mathbf{12.5329\ m^3/s.\ Ans.}$$

Problem *A circular tank of diameter 4 m contains water upto a height of 5 m. The tank is provided with an orifice of diameter 0.5 m at the bottom. Find the time taken by water (i) to fall from 5 m to 2 m (ii) for completely emptying the tank. Take $C_d = 0.6$.*

Solution. Given :

Dia. of tank, $D = 4$ m

∴ Area, $A = \frac{\pi}{4}(4)^2 = 12.566 \text{ m}^2$

Dia. of orifice, $d = 0.5$ m

∴ Area, $a = \frac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

Initial height of water, $H_1 = 5$ m

Final height of water, (*i*) $H_2 = 2$ m (*ii*) $H_2 = 0$

First Case. When $H_2 = 2$ m

Using equation we have $T = \frac{2A}{C_d \,.\, a \,.\, \sqrt{2g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$

$$= \frac{2 \times 12.566}{0.6 \times .1963 \times \sqrt{2 \times 9.81}}\left[\sqrt{5} - \sqrt{2.0}\right] \text{ seconds}$$

$$= \frac{20.653}{0.5217} = \mathbf{39.58\ seconds.\ Ans.}$$

Second Case. When $H_2 = 0$

$$T = \frac{2A}{C_d \,.\, a \,.\, \sqrt{2g}}\sqrt{H_1} = \frac{2 \times 12.566 \times \sqrt{5}}{0.6 \times .1963 \times \sqrt{2 \times 9.81}}$$

$$= \mathbf{107.7\ seconds.\ Ans.}$$

Problem *A hemispherical tank of diameter 4 m contains water upto a height of 1.5 m. An orifice of diameter 50 mm is provided at the bottom. Find the time required by water (i) to fall from 1.5 m to 1.0 m (ii) for completely emptying the tank. Tank C_d = 0.6.*

Solution. Given :

Dia. of hemispherical tank, $D = 4$ m

$\therefore$ Radius, $R = 2.0$ m

Dia. of orifice, $d = 50$ mm $= 0.05$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.05)^2 = 0.001963 \text{ m}^2$

Initial height of water, $H_1 = 1.5$ m

$C_d = 0.6$

First Case. $H_2 = 1.0$

Time T is given by equation

$$\therefore \quad T = \frac{\pi}{C_d \times a \times \sqrt{2g}}\left[\frac{4}{3}R\left(H_1^{3/2} - H_2^{3/2}\right) - \frac{2}{5}\left(H_1^{5/2} - H_2^{5/2}\right)\right]$$

$$= \frac{\pi}{0.6 \times .001963 \times \sqrt{2 \times 9.81}} \times \left[\frac{4}{3} \times 2.0\left(1.5^{3/2} - 1.0^{3/2}\right) - \frac{2}{5}\left(1.5^{5/2} - 1.0^{5/2}\right)\right]$$

$= 602.189$ [2.2323 − 0.7022] $= 921.4$ second

= 15 min 21.4 sec. Ans.

Second Case. $H_2 = 0$ and hence time T is given by equation

$$\therefore \quad T = \frac{\pi}{C_d \,.\, a \,.\, \sqrt{2g}}\left[\frac{4}{3}RH_1^{3/2} - \frac{2}{5}H_1^{5/2}\right]$$

$$= \frac{\pi}{0.6 \times .001963\sqrt{2 \times 9.81}}\left[\frac{4}{3} \times 2.0 \times 1.5^{3/2} - \frac{2}{5} \times 1.5^{5/2}\right]$$

$= 602.189$ [4.8989 − 1.1022] sec $= 2286.33$ sec

= 38 min 6.33 sec. Ans.

Problem *An orifice of diameter 150 mm is fitted at the bottom of a boiler drum of length 8 m and of diameter 3 metres. The drum is horizontal and contains water upto a height of 2.4 m. Find the time required to empty the boiler. Take C_d = 0.6.*

Solution. Given :

Dia. of orifice, $d = 150$ mm $= 0.15$ m

$\therefore$ Area, $a = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Length, $L = 8.0$ m

Dia. of boiler, $D = 3.0$ m

$\therefore$ Radius, $R = 1.5$ m

Initial height of water, $H_1 = 2.4$ m

Find height of water, $H_2 = 0$

$C_d = 0.6$.

For completely emptying the tank, T is given by equation

$$T = \frac{4L}{3C_d \times a \times \sqrt{2g}} [(2R)^{3/2} - (2R - H_1)^{3/2}]$$

$$= \frac{4 \times 8.0}{3 \times .6 \times .01767 \times \sqrt{2 \times 9.81}} [(2 \times 1.5)^{3/2} - (2 \times 1.5 - 2.4)^{3/2}]$$

$$= 227.14\ [5.196 - 0.4647] = 1074.66 \text{ sec}$$

$$= \mathbf{17\ min\ 54.66\ sec.\ Ans.}$$

Problem *Find the discharge from a 100 mm diameter external mouthpiece, fitted to a side of a large vessel if the head over the mouthpiece is 4 metres.*

Solution. Given :

Dia. of mouthpiece = 100 m = 0.1 m

$\therefore$ Area, $a = \frac{\pi}{4}(0.1)^2 = 0.007854 \text{ m}^2$

Head, $H = 4.0$ m

C_d for mouthpiece $= 0.855$

$\therefore$ Discharge $= C_d \times \text{Area} \times \text{Velocity} = 0.855 \times a \times \sqrt{2gH}$

$= .855 \times .007854 \times \sqrt{2 \times 9.81 \times 4.0} =$ **.05948 m³/s. Ans.**

Problem *A convergent-divergent mouthpiece having throat diameter of 4.0 cm is discharging water under a constant head of 2.0 m, determine the maximum outer diameter for maximum discharge. Find maximum discharge also. Take H_a = 10.3 m of water and H_{sep} = 2.5 m of water (absolute).*

Solution. Given :

Dia. of throat, $d_c = 4.0$ cm

$\therefore$ Area, $a_c = \frac{\pi}{4}(4)^2 = 12.566 \text{ cm}^2$

Constant head, $H = 2.0$ m

Find max. dia. at outlet, d_1 and Q_{max}

$H_a = 10.3$ m

$H_{sep} = 2.5$ m (absolute)

The discharge, Q in convergent-divergent mouthpiece depends on the area at throat.

$\therefore$ $Q_{max} = a_c \times \sqrt{2gH} = 12.566 \times \sqrt{2 \times 9.81 \times 200} =$ **7871.5 cm³/s. Ans.**

Now ratio of areas at outlet and throat is given by equation

$$\frac{a_1}{a_c} = \sqrt{1 + \frac{H_a - H_c}{H}} = \sqrt{1 + \frac{10.3 - 2.5}{2.0}} \quad \{\because H_c = H_{sep} = 2.5\}$$

$$= 2.2135$$

$$\frac{\pi}{4} d_1^2 \Big/ \frac{\pi}{4} d_c^2 = 2.2135 \text{ or } \left(\frac{d_1}{d_c}\right)^2 = 2.2135$$

$\therefore$ $\frac{d_1}{d_c} = \sqrt{2.2135} = 1.4877$

$\therefore$ $d_1 = 1.4877 \times d_c = 1.4877 \times 4.0 =$ **5.95 cm. Ans.**

Problem *Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.*

Solution. Given :

Length of the notch, $L = 2.0$ m

Head over notch, $H = 300 \text{ m} = 0.30 \text{ m}$

$C_d = 0.60$

Discharge, $$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\left[H^{3/2}\right]$$

$$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times [0.30]^{1.5} \text{ m}^3/\text{s}$$

$$= 3.5435 \times 0.1643 = \mathbf{0.582\ m^3/s.\ Ans.}$$

Problem *Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 litres/s. Take $C_d = 0.6$ and neglect end contractions.*

Solution. Given :

Length of weir, $L = 6$ m

Depth of water, $H_1 = 1.8$ m

Discharge, $Q = 2000 \text{ lit/s} = 2 \text{ m}^3/\text{s}$

$C_d = 0.6$

Let H is height of water above the crest of weir, and H_2 = height of weir

The discharge over the weir is given by the equation

$$Q = \frac{2}{3} C_d \times L \times \sqrt{2g}\ H^{3/2}$$

or $$2.0 = \frac{2}{3} \times 0.6 \times 6.0 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$= 10.623\ H^{3/2}$$

$$\therefore \quad H^{3/2} = \frac{2.0}{10.623}$$

$$\therefore \quad H = \left(\frac{2.0}{10.623}\right)^{2/3} = 0.328 \text{ m}$$

$\therefore$ Height of weir, $H_2 = H_1 - H$

= Depth of water on upstream side $- H$

$= 1.8 - .328 =$ **1.472 m. Ans.**

Problem *Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m. Assume $C_d = 0.6$.*

Solution. Given :

Angle of V-notch, $\theta = 60°$

Head over notch, $H = 0.3$ m

$C_d = 0.6$

Discharge, Q over a V-notch is given by equation

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \tan\frac{60^\circ}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.8182 \times 0.0493 = \mathbf{0.040\ m^3/s.\ Ans.}$$

Problem *Water flows over a rectangular weir 1 m wide at a depth of 150 mm and afterwards passes through a triangular right-angled weir. Taking C_d for the rectangular and triangular weir as 0.62 and 0.59 respectively, find the depth over the triangular weir.*

Solution. Given :

For rectangular weir, length, $L = 1$ m

Depth of water, $H = 150$ mm $= 0.15$ m

$C_d = 0.62$

For triangular weir, $\theta = 90°$

$C_d = 0.59$

Let depth over triangular weir $= H_1$

The discharge over the rectangular weir is given by equation

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.62 \times 1.0 \times \sqrt{2 \times 9.81} \times (.15)^{3/2} \text{ m}^3\text{/s} = 0.10635 \text{ m}^3\text{/s}$$

The same discharge passes through the triangular right-angled weir. But discharge, Q, is given by equation for a triangular weir as

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$\therefore \quad 0.10635 = \frac{8}{15} \times .59 \times \tan\frac{90°}{2} \times \sqrt{2g} \times H_1^{5/2} \qquad \{\because \theta = 90° \text{ and } H = H_1\}$$

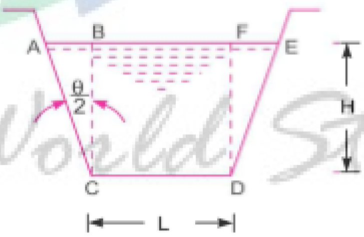
$$= \frac{8}{15} \times .59 \times 1 \times 4.429 \times H_1^{5/2} = 1.3936\, H_1^{5/2}$$

$$\therefore \quad H_1^{5/2} = \frac{0.10635}{1.3936} = 0.07631$$

$$\therefore \quad H_1 = (.07631)^{0.4} = \mathbf{0.3572 \text{ m. Ans.}}$$

Problem *Find the discharge through a trapezoidal notch which is 1 m wide at the top and 0.40 m at the bottom and is 30 cm in height. The head of water on the notch is 20 cm. Assume C_d for rectangular portion = 0.62 while for triangular portion = 0.60.*

Solution. Given :



Top width, $AE = 1$ m

Base width, $CD = L = 0.4$ m

Head of water, $H = 0.20$ m

For rectangular portion, $C_{d_1} = 0.62$

For triangular portion, $C_{d_2} = 0.60$

From ΔABC, we have

$$\tan\frac{\theta}{2} = \frac{AB}{BC} = \frac{(AE - CD)/2}{H}$$

$$= \frac{(1.0 - 0.4)/2}{0.3} = \frac{0.6/2}{0.3} = \frac{0.3}{0.3} = 1$$

Discharge through trapezoidal notch is given by equation

$$Q = \frac{2}{3} C_{d_1} \times L \times \sqrt{2g} \times H^{3/2} + \frac{8}{15} C_{d_2} \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{2}{3} \times 0.62 \times 0.4 \times \sqrt{2 \times 9.81} \times (0.2)^{3/2} + \frac{8}{15} \times .60 \times 1 \times \sqrt{2 \times 9.81} \times (0.2)^{5/2}$$

$$= 0.06549 + 0.02535 = 0.09084 \text{ m}^3\text{/s} = \mathbf{90.84 \text{ litres/s. Ans.}}$$

Problem *(a) A broad-crested weir of 50 m length, has 50 cm height of water above its crest. Find the maximum discharge. Take $C_d = 0.60$. Neglect velocity of approach. (b) If the velocity of approach is to be taken into consideration, find the maximum discharge when the channel has a cross-sectional area of 50 m^2 on the upstream side.*

Solution. Given :

Length of weir, $L = 50$ m

Head of water, $H = 50$ cm $= 0.5$ m

$C_d = 0.60$

(*i*) **Neglecting velocity of approach.** Maximum discharge is given by equation

$$Q_{max} = 1.705 \times C_d \times L \times H^{3/2}$$

$$= 1.705 \times 0.60 \times 50 \times (.5)^{3/2} = \mathbf{18.084 \text{ m}^3\text{/s. Ans.}}$$

(*ii*) **Taking velocity of approach into consideration**

Area of channel, $A = 50 \text{ m}^2$

Velocity of approach, $V_a = \frac{Q}{A} = \frac{18.084}{50} = 0.36$ m/s

$\therefore$ Head due to V_a, $h_a = \frac{V_a^2}{2g} = \frac{0.36 \times .36}{2 \times 9.81} = .0066$ m

Maximum discharge, Q_{max} is given by

$$Q_{max} = 1.705 \times C_d \times L \times [(H + h_a)^{3/2} - h_a^{3/2}]$$

$$= 1.705 \times 0.6 \times 50 \times [(.50 + .0066)^{1.5} - (.0066)^{1.5}]$$

$$= 51.15[0.3605 - .000536] = \mathbf{18.412 \text{ m}^3\text{/s. Ans.}}$$

Problem *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.*

Take ν for water = 0.01 stoke.

Solution. Given :

Dia. of pipe, $d = 300$ mm $= 0.30$ m

Length of pipe, $L = 50$ m

Velocity of flow, $V = 3$ m/s

Chezy's constant, $C = 60$

Kinematic viscosity, $\nu = 0.01$ stoke $= 0.01$ cm²/s

$= 0.01 \times 10^{-4}$ m²/s.

(*i*) **Darcy Formula** is given by equation

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by $$R_e = \frac{V \times d}{\nu} = \frac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$$

$\therefore$ Value of $$f = \frac{0.079}{R_e^{1/4}} = \frac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$$

$\therefore$ Head lost, $$h_f = \frac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81} = \mathbf{.7828\ m.\ Ans.}$$

(*ii*) **Chezy's Formula.** Using equation

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \frac{d}{4} = \frac{0.30}{4} = 0.075$ m

$$\therefore \quad 3 = 60\sqrt{.075 \times i} \text{ or } i = \left(\frac{3}{60}\right)^2 \times \frac{1}{.075} = 0.0333$$

But $$i = \frac{h_f}{L} = \frac{h_f}{50}$$

Equating the two values of i, we have $\frac{h_f}{50} = .0333$

$$\therefore \quad h_f = 50 \times .0333 = \mathbf{1.665\ m.\ Ans.}$$

Problem *Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of C = 50 in Chezy's formulae.*

Solution. Given :

Length of pipe, $L = 2000$ m

Discharge, $Q = 200$ litre/s $= 0.2\ m^3/s$

Head lost due to friction, $h_f = 4$ m

Value of Chezy's constant, $C = 50$

Let the diameter of pipe $= d$

Velocity of flow, $$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $$m = \frac{d}{4}$$

Loss of head per unit length, $$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

Chezy's formula is given by equation as $V = C\sqrt{mi}$

Substituting the values of V, m, i and C, we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times .002} \text{ or } \sqrt{\frac{d}{4} \times .002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{.00509}{d^2}$$

Squaring both sides, $$\frac{d}{4} \times .002 = \frac{.00509^2}{d^4} = \frac{.0000259}{d^4} \text{ or } d^5 = \frac{4 \times .0000259}{.002} = 0.0518$$

$$\therefore \quad d = \sqrt[5]{0.0518} = (.0518)^{1/5} = 0.553 \text{ m} = \mathbf{553\ mm.\ Ans.}$$

Problem *An oil of sp. gr. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take* $\nu = .29$ *stokes.*

Solution. Given :

Sp. gr. of oil, $S = 0.7$

Dia. of pipe, $d = 300$ mm $= 0.3$ m

Discharge, $Q = 500$ litres/s $= 0.5\ m^3/s$

Length of pipe, $L = 1000$ m

Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0.5}{\frac{\pi}{4}d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073 \text{ m/s}$$

$\therefore$ Reynolds number, $$R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times (10)^4$$

$\therefore$ Co-efficient of friction, $$f = \frac{.079}{R_e^{1/4}} = \frac{0.79}{\left(7.316 \times 10^4\right)^{1/4}} = .0048$$

$\therefore$ Head lost due to friction, $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18 \text{ m}$$

Power required $$= \frac{\rho g \,.\, Q \,.\, h_f}{1000} \text{ kW}$$

where ρ = density of oil $= 0.7 \times 1000 = 700\ kg/m^3$

$\therefore$ Power required $$= \frac{700 \times 9.81 \times 0.5 \times 163.18}{1000} = \mathbf{560.28\ kW.\ Ans.}$$

Problem *Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.*

Solution. Given :

Dia. of smaller pipe, $D_1 = 200$ mm = 0.20 m

∴ Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400$ mm = 0.4 m

∴ Area, $A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$

Discharge, $Q = 250$ litres/s = 0.25 m^3/s

Velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96$ m/s

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99$ m/s

Loss of head due to enlargement is given by equation

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \mathbf{1.816 \text{ m of water}}$$ **Ans.**

Problem *The rate of flow of water through a horizontal pipe is 0.25 m^3/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm^2. Determine :*

(*i*) *loss of head due to sudden enlargement,* (*ii*) *pressure intensity in the large pipe,*
(*iii*) *power lost due to enlargement.*

Solution. Given :

Discharge, $Q = 0.25 \text{ m}^3/\text{s}$

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4}(.2)^2 = 0.03141 \text{ m}^2$

Dia. of large pipe, $D_2 = 400 \text{ mm} = 0.40 \text{ m}$

∴ Area, $A_2 = \frac{\pi}{4}(0.4)^2 = 0.12566 \text{ m}^2$

Pressure in smaller pipe, $p_1 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

Now velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12566} = 1.99 \text{ m/s}$

(*i*) Loss of head due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \textbf{1.816 m. Ans.}$$

(*ii*) Let the pressure intensity in large pipe = p_2.

Then applying Bernoulli's equation before and after the sudden enlargement,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$ (Given horizontal pipe)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e \text{ or } \frac{p_2}{\rho g} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178 = 13.21 \text{ m of water}$$

$$\therefore \quad p_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= 13.21 \times 1000 \times 9.81 \times 10^{-4} \text{ N/cm}^2 = \textbf{12.96 N/cm}^2\textbf{. Ans.}$$

(*iii*) Power lost due to sudden enlargement,

$$P = \frac{\rho g \cdot Q \cdot h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000} = \textbf{4.453 kW. Ans.}$$

Problem *A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm² and 11.772 N/cm² respectively. Find the loss of head due to contraction if C_c = 0.62. Also determine the rate of flow of water.*

Solution. Given :

Dia. of large pipe, $D_1 = 500 \text{ mm} = 0.5 \text{ m}$

Area, $A_1 = \frac{\pi}{4}(0.5)^2 = 0.1963 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 250 \text{ mm} = 0.25 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4}(.25)^2 = 0.04908 \text{ m}^2$

Pressure in large pipe, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

Pressure in smaller pipe, $p_2 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

$C_c = 0.62$

Head lost due to contraction $= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1.0\right]^2 = \frac{V_2^2}{2g}\left[\frac{1}{0.62} - 1.0\right]^2 = 0.375\frac{V_2^2}{2g}$

From continuity equation, we have $A_1V_1 = A_2V_2$

or
$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ (pipe is horizontal)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

But $h_c = 0.375\frac{V_2^2}{2g}$ and $V_1 = \frac{V_2}{4}$

Substituting these values in the above equation, we get

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(V_2/4)^2}{2g} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{V_2^2}{2g} + 0.375\frac{V_2^2}{2g}$$

or
$$14.0 + \frac{V_2^2}{16 \times 2g} = 12.0 + 1.375\frac{V_2^2}{2g}$$

or
$$14 - 12 = 1.375\frac{V_2^2}{2g} - \frac{1}{16}\frac{V_2^2}{2g} = 1.3125\frac{V_2^2}{2g}$$

or
$$2.0 = 1.3125 \times \frac{V_2^2}{2g} \text{ or } V_2 = \sqrt{\frac{2.0 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s.}$$

(*i*) Loss of head due to contraction, $h_c = 0.375\frac{V_2^2}{2g} = \frac{0.375 \times (5.467)^2}{2 \times 9.81}$ = **0.571 m. Ans.**

(*ii*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 5.467 = 0.2683 \text{ m}^3/\text{s}$ = **268.3 lit/s. Ans.**

Problem *A syphon of diameter 200 mm connects two reservoirs having a difference in elevation of 15 m. The total length of the syphon is 600 m and the summit is 4 m above the water level in the upper reservoir. If the separation takes place at 2.8 m of water absolute, find the maximum length of syphon from upper reservoir to the summit. Take f = .004 and atmospheric pressure = 10.3 m of water.*

Solution. Given :

Dia. of syphon, $d = 200 \text{ mm} = 0.2 \text{ m}$

Difference of level in two reservoirs $= 15 \text{ m}$

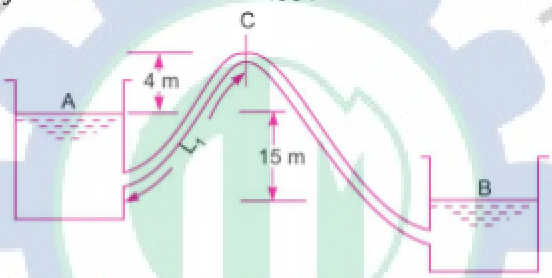
Total length of pipe $= 600 \text{ m}$

Height of summit from upper reservoir $= 4 \text{ m}$

Pressure head at summit, $\frac{p_c}{\rho g} = 2.8$ m of water absolute

Atmospheric pressure head, $\frac{p_c}{\rho g} = 10.3$ m of water absolute

Co-efficient of friction, $f = .004$



Applying Bernoulli's equation to points A and C and taking the datum line passing through, A,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c + \text{Loss of head due to friction between } A \text{ and } C$$

Substituting the values of pressures in terms of absolute, we have

$$10.3 + 0 + 0 = 2.8 + \frac{V^2}{2g} + 4.0 + h_{f_1} \qquad [\because V_c = \text{velocity in pipe} = V]$$

$$\therefore \qquad h_{f_1} = 10.3 - 2.8 - 4.0 - \frac{V^2}{2g} = 3.5 - \frac{V^2}{2g} \qquad ...(i)$$

Applying Bernoulli's equation to points A and B and taking datum line passing through B,

$$\frac{p_A}{\rho g} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + z_B + \text{Loss of head due to friction from } A \text{ to } B$$

But $\frac{p_A}{\rho g} = \frac{p_B}{\rho g}$ = atmospheric pressure

$V_A = 0, V_B = 0, z_A = 15, z_B = 0$

$$\therefore \qquad 0 + 0 + 15 = 0 + 0 + 0 + h_f$$

$$\therefore \qquad h_f = 15 \text{ or } \frac{4 \times f \times L \times V^2}{d \times 2g} = 15$$

or $$\frac{4 \times .004 \times 600 \times V^2}{0.2 \times 2 \times 9.81} = 15 \text{ or } V = \sqrt{\frac{15 \times 0.2 \times 2 \times 9.81}{4 \times .004 \times 600}} = 2.47 \text{ m/s}$$

Substituting this value of V in equation (i), we get

$$h_{f_1} = 3.5 - \frac{2.47^2}{2 \times 9.81} = 3.5 - 0.311 = 3.189 \text{ m}$$

But
$$h_{f_2} = \frac{4 \times f \times L_1 \times V^2}{d \times 2g}$$

where L_1 = inlet leg of syphon or length of syphon from upper reservoir to the summit.

$$h_{f_1} = \frac{4 \times .004 \times L_1 \times (2.47)^2}{0.2 \times 2 \times 9.81} = 0.0248 \times L_1$$

Substituting this value in equation (*ii*),

$$0.0248\ L_1 = 3.189$$

$$\therefore \quad L_1 = \frac{3.189}{.0248} = \mathbf{128.58\ m.\ Ans.}$$

Problem *The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 300 m, 170 m and 210 m and of diameters 300 mm, 200 mm and 400 mm respectively, is 12 m. Determine the rate of flow of water if co-efficient of friction are .005, .0052 and .0048 respectively, considering : (i) minor losses also (ii) neglecting minor losses.*

Solution. Given :

Difference of water level, $H = 12$ m

Length of pipe 1, $L_1 = 300$ m and dia., $d_1 = 300$ mm= 0.3 m

Length of pipe 2, $L_2 = 170$ m and dia., $d_2 = 200$ mm = 0.2 m

Length of pipe 3, $L_3 = 210$ m and dia., $d_3 = 400$ mm $= 0.4$ m

Also, $f_1 = .005$, $f_2 = .0052$ and $f_3 = .0048$

(*i*) **Considering Minor Losses.** Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$\therefore \quad V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2}V_1 = \frac{d_1^2}{d_2^2}V_1 = \left(\frac{0.3}{.2}\right)^2 \times V_1 = 2.25\ V_1$$

and
$$V_3 = \frac{A_1V_1}{A_3} = \frac{d_1^2}{d_3^2}V_1 = \left(\frac{0.3}{0.4}\right)^2 V_1 = 0.5625\ V_1$$

Now using equation (11.12), we have

$$H = \frac{0.5\,V_1^2}{2g} + \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{0.5\,V_2^2}{2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{(V_2 - V_3)^2}{2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g} + \frac{V_3^2}{2g}$$

Substituting V_2 and V_3,
$$12.0 = \frac{0.5\,V_1^2}{2g} + \frac{4 \times .005 \times 300 \times V_1^2}{0.3 \times 2g} + \frac{0.5 \times \left(2.25V_1^2\right)^2}{2g}$$

$$+\ 4 \times 0.0052 \times 170 \times \frac{(2.25\,V_1)^2}{0.2 \times 2g} + \frac{(2.25\,V_1 - .562\,V_1)^2}{2g} + \frac{4 \times .0048 \times 210 \times (.5625\,V_1)^2}{0.4 \times 2g} + \frac{(.5625\,V_1)^2}{2g}$$

$$12.0 = \frac{V_1^2}{2g}\,[0.5 + 20.0 + 2.53 + 89.505 + 2.847 + 3.189 + 0.316]$$

$$= \frac{V_1^2}{2g}\,[118.887]$$

$$\therefore \quad V_1 = \sqrt{\frac{12 \times 2 \times 9.81}{118.887}} = 1.407 \text{ m/s}$$

$\therefore$ Rate of flow, Q = Area $\times$ Velocity $= A_1 \times V_1$

$$= \frac{\pi}{4}(d_1)^2 \times V_1 = \frac{\pi}{4}(.3)^2 \times 1.407 = 0.09945 \text{ m}^3\text{/s}$$

= **99.45 litres/s. Ans.**

(*ii*) **Neglecting Minor Losses.** Using equation we have

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

or
$$12.0 = \frac{V_1^2}{2g}\left[\frac{4 \times .005 \times 300}{0.3} + \frac{4 \times .0052 \times 170 \times (2.25)^2}{0.2} + \frac{4 \times .0048 \times 210 \times (.5625)^2}{0.4}\right]$$

$$= \frac{V_1^2}{2g}\,[20.0 + 89.505 + 3.189] = \frac{V_1^2}{2g} \times 112.694$$

$$V_1 = \sqrt{\frac{2 \times 9.81 \times 12.0}{112.694}} = 1.445 \text{ m/s}$$

Discharge, $Q = V_1 \times A_1 = 1.445 \times \frac{\pi}{4}(.3)^2 = 0.1021$ m^3/s = **102.1 litres/s. Ans.**

Problem *Three pipes of lengths 800 m, 500 m and 400 m and of diameters 500 mm, 400 mm and 300 mm respectively are connected in series. These pipes are to be replaced by a single pipe of length 1700 m. Find the diameter of the single pipe.*

Solution. Given :

Length of pipe 1, L_1 = 800 m and dia., d_1 = 500 mm = 0.5 m

Length of pipe 2, L_2 = 500 m and dia., d_2 = 400 mm = 0.4 m

Length of pipe 3, L_3 = 400 m and dia., d_3 = 300 mm = 0.3 m

Length of single pipe, L = 1700 m

Let the diameter of equivalent single pipe = d

Applying equation $$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

or $$\frac{1700}{d^5} = \frac{800}{.5^5} + \frac{500}{.4^5} + \frac{400}{0.3^5} = 25600 + 48828.125 + 164609 = 239037$$

$$\therefore \quad d^5 = \frac{1700}{239037} = .007118$$

$$\therefore \quad d = (.007188)^{0.2} = 0.3718 = \mathbf{371.8\ mm.\ Ans.}$$

Problem 11.32 *A main pipe divides into two parallel pipes which again forms one pipe as shown in Fig. The length and diameter for the first parallel pipe are 2000 m and 1.0 m respectively, while the length and diameter of 2nd parallel pipe are 2000 m and 0.8 m. Find the rate of flow in each parallel pipe, if total flow in the main is 3.0 m*^{3}*/s. The co-efficient of friction for each parallel pipe is same and equal to .005.*

Solution. Given :

Length of pipe 1, $L_1 = 2000$ m
Dia. of pipe 1, $d_1 = 1.0$ m
Length of pipe 2, $L_2 = 2000$ m
Dia. of pipe 2, $d_2 = 0.8$ m
Total flow, $Q = 3.0\ m^3/s$
$f_1 = f_2 = f = .005$

Let Q_1 = discharge in pipe 1
Q_2 = discharge in pipe 2

From equation $Q = Q_1 + Q_2 = 3.0$ ...(*i*)

Using equation , we have

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\frac{4 \times .005 \times 2000 \times V_1}{1.0 \times 2 \times 9.81} = \frac{4 \times .005 \times 2000 \times V_2^2}{0.8 \times 2 \times 9.81}$$

or
$$\frac{V_1^2}{1.0} = \frac{V_2^2}{0.8} \text{ or } V_1{}^2 = \frac{V_2^2}{0.8}$$

$\therefore$
$$V_1 = \frac{V_2}{\sqrt{0.8}} = \frac{V_2}{.894} \quad ...(ii)$$

Now
$$Q_1 = \frac{\pi}{4} d_1{}^2 \times V_1 = \frac{\pi}{4} (1)^2 \times \frac{V_2}{.894} \quad \left[\because V_1 = \frac{V_2}{.894}\right]$$

and
$$Q_2 = \frac{\pi}{4} d_2{}^2 \times V_2 = \frac{\pi}{4} (.8)^2 \times V_2 = \frac{\pi}{4} \times .64 \times V_2$$

Substituting the value of Q_1 and Q_2 in equation (*i*), we get

$$\frac{\pi}{4} \times \frac{V_2}{0.894} + \frac{\pi}{4} \times .64\ V_2 = 3.0 \text{ or } 0.8785\ V_2 + 0.5026\ V_2 = 3.0$$

$$V_2[.8785 + .5026] = 3.0 \text{ or } V = \frac{3.0}{1.3811} = 2.17 \text{ m/s.}$$

Substituting this value in equation (*ii*),

$$V_1 = \frac{V_2}{.894} = \frac{2.17}{0.894} = 2.427 \text{ m/s}$$

Hence
$$Q_1 = \frac{\pi}{4} d_1{}^2 \times V_1 = \frac{\pi}{4} \times 1^2 \times 2.427 = \mathbf{1.906\ m^3/s.\ Ans.}$$

$\therefore$
$$Q_2 = Q - Q_1 = 3.0 - 1.906 = \mathbf{1.094\ m^3/s.\ Ans.}$$

Problem *A pipe of diameter 0.4 m and of length 2000 m is connected to a reservoir at one end. The other end of the pipe is connected to a junction from which two pipes of lengths 1000 m and diameter 300 mm run in parallel. These parallel pipes are connected to another reservoir, which is having level of water 10 m below the water level of the above reservoir. Determine the total discharge if f = 0.015. Neglect minor losses.*

Solution. Given :

Dia. of pipe, $d = 0.4$ m
Length of pipe, $L = 2000$ m

Dia. of parallel pipes, $d_1 = d_2 = 300$ mm = 0.30 m

Length of parallel pipes, $L_1 = L_2 = 1000$ m

Difference of water level in two reservoir, $H = 10$ m, $f = .015$

Applying Bernoulli's equation to points E and F. Taking flow through ABC.

$$10 = \frac{4fLV^2}{d \times 2g} + \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g}$$

$$= \frac{4 \times .015 \times 2000 \times V^2}{0.4 \times 2 \times 9.81} + \frac{4 \times .015 \times 1000 \times V_1^2}{0.3 \times 2 \times 9.81}$$

$$= 15.29\ V^2 + 10.19\ V_1^2 \qquad ...(i)$$

From continuity equation

Discharge through AB = discharge through BC + discharge through BD

or $$\frac{\pi}{4}\ d^2 \times V = \frac{\pi}{4}\ d_1^2 \times V_1 + \frac{\pi}{4}\ d_1^2 V_2$$

But $d_1 = d_2$ and also the lengths of pipes BC and BD are equal and hence discharge through BC and BD will be same. This means $V_1 = V_2$ also

$\therefore$ $$\frac{\pi}{4}\ d^2 V = \frac{\pi}{4}\ d_1^2 \times V_1 + \frac{\pi}{4}\ d_1^2 \times V_1 \qquad [\because\ d_1 = d_2, V_1 = V_2]$$

$$= 2 \times \frac{\pi}{4}\ d_1^2 \times V_1 \text{ or } d^2 V = 2 d_1^2 V_1$$

or $$(0.4)^2 \times V = 2 \times (0.3)^2 V_1 \text{ or } .16V = 0.18\ V_1$$

$\therefore$ $$V_1 = \frac{0.16}{0.18}\ V = 0.888\ V$$

Substituting this value of V_1 in equation (i), we get

$$10 = 15.29\ V^2 + (10.19)(.888)^2 V^2 = 15.29\ V^2 + 8.035\ V^2 = 23.325\ V^2$$

$\therefore$ $$V = \sqrt{\frac{10}{23.325}} = 0.654 \text{ m/s}$$

$\therefore$ Discharge $= V \times$ Area

$$= 0.654 \times \frac{\pi}{4}\ d^2 = 0.654 \times \frac{\pi}{4}\ (0.4)^2 = \mathbf{.0822\ m^3/s.\ Ans.}$$

Problem *Three reservoirs A, B and C are connected by a pipe system shown in Fig. Find the discharge into or from the reservoirs B and C if the rate of flow from reservoirs A is 60 litres/s. Find the height of water level in the reservoir C. Take f = .006 for all pipes.*

Solution. Given :

Length of pipe AD, $L_1 = 1200$ m

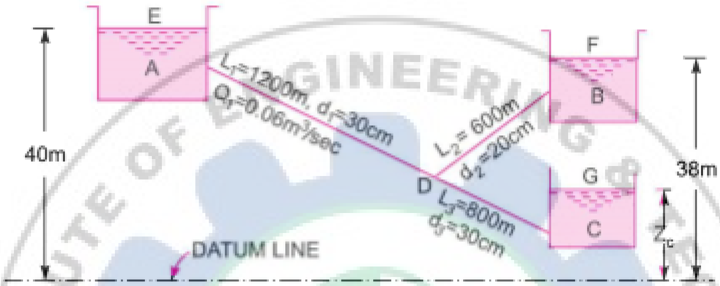
Dia. of pipe AD, $d_1 = 30$ cm = 0.30 m

Discharge through AD, $Q_1 = 60$ litres/s = 0.06 m^3/s

Height of water level in A from reference line, $Z_A = 40$ m

For pipe DB, length $L_2 = 600$ m, dia., $d_2 = 20$ cm = 0.20 m, $Z_B = 38.0$

For pipe DC, length $L_3 = 800$ m, dia., $d_3 = 30$ cm = 0.30 m



Applying Bernoulli's equations to points E and D, $Z_A = Z_D + \frac{p_D}{\rho g} + h_{f_1}$

where $h_{f_1} = \frac{4 \,.\, f \,.\, L_1 \,.\, V_1^2}{d_1 \times 2g}$, where $V_1 = \frac{Q_1}{\text{Area}} = \frac{0.06}{\frac{\pi}{4}(.3)^2} = 0.848$ m/sec

$$h_{f_1} = \frac{4 \times .006 \times 1200 \times .848^2}{0.3 \times 2 \times 9.81} = 3.518 \text{ m}$$

$\therefore$ $$Z_A = Z_D + \frac{p_D}{\rho g} + 3.518 \text{ or } 40.0 = Z_D + \frac{p_D}{\rho g} + 3.518$$

$\therefore$ $$\left(Z_D + \frac{p_1}{\rho g}\right) = 40.0 - 3.518 = 36.482 \text{ m}$$

Hence piezometric head at $D = 36.482$. But $Z_B = 38$ m. Hence water flows from B to D.

Applying Bernoulli's equation to points B and D

$$Z_B = \left(Z_D + \frac{p_D}{\rho g}\right) + h_{f_2} \text{ or } 38 = 36.482 + h_{f_2}$$

$\therefore$ $$h_{f_2} = 38 - 36.482 = 1.518 \text{ m}$$

But $$h_{f_2} = \frac{4 \,.\, f \,.\, L_2 \,.\, V_2^2}{d_2 \times 2g} = \frac{4 \times .006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$$

$\therefore$ $$1.518 = \frac{4 \times .006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81}$$

$\therefore$ $$V_2 = \sqrt{\frac{1.518 \times 0.2 \times 2 \times 9.81}{4 \times .006 \times 600}} = 0.643 \text{ m/s.}$$

$\therefore$ Discharge, $$Q_2 = V_2 \times \frac{\pi}{4} (d_2)^2 = 0.643 \times \frac{\pi}{2} \times (.2)^2$$

$$= 0.0202 \text{ m}^3\text{/s} = \mathbf{20.2 \text{ litres/s. Ans.}}$$

Applying Bernoulli's equation to points D and C

$$Z_D + \frac{p_D}{\rho g} = Z_C + h_{f_3}$$

or $$36.482 = Z_C + \frac{4f \cdot L_3 \cdot V_3^2}{d_3 \times 2g}, \text{ where } V_3 = \frac{Q_3}{\frac{\pi}{4} d_3^2}$$

But from continuity $Q_1 + Q_2 = Q_3$

$\therefore$ $$Q_3 = Q_1 + Q_2 = 0.06 + 0.0202 = 0.0802 \text{ m}^3\text{/s}$$

$\therefore$ $$V_3 = \frac{Q_3}{\frac{\pi}{4}(.3)^2} = \frac{0.0802}{\frac{\pi}{4}(.09)} = 1.134 \text{ m/s}$$

$\therefore$ $$36.482 = Z_C + \frac{4 \times .006 \times 800 \times 1.134^2}{0.3 \times 2 \times 9.81} = Z_C + 4.194$$

$\therefore$ $$Z_C = 36.482 - 4.194 = \mathbf{32.288 \text{ m. Ans.}}$$

Problem *Find the maximum power transmitted by a jet of water discharging freely out of nozzle fitted to a pipe = 300 m long and 100 mm diameter with co-efficient of friction as 0.01. The available head at the nozzle is 90 m.*

Solution. Given :

Length of pipe, $L = 300$ m

Dia. of pipe, $D = 100$ mm $= 0.1$ m

Co-efficient of friction, $f = .01$

Head available at nozzle, $= 90$ m

For maximum power transmission through the nozzle, the diameter at the outlet of nozzle is given by equation

$$d = \left(\frac{D^5}{8fL}\right)^{1/4} = \left[\frac{(0.1)^5}{8 \times .01 \times 300}\right]^{1/4} = .0254 \text{ m}$$

$\therefore$ Area at the nozzle, $$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.0254)^2 = .0005067 \text{ m}^2.$$

The nozzle at the outlet, discharges water into atmosphere and hence the total head available at the nozzle is converted into kinetic head.

$\therefore$ Head available at outlet $= v^2/2g$ or $90 = v^2/2g$

$\therefore \qquad v = \sqrt{2 \times 9.81 \times 90} = 42.02$ m/s

Discharge through nozzle, $\quad Q = a \times v = .0005067 \times 42.02 = 0.02129\ m^3/s$

$\therefore$ Maximum power transmitted $= \dfrac{\rho g \times Q \times \text{Head at outlet of nozzle}}{1000}$

$$= \frac{1000 \times 9.81 \times 0.02129 \times 90}{1000} = \mathbf{18.796\ kW.\ Ans.}$$

Problem *The water is flowing with a velocity of 1.5 m/s in a pipe of length 2500 m and of diameter 500 mm. At the end of the pipe, a valve is provided. Find the rise in pressure if the valve is closed in 25 seconds. Take the value of C = 1460 m/s.*

Solution. Given :

Velocity of water, $V = 1.5$ m/s

Length of pipe, $L = 2500$ m

Diameter of pipe, $D = 500$ mm $= 0.5$ m

Time to close the valve, $T = 25$ seconds

Value of, $C = 1460$ m/s

Let the rise in pressure $= p$

The ratio, $\dfrac{2L}{C} = \dfrac{2 \times 2500}{1460} = 3.42$

From equation we have if $T > \dfrac{2L}{C}$, the closure of valve is said to be gradual.

Here $T = 25$ sec and $\dfrac{2L}{C} = 3.42$

$\therefore \qquad T > \dfrac{2L}{C}$ and hence valve is closed gradually.

For gradually closure of valve, the rise in pressure is given by equation

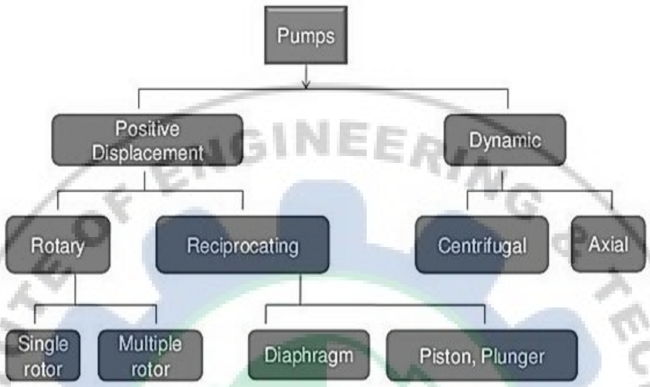
$$p = \frac{\rho V L}{T} = 1000 \times 2500 \times \frac{1.5}{25} = 150000\ N/m^2$$

$$= \frac{150000}{10^4} \frac{N}{cm^2} = \mathbf{15.0\ \frac{N}{cm^2}.\ Ans.}$$

Hydraulic Pump

A hydraulic pump is a mechanical source of power that converts mechanical power into hydraulic energy. It generates flow with enough power to overcome pressure induced by the load at the pump outlet. When a hydraulic pump operates, it creates a vacuum at the pump inlet, which forces liquid from the reservoir into the inlet line to the pump and by mechanical action delivers this liquid to the pump outlet and forces it into the hydraulic system.

Classifications of Pump



Centrifugal Pump

The main components of a centrifugal pump are:

i) Impeller
ii) Casing
iii) Suction pipe
iv) Foot valve with strainer,
v) Delivery pipe
vi) Delivery valve.

Impeller is the rotating component of the pump. It is made up of a series of curved vanes. The impeller is mounted on the shaft connecting an electric motor.

Casing is an air tight chamber surrounding the impeller. The shape of the casing is designed in such a way that the kinetic energy of the impeller is gradually changed to potential energy. This is achieved by gradually increasing the area of cross section in the direction of flow.

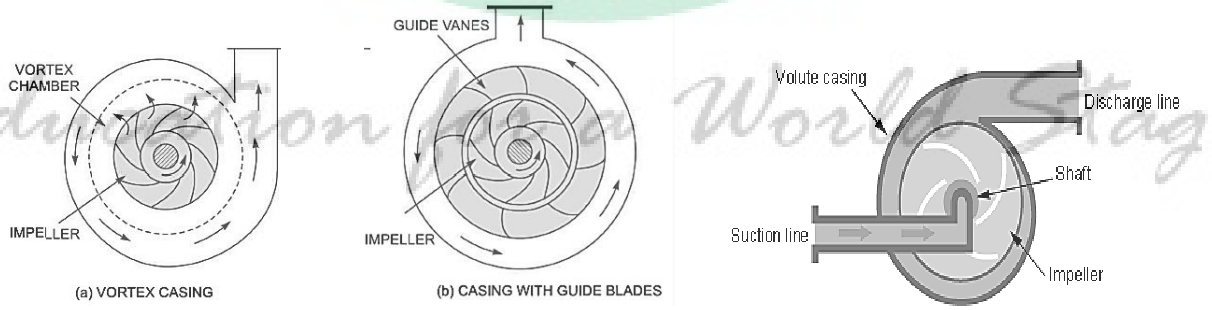


Fig. 1 Types of Casing

Suction pipe: It is the pipe connecting the pump to the sump, from where the liquid has to be lifted up.

Foot valve with strainer: The foot valve is a non-return valve which permits the flow of the liquid from the other words the foot valve opens only in the upward direction. The strainer is a mesh surrounding the valve, it p debris and silt into the pump.

Delivery pipe is a pipe connected to the pump to the overhead tank. Delivery valve is a valve which can regulate the pump.

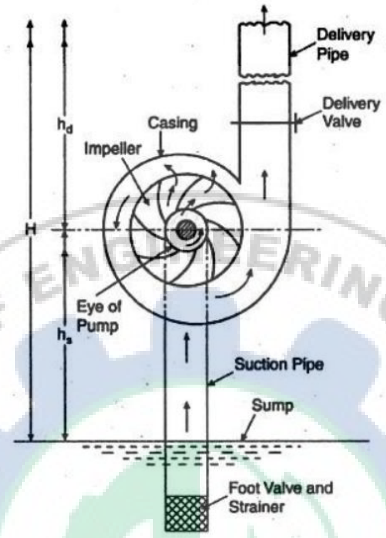


Fig. 2 Main parts of a centrifugal pump

Working

A centrifugal pump works on the principle that when a certain mass of fluid is rotated by an external source, it is thrown away from the central axis of rotation and a centrifugal head is impressed which enables it to rise to a higher level.

Working operation of a centrifugal pump is explained in the following steps:

1. Close the delivery valve and prime the pump.
2. Start the motor connected to the pump shaft, this causes an increase in the impeller pressure.
3. Open the delivery valve gradually, so that the liquid starts flowing into the deliver pipe.
4. A partial vacuum is created at the eye of the centrifugal action, the liquid rushed from the sump to the pump due to pressure difference at the two ends of the suction pipe.
5. As the impeller continues to run, move & more liquid are made available to the pump at its eye. Therefore impeller increases the energy of the liquid and delivers it to the reservoir.
6. While stopping the pump, the delivery valve should be closed first; otherwise there may be back flow from the reservoir.

It may be noted that a uniform velocity of flow is maintained in the delivery pipe. This is due to the special design of the casing. As the flow proceeds from the tongue of the casing to the delivery pipe, the area of the casing increases. There is a corresponding change in the quantity of the liquid from the impeller. Thus a uniform flow occurs in the delivery pipe.

Centrifugal pump converts rotational energy, often from a motor, to energy in a moving fluid. A portion of the energy goes into kinetic energy of the fluid. Fluid enters axially through eye of the casing, is caught up in the impeller blades, and is whirled tangentially and radially outward until it leaves through all circumferential parts of the impeller into the diffuser part of the casing. The fluid gains both velocity and pressure while passing through the impeller. The doughnut-shaped diffuser, or scroll, section of the casing decelerates the flow and further increases the pressure. The negative pressure at the eye of the impeller helps to maintain the flow in the system. If no water is present initially, the negative pressure developed by the rotating air, at the eye will be negligibly small to suck fresh stream of water. As a result the impeller will rotate without sucking and discharging any water content. So the pump should be initially filled with water before starting it. This process is known as priming.

Use of the Casing

From the illustrations of the pump so far, one speciality of the casing is clear. It has an increasing area along the flow direction. Such increasing area will help to accommodate newly added water stream, and will also help to reduce the exit flow velocity. Reduction in the flow velocity will result in increase in the static pressure, which is required to overcome the resistance of pumping system.

NPSH - Overcoming the problem of Cavitation

If pressure at the suction side of impeller goes below vapour pressure of the water, a dangerous phenomenon could happen. Water will start to boil forming vapour bubbles. These bubbles will move along with the flow and will break in a high pressure region. Upon breaking the bubbles will send high impulsive shock waves and spoil impeller material overtime. This phenomenon is known as cavitation. More the suction head, lesser should be the pressure at suction side to lift the water. This fact puts a limit to the maximum suction head a pump can have. However Cavitation can be completely avoided by careful pump selection. The term NPSH (Net Positive Suction Head) helps the designer to choose the right pump which will completely avoid Cavitation. NPSH is defined as follows:

$$NPSH = \left(\frac{P}{\rho g} + \frac{V^2}{2g}\right)_{Suction} - \frac{P_v}{\rho g}$$

Where P_v is vapour pressure of water

V is speed of water at suction side

Work done by the centrifugal pump (or by impeller) on water

Velocity triangles at inlet and outlet

Let,

D_1 : *Diameter of impeller at inlet* $= 2 \times R_1$

D_2 : *Diameter of impeller at outlet* $= 2 \times R_2$

N : *Speed ofimpeller in rpm*

u_1 : *Tangential blade velocity at inlet* $= wR_1 = (\frac{2\pi N}{60})R_1$

u_2 : *Tangential blade velocity at outlet* $= wR_2 = (\frac{2\pi N}{60})R_2$

V : *Absolute velocity*

V_r : *Relative velocity*

V_f : *Velocity of flow*

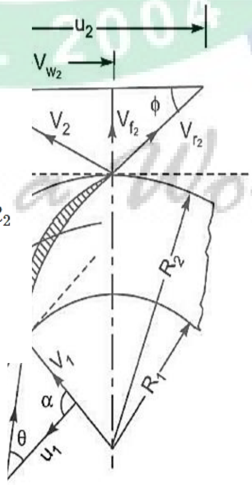
V_w; *Velocity of whirl*

α_1 : *Angle mode by absolute velocity V_1 at inlet*

θ : *Inlet angle of vane*

ϕ : *Outlet angle of vane*

β : *Discharge angle of absolute velocity at outlet*



$$Angular\ momentum = mass \times tangential velocity \times Radius$$

$$Angular\ momentum\ entering\ the\ impeller\ per\ sec = m.V_{w1}.R_1$$

$$Angular\ momentum\ leaving\ the\ impeller\ per\ sec = m.V_{w2}.R_2$$

$$Torque\ transmitted = rate\ of\ change\ of\ angular\ momentum$$
$$= m.V_{w2}.R_2 - m.V_{w1}.R_1$$
$$= \frac{w}{g}(V_{w2}.R_2 - V_{w1}.R_1)$$

Since the work done in unit time is given by the product of torque and angular velocity

W.D per sec = Torque x W

$$= \frac{w}{g}(V_{w2}.R_2 w - V_{w1}.R_1 w)$$

But $R_2 w = u_2$ and $R_1 w = u_1$

W.D per sec = $\frac{w}{g}(V_{w2}u_2 . V_{w1}u_1)$

Work done by impeller per N weight of liquid per sec,

W.D = $\frac{1}{g}(V_{w2}u_2 - V_{w1}u_1)$

But $V_{w1} = 0$ since entry is radial

W.D per N weight per sec = $\frac{V_{w2}.u_2}{g}$

Definitions of Heads and Efficiencies of a centrifugal pump

1. Suction Head (h_s). It is the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted as shown in Fig. This height is also called suction lift and is denoted by 'h_s'.

2. Delivery Head (h_d). The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head. This is denoted by 'h_d'.

3. Static Head (H_s). The sum of suction head and delivery head is known as static head. This is represented by 'H_s' and is written as

$$H_s = h_s + h_d.$$

4. Manometric Head (H_m). The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted by 'H_m'. It is given by the following expressions :

(*a*) H_m = Head imparted by the impeller to the water – Loss of head in the pump

$$= \frac{V_{w_2} u_2}{g} - \text{Loss of head in impeller and casing}$$

$$= \frac{V_{w_2} u_2}{g} \quad \text{...if loss of pump is zero}$$

(*b*) H_m = Total head at outlet of the pump – Total head at the inlet of the pump

$$= \left(\frac{P_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right)$$

(c) $$H_m = h_s + h_d + h_{f_s} + h_{f_d} + \frac{V_d^2}{2g}$$

where h_s = Suction head, h_d = Delivery head,
h_{f_s} = Frictional head loss in suction pipe, h_{f_d} = Frictional head loss in delivery pipe,
V_d = Velocity of water in delivery pipe.

(*a*) **Manometric Efficiency (η_{man}).**

$$\eta_{man} = \frac{\text{Manometric head}}{\text{Head imparted by impeller to water}}$$

$$= \frac{H_m}{\left(\frac{V_{w_2} u_2}{g}\right)} = \frac{gH_m}{V_{w_2} u_2}$$

The power at the impeller of the pump is more than the power given to the water at outlet of the pump. The ratio of the power given to water at outlet of the pump to the power available at the impeller, is known as manometric efficiency.

(*b*) **Mechanical Efficiency (η_m).**

$$\eta_m = \frac{\text{Power at the impeller}}{\text{Power at the shaft}}$$

The power at the impeller in kW $= \frac{\text{Work done by impeller per second}}{1000}$

$$= \frac{W}{g} \times \frac{V_{w_2} u_2}{1000}$$

ıe pump to the power input to

$$\eta_m = \frac{\frac{W}{g}\left(\frac{V_{w_2} u_2}{1000}\right)}{\text{S.P.}}$$

H_m

00

or

where S.P. = Shaft power.

= S.P. of the pump.

$\therefore$ $$\eta_o = \frac{\left(\frac{WH_m}{1000}\right)}{\text{S.P.}}$$

Also $\eta_o = \eta_{man} \times \eta_m$.

PRIMING OF A CENTRIFUGAL PUMP

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

CAVITATION

Cavitation includes formation of vapour bubbles of the flowing liquid and collapsing of the vapour bubbles. Formation of vapour bubbles of the flowing liquid take place only whenever the pressure in any region falls below vapour pressure. When the pressure of the flowing liquid is less than its vapour pressure, the liquid starts boiling and vapour bubbles are formed. These vapour bubbles are carried along with the flowing liquid to higher pressure zones where these vapours condense and bubbles collapse. Due to sudden collapsing of the bubbles on the metallic surface, high pressure is produced and metallic surfaces are subjected to high local stresses. Thus the surfaces are damaged.

Cavitation in Centrifugal Pumps. In centrifugal pumps the cavitation may occur at the inlet of the impeller of the pump, or at the suction side of the pumps, where the pressure is considerably reduced. Hence if the pressure at the suction side of the pump drops below the vapour pressure of the liquid then the cavitation may occur. The cavitation in a pump can be noted by a sudden drop in efficiency and head. In order to determine whether cavitation will occur in any portion of the suction side of the pump, the critical value of Thoma's cavitation factor (σ) is calculated.

Precaution Against Cavitation.

(*i*) The pressure of the flowing liquid in any part of the hydraulic system should not be allowed to fall below its vapour pressure. If the flowing liquid is water, then the absolute pressure head should not be below 2.5 m of water.

(*ii*) The special materials or coatings such as aluminium-bronze and stainless steel, which are cavitation resistant materials, should be used.

Effects of Cavitation.

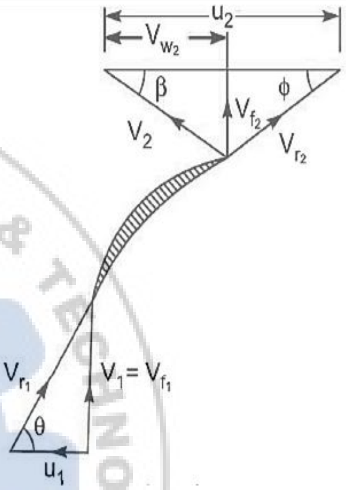
(*i*) The metallic surfaces are damaged and cavities are formed on the surfaces.

(*ii*) Due to sudden collapse of vapour bubble, considerable noise and vibrations are produced.

(*iii*) The efficiency of a turbine decreases due to cavitation. Due to pitting action, the surface of the turbine blades becomes rough and the force exerted by water on the turbine blades decreases. Hence, the work done by water or output horse power becomes less and thus efficiency decreases.

Example *The internal and external diameters of the impeller of a centrifugal pump are 200 and 400 mm respectively. The pump is running at 1200 rpm. The vane angles of the impeller at inlet and outlet are 20 and 30 respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.*

Given:



Internal diameter of impeller, $D_1 = 200$ mm $= 0.20$ m

External diameter of impeller, $D_2 = 400$ mm $= 0.40$ m

Speed, $N = 1200$ r.p.m.

Vane angle at inlet, $\theta = 20°$

Vane angle at outlet, $\phi = 30°$

Water enters radially* means, $\alpha = 90°$ and $V_{w_1} = 0$

Velocity of flow, $V_{f_1} = V_{f_2}$

Tangential velocity of impeller at inlet and outlet are,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.20 \times 1200}{60} = 12.56 \text{ m/s}$$

d

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 1200}{60} = 25.13 \text{ m/s.}$$

From inlet velocity triangle, $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{V_{f_1}}{12.56}$

$\therefore \quad V_{f_1} = 12.56 \tan\theta = 12.56 \times \tan 20° = 4.57$ m/s

$\therefore \quad V_{f_2} = V_{f_1} = 4.57$ m/s.

From outlet velocity triangle, $\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{4.57}{25.13 - V_{w_2}}$

$$25.13 - V_{w_2} = \frac{4.57}{\tan\phi} = \frac{4.57}{\tan 30°} = 7.915$$

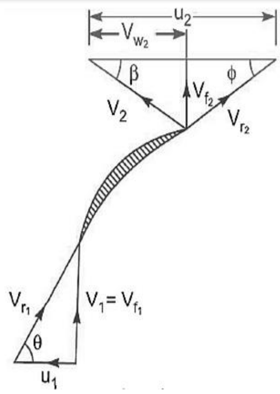
$\therefore \quad V_{w_2} = 25.13 - 7.915 = 17.215$ m/s.

The work done by impeller per kg of water per second is given by equation (

$$= \frac{1}{g} V_{w_2} u_2 = \frac{17.215 \times 25.13}{9.81} = \mathbf{44.1\ Nm/N}.$$

Example *A centrifugal pump is to discharge 0.118 m³/s at a speed of 1450 rpm against a head of 25m. the impeller diameter is 250 mm, its width at outlet is 50 mm and manometric efficiency is 75%. Determine the vane angle at the outer periphery of the impeller.*

Given:



Discharge, $Q = 0.118$ m³/s

Speed, $N = 1450$ r.p.m.

Head, $H_m = 25$ m

Diameter at outlet, $D_2 = 250$ mm $= 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\eta_{man} = 75\% = 0.75$.

Let vane angle at outlet $= \phi$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.25 \times 1450}{60} = 18.98 \text{ m/s}$$

Discharge is given by $\quad Q = \pi D_2 B_2 \times V_{f_2}$

$$\therefore \quad V_{f_2} = \frac{Q}{\pi D_2 B_2} = \frac{0.118}{\pi \times 0.25 \times .05} = 3.0 \text{ m/s.}$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25}{V_{w_2} \times 18.98}$$

$$V_{w_2} = \frac{9.81 \times 25}{\eta_{man} \times 18.98} = \frac{9.81 \times 25}{0.75 \times 18.98} = 17.23.$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} = \frac{3.0}{(18.98 - 17.23)} = 1.7143$$

$$\therefore \quad \phi = \tan^{-1} 1.7143 = 59.74° \text{ or } \mathbf{59° \, 44'. \, Ans.}$$

Example *A centrifugal pump delivers water against a net head of 14.5 m and a design speed of 1000 rpm. The vanes are curved back at an angle of 30° with the periphery. The impeller diameter is 300 mm and outlet width is 50 mm. determine the discharge of the pump if manometric efficiency is 95%.*

Given:

Net head, $\quad H_m = 14.5$ m

Speed, $\quad N = 1000$ r.p.m.

Vane angle at outlet, $\quad \phi = 30°$

Impeller diameter means the diameter of the impeller at outlet

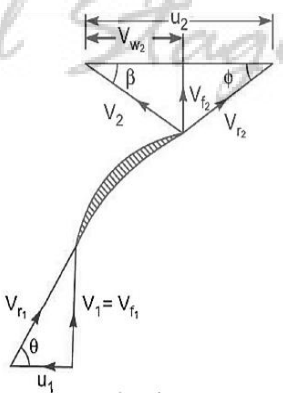
$\therefore$ Diameter, $\quad D_2 = 300$ mm $= 0.30$ m

Outlet width, $\quad B_2 = 50$ mm $= 0.05$ m

Manometric efficiency, $\quad \eta_{man} = 95\% = 0.95$

Tangential velocity of impeller at outlet,

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.30 \times 1000}{60} = 15.70 \text{ m/s.}$$



$$\eta_{man} = \frac{gH_m}{V_{w_2} \times u_2}$$

$$0.95 = \frac{9.81 \times 14.5}{V_{w_2} \times 15.70}$$

$$V_{w_2} = \frac{0.95 \times 14.5}{0.95 \times 15.70} = 9.54 \text{ m/s.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{(u_2 - V_{w_2})} \text{ or } \tan 30° = \frac{V_{f_2}}{(15.70 - 9.54)} = \frac{V_{f_2}}{6.16}$$

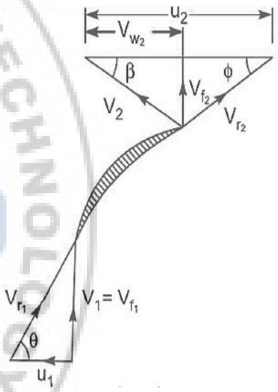
$$V_{f_2} = 6.16 \times \tan 30° = 3.556 \text{ m/s.}$$

$$Q = \pi D_2 B_2 \times V_{f_2}$$

$$= \pi \times 0.30 \times 0.05 \times 3.556 \text{ m}^3/\text{s} = \mathbf{0.1675 \text{ m}^3/\text{s}.}\ \textbf{Ans.}$$

Example *A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1000 rpm works against a total head of 40 m. the velocity of flow through the impeller is constant and equal to 2.5 m/s. the vanes are set back at an angle of 40° at outlet. If the outer diameter of the impeller is 500 mm and width at the outlet is 50 mm, determine: i) Vane angle at inlet, ii) work done by impeller on water per second iii) manometric efficiency*

Given:



Speed, $N = 1000$ r.p.m.

Head, $H_m = 40$ m

Velocity of flow, $V_{f_1} = V_{f_2} = 2.5$ m/s

Vane angle at outlet, $\phi = 40°$

Outer dia. of impeller, $D_2 = 500$ mm $= 0.50$ m

Inner dia. of impeller, $D_1 = \frac{D_2}{2} = \frac{0.50}{2} = 0.25$ m

Width at outlet, $B_2 = 50$ mm $= 0.05$ m

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09 \text{ m/s}$$

and $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.50 \times 1000}{60} = 26.18 \text{ m/s.}$$

Discharge is given by, $Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.50 \times .05 \times 2.5 = 0.1963 \text{ m}^3/\text{s}.$

(*i*) **Vane angle at inlet (θ).**

From inlet velocity triangle $\tan \theta = \frac{V_{f_1}}{u_1} = \frac{2.5}{13.09} = 0.191$

$\therefore$ $\theta = \tan^{-1} .191 = 10.81°$ or $\mathbf{10° \, 48'}$.

(*ii*) **Work done by impeller on water per second** is given by equation

$$= \frac{W}{g} \times V_{w_2} u_2 = \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times V_{w_2} \times 26.18$$

But from outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.5}{(26.18 - V_{w_2})}$$

$\therefore$ $$26.18 - V_{w_2} = \frac{2.5}{\tan \phi} = \frac{2.5}{\tan 40°} = 2.979$$

$\therefore$ $$V_{w_2} = 26.18 - 2.979 = 23.2 \text{ m/s.}$$

Substituting this value of V_{w_2} in equation (i), we get the work done by impeller as

$$= \frac{1000 \times 9.81 \times 0.1963}{9.81} \times 23.2 \times 26.18$$

$$= \mathbf{119227.9\ Nm/s.} \quad \textbf{Ans.}$$

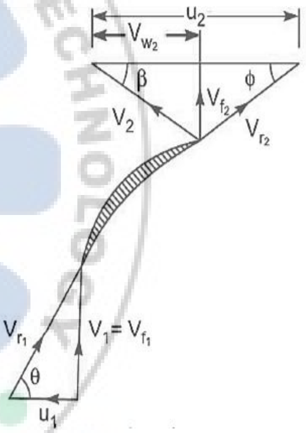
(iii) **Manometric efficiency (η_{man}).** Using equation (19.8), we have

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 40}{23.2 \times 26.18} = 0.646 = \mathbf{64.4\%}.$$

Example *The outer diameter of an impeller of a centrifugal pump is 400 mm and outlet width is 50 mm. the pump is running at 800 rpm and is working against a total head of 15 m. the vanes angle at outlet is 40° and manometric efficiency is 75%. Determine: i) Velocity of flow at outlet, ii) velocity of water leaving the vane, iii) angle made by the absolute velocity at outlet with the direction of motion at outlet and iv) discharge*

Given:

Outer diameter, $D_2 = 400$ mm $= 0.4$ m
Width at outlet, $B_2 = 50$ mm $= 0.05$ m
Speed, $N = 800$ r.p.m.
Head, $H_m = 15$ m
Vane angle at outlet, $\phi = 40°$
Manometric efficiency, $\eta_{man} = 75\% = 0.75$
Tangential velocity of impeller at outlet,



$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60} = 16.75 \text{ m/s.}$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$$

$$0.75 = \frac{9.81 \times 15}{V_{w_2} \times 16.75}$$

$$V_{w_2} = \frac{9.81 \times 15}{0.75 \times 16.75} = 11.71 \text{ m/s.}$$

From the outlet velocity triangle, we have

$$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{V_{f_2}}{(16.75 - 11.71)} = \frac{V_{f_2}}{5.04}$$

(i) $\therefore$ $V_{f_2} = 5.04 \tan\phi = 5.04 \times \tan 40° = \mathbf{4.23\ m/s.}$

(ii) **Velocity of water leaving the vane (V_2).**

$$V_2 = \sqrt{V_{f_2}^2 + V_{w_2}^2} = \sqrt{4.23^2 + 11.71^2}$$

$$= \sqrt{17.89 + 137.12} = \mathbf{12.45\ m/s.}$$

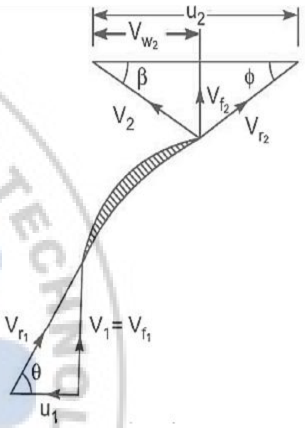
(iii) **Angle made by absolute velocity at outlet (β),**

$$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{4.23}{11.71} = 0.36$$

$\therefore$ $\beta = \tan^{-1} 0.36 = 19.80°$ or $\mathbf{19° 48'}$..23 = $\mathbf{0.265\ m^3/s.}$

Example *The internal diameter and external diameter of an impeller of a centrifugal pump which is running at 1000 rpm are 200 and 40 mm respectively. The discharge through pump is 0.04 m3/s and velocity of flow is constant and equal to 2.0 m/s. the diameter of the suction and delivery pipes are 150 and 100 mm respectively and suction and delivery heads are 6 m (abs.) and 30 m (abs.) of water respectively. If the outlet vane angle is 45° and power required to drive the pump is 16.168 kW, determine: i) Vane angle of the impeller at inlet, ii) the overall efficiency of the pump and iii) manometric efficiency of the pump*

Given:



Speed, $N = 1000$ r.p.m.

Internal dia., $D_1 = 200$ mm $= 0.2$ m

External dia., $D_2 = 400$ mm $= 0.4$ m

Discharge, $Q = 0.04$ m^3/s

Velocity of flow, $V_{f_1} = V_{f_2} = 2.0$ m/s

Dia. of suction pipe, $D_s = 150$ mm $= 0.15$ m

Dia. of delivery pipe, $D_d = 100$ mm $= 0.10$ m

Suction head, $h_s = 6$ m (abs.)

Delivery head, $h_d = 30$ m (abs.)

Outlet vane angle, $\phi = 45°$

Power required to drive the pump, $P = 16.186/$ kW

From inlet velocity, we have $\tan\theta = \frac{V_{f_1}}{u_1} = \frac{2.0}{u_1}$, where $u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.2 \times 1000}{60} = 10.47$ m/s

$$\therefore \quad \tan\theta = \frac{2.0}{10.47} = 0.191 \text{ or } \theta = \tan^{-1} .191 = \mathbf{10° \; 48'. \; Ans.}$$

(*ii*) **Overall efficiency of the pump (η_o).**

Using equation (19.10), we have $\eta_o = \dfrac{\left(\dfrac{WH_m}{1000}\right)}{\text{S.P.}}$

where S.P. = Power required to drive the pump and equal to P here.

$$\eta_o = \frac{\left(\dfrac{\rho \times g \times Q \times H_m}{1000}\right)}{P} = \frac{\rho g \times Q \times H_m}{1000 \times P}$$

$$= \frac{1000 \times 9.81 \times .04 \times H_m}{1000 \times 16.186} = 0.02424\, H_m \qquad \ldots(i)$$

Now H_m is given by equation (19.6) as

$$H_m = \left(\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right) - \left(\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right) \qquad \ldots(ii)$$

$$H_m = \left(30 + \frac{V_d^2}{2g}\right) - \left(6 + \frac{V_s^2}{2g}\right) \qquad \ldots(iii)$$

$$V_d = \frac{\text{Discharge}}{\text{Area of delivery pipe}} = \frac{0.04}{\frac{\pi}{4}(D_d)^2} = \frac{.04}{\frac{\pi}{4} \times .1^2} = 5.09 \text{ m/s}$$

$$V_s = \frac{.04}{\text{Area of suction pipe}} = \frac{.04}{\frac{\pi}{4} D_s^2} = \frac{.04}{\frac{\pi}{4} \times .15^2} = 2.26 \text{ m/s.}$$

$$H_m = \left(30 + \frac{5.09^2}{2 \times 9.81}\right) - \left(6 + \frac{2.26^2}{2 \times 9.81}\right)$$

$$= (30 + 1.32) - (6 + .26) = 31.32 - 6.26 = 25.06 \text{ m.}$$

Substituting the value of 'H_m' in equation (i), we get

$$\eta_o = .02424 \times 25.06 = 0.6074 = \mathbf{60.74\%.}$$

(iii) **Manometric efficiency of the pump (η_{man}).**

Tangential velocity at outlet is given by

$$u_2 = \frac{\pi D_2 \times N}{60} = \frac{\pi \times 0.4 \times 1000}{60} = 20.94 \text{ m/s.}$$

From outlet velocity triangle, we have

$$\tan \phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{2.0}{20.94 - V_{w_2}}$$

$$\therefore \quad 20.94 - V_{w_2} = \frac{2.0}{\tan \phi} = \frac{2.0}{\tan 45} = 2.0$$

$$\therefore \quad V_{w_2} = 20.94 - 2.0 = 18.94.$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2} = \frac{9.81 \times 25.06}{18.94 \times 20.94} = 0.6198 = \mathbf{61.98\%}$$

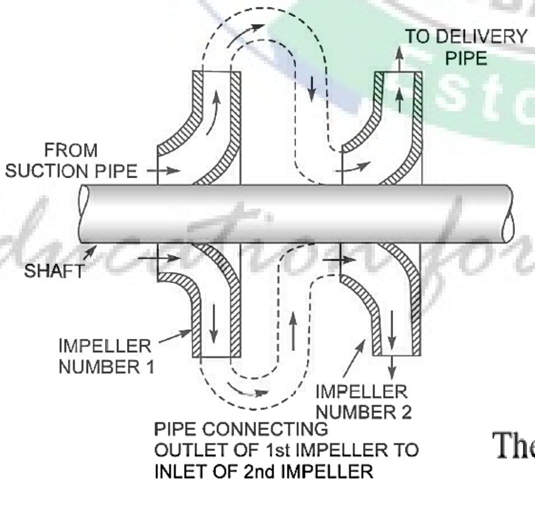
MULTISTAGE CENTRIFUGAL PUMPS

If a centrifugal pump consists of two or more impellers, the pump is called a multistage centrifugal pump. The impellers may be mounted on the same shaft or on different shafts. A multistage pump is having the following two important functions :

1. To produce a high head, and
2. To discharge a large quantity of liquid.

If a high head is to be developed, the impellers are connected in series (or on the same shaft) while for discharging large quantity of liquid, the impellers (or pumps) are connected in parallel.

Multistage Centrifugal Pumps for High Heads.



Then total head developed

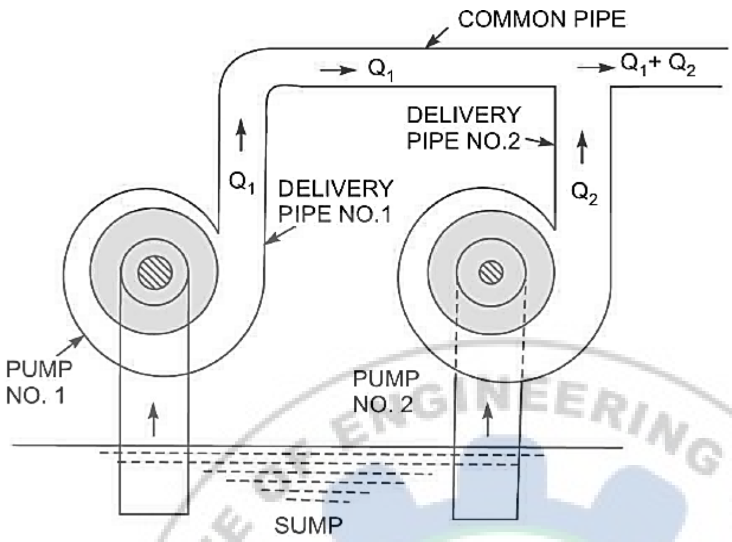
$$= n \times H_m$$

The discharge passing through each impeller is same

n = Number of identical impellers mounted on the same shaft,

H_m = Head developed by each impeller.

Multistage Centrifugal Pumps for High Discharge.



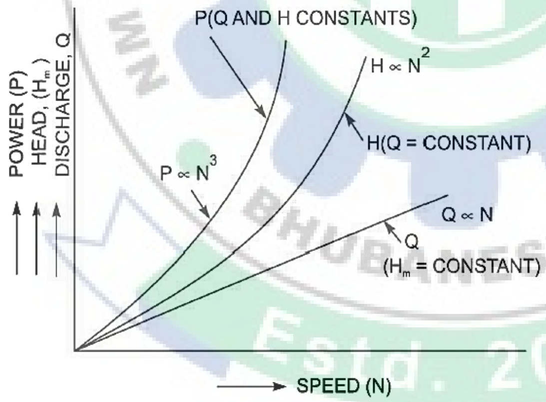
Let n = Number of identical pumps arranged in parallel.

Q = Discharge from one pump.

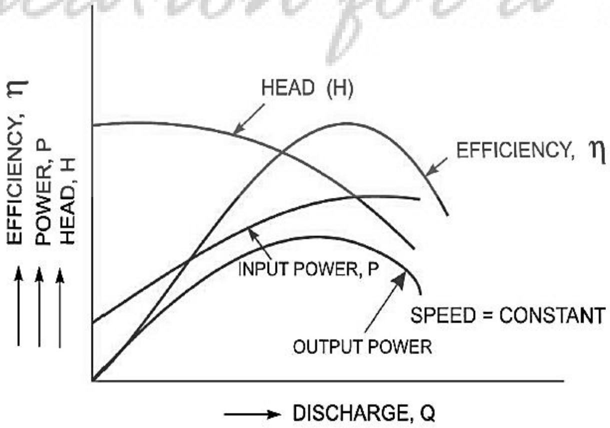
∴ Total discharge $= n \times Q$

CHARACTERISTIC CURVES OF CENTRIFUGAL PUMPS

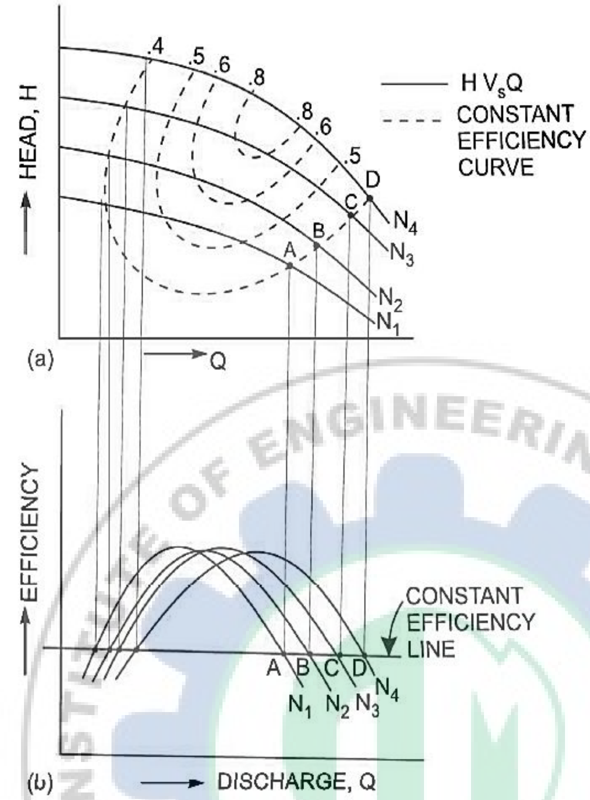
Main Characteristic Curves.



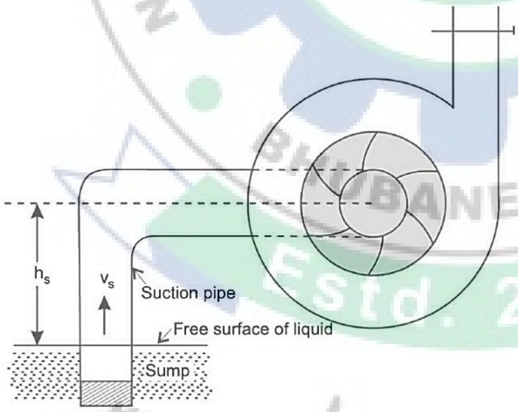
Operating Characteristic Curves.



Constant Efficiency Curves.



MAXIMUM SUCTION LIFT (or SUCTION HEIGHT)



Applying Bernoulli's equation at the free surface of liquid in the sump and section 1 in the suction pipe just at the inlet of the pump and taking the free surface of liquid as datum line, we get

$$\frac{p_a}{\rho g} + \frac{V_a^2}{2g} + Z_a = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 + h_L \qquad ...(i)$$

$$\frac{p_a}{\rho g} + 0 + 0 = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$\frac{p_a}{\rho g} = \frac{p_1}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right) \qquad ...(ii)$$

For finding the maximum suction lift, the pressure at the inlet of the pump should not be less than the vapour pressure of the liquid. Hence for the limiting case, taking the pressure at the inlet of pump equal to vapour pressure of the liquid, we get

$p_1 = p_v$, where p_v = vapour pressure of the liquid in absolute units.

Now the equation (*ii*) becomes as

$$\frac{p_v}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\frac{p_a}{\rho g} = \frac{p_v}{\rho g} + \frac{v_s^2}{2g} + h_s + h_{f_s} \qquad (\because \quad p_1 = p_v) \ ...(iii)$$

$$\frac{p_a}{\rho g} = \text{Atmospheric pressure head } = H_a \text{ (meter of liquid)}$$

$$\frac{p_v}{\rho g} = \text{Vapour pressure head} = H_v \text{ (meter of liquid)}$$

Now, equation (*iii*) becomes as

$$H_a = H_v + \frac{v_s^2}{2g} + h_s + h_{f_s}$$

$$h_s = H_a - H_v - \frac{v_s^2}{2g} - h_{f_s}$$

Equation (19.31) gives the value of maximum suction lift (or maximum suction height) for a centrifugal pump. Hence, the suction height of any pump should not be more than that given by equation (19.31). If the suction height of the pump is more, then vaporization of liquid at inlet of pump will take place and there will be a possibility of cavitation.

NET POSITIVE SUCTION HEAD (NPSH)

The term NPSH (Net Positive Suction Head) is very commonly used in the pump industry. Actually the minimum suction conditions are more frequently specified in terms of NPSH.

The net positive suction head (NPSH) is defined as the *absolute* pressure head at the inlet to the pump, minus the vapour pressure head (in absolute units) plus the velocity head.

∴ NPSH = Absolute pressure head at inlet of the pump – vapour pressure head (absolute units) + velocity head

$$= \frac{p_1}{\rho g} - \frac{p_v}{\rho g} + \frac{v_s^2}{2g} \qquad (\because \text{ Absolute pressure at inlet of pump} = p_1)\ .$$

the absolute pressure head at inlet of the pump is given by as

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)$$

$$\text{NPSH} = \left[\frac{p_a}{\rho g} - \left(\frac{v_s^2}{2g} + h_s + h_{f_s}\right)\right] - \frac{p_v}{\rho g} + \frac{v_s^2}{2g}$$

$$= \frac{p_a}{\rho g} - \frac{p_v}{\rho g} - h_s - h_{f_s}$$

$$= H_a - H_v - h_s - h_{f_s}$$

RECIPROCATING PUMP

If the mechanical energy is converted into hydraulic energy by sucking the liquid into a cylinder in which a piston is reciprocating, which exerts the thrust on the liquid and increases its hydraulic energy is known as reciprocating pump. A reciprocating pump is a positive displacement pump. It is often used where relatively small quantity of liquid is to be handled and where delivery pressure is quite large.

Reciprocating pump consists of following parts.

1. A cylinder with a piston
2. piston rod
3. connecting rod
4. crank
5. suction pipe
6. delivery pipe
7. suction valve
8. delivery valve

WORKING OF A SINGLE-ACTING RECIPROCATING PUMP

Single acting reciprocating pump:-

A single acting reciprocating pump, which consists of a piston which moves forwards and backwards in a close fitting cylinder. The movement of the piston is obtained by connecting the piston rod to crank by means of a connecting rod. The crank is rotated by means of an electric motor. Suction and delivery pipes with suction valve and delivery valve are connected to the cylinder. The suction and delivery valves are one way valves or non-return valves, which allow the water to flow in one direction only. Suction valve allows water from suction pipe to the cylinder which delivery valve allows water from cylinder to delivery pipe.

The rotation of the crank brings about an outward and inward movement of the piston in the cylinder. During the suction stroke the piston is moving towards right in the cylinder, this movement of piston causes vacuum in the cylinder. The pressure of the atmosphere acting on the sump water surface forces the water up in the suction pipe. The forced water opens the suction valve and the water enters the cylinder. The piston from its extreme right position starts moving towards left in the cylinder. The movement of the piston towards left increases the pressure of the liquid inside the cylinder more than atmospheric pressure. Hence suction valve closes and delivery valve opens. The liquid is forced into the delivery pipe and is raised to a required height.

For one revolution of the crank, the quantity of water raised up in the delivery pipe is equal to the stroke volume in the cylinder in the single acting pump and twice this volume in the double acting pump. Discharge through a single acting reciprocating pump.

D = diameter of the cylinder
A = cross section are of the piston or cylinder
r = radius of crank
N = r.p.m of the crank
L = Length of the stroke = 2 x r
h_s = Suction head or height of axis of the cylinder from water surface in sump.
h_d = Delivery head or height of the delivery outlet above the cylinder axis.
Discharge of water in one revolution = Area x Length of stroke
= A x L
Number of revolution per second = N/60
Discharge of the pump per second
Q = Discharge in one revolution x No.of revolution per second

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60} \text{ m}^3\text{/sec}$$

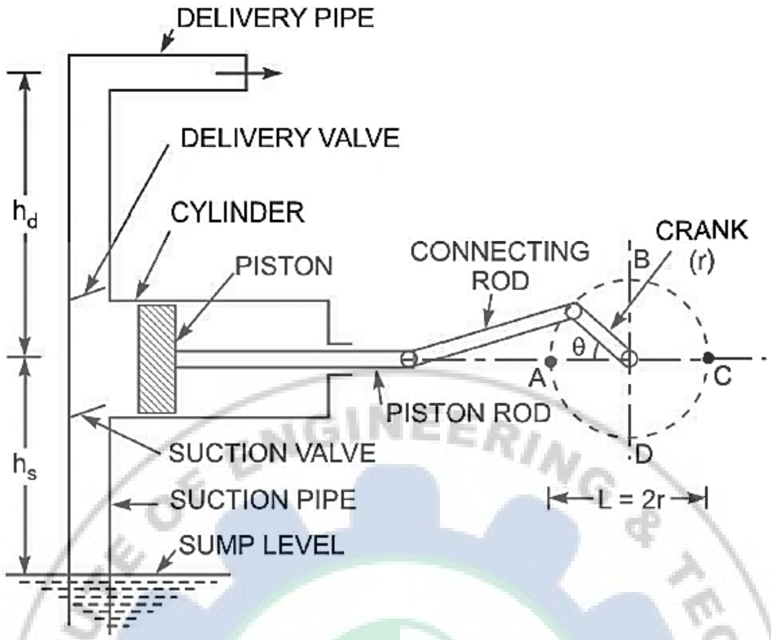


Fig.3

Double acting reciprocating pump

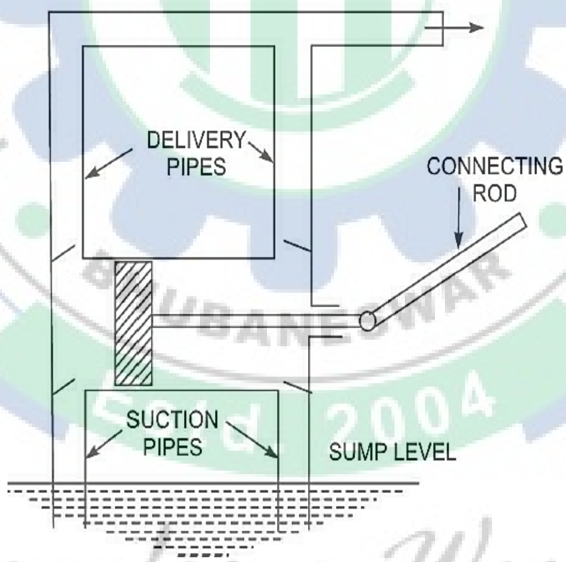


Fig.4

Discharge Through a Reciprocating Pump.

Let D = Diameter of the cylinder

A = Cross-sectional area of the piston or cylinder

$$= \frac{\pi}{4} D^2$$

r = Radius of crank

N = r.p.m. of the crank

L = Length of the stroke = $2 \times r$

h_s = Height of the axis of the cylinder from water surface in sump.

h_d = Height of delivery outlet above the cylinder axis (also called delivery head)

Volume of water delivered in one revolution or discharge of water in one revolution

$= \text{Area} \times \text{Length of stroke} = A \times L$

Number of revolution per second, $= \frac{N}{60}$

$\therefore$ Discharge of the pump per second,

Q = Discharge in one revolution × No. of revolution per second

$$= A \times L \times \frac{N}{60} = \frac{ALN}{60} \quad ...(20.1)$$

Weight of water delivered per second,

$$W = \rho \times g \times Q = \frac{\rho g ALN}{60}.$$

Work done by Reciprocating Pump.

Work done per second = Weight of water lifted per second × Total height through which water is lifted

$$= W \times (h_s + h_d) \quad ...(i)$$

where $(h_s + h_d)$ = Total height through which water is lifted.

From equation (20.2), Weight, W, is given by

$$W = \frac{\rho g \times ALN}{60}.$$

Substituting the value of W in equation (i), we get

$$\text{Work done per second} = \frac{\rho g \times ALN}{60} \times (h_s + h_d) \quad ...(20.3)$$

$\therefore$ Power required to drive the pump, in kW

$$P = \frac{\text{Work done per second}}{1000} = \frac{\rho g \times ALN \times (h_s + h_d)}{60 \times 1000}$$

$$= \frac{\rho g \times ALN \times (h_s + h_d)}{60{,}000} \text{ kW} \quad ...(20.4)$$

Discharge, Work done and Power Required to Drive a Double-acting Pump.

Let D = Diameter of the piston,

d = Diameter of the piston rod

$\therefore$ Area on one side of the piston,

$$A = \frac{\pi}{4} D^2$$

Area on the other side of the piston, where piston rod is connected to the piston,

$$A_1 = \frac{\pi}{4} D^2 - \frac{\pi}{4} d^2 = \frac{\pi}{4} (D^2 - d^2).$$

∴ Volume of water delivered in one revolution of crank

$$= A \times \text{Length of stroke} + A_1 \times \text{Length of stroke}$$

$$= AL + A_1L = (A + A_1)L = \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L$$

∴ Discharge of pump per second

= Volume of water delivered in one revolution × No. of revolution per second

$$= \left[\frac{\pi}{4}D^2 + \frac{\pi}{4}\left(D^2 - d^2\right)\right] \times L \times \frac{N}{60}$$

If 'd' the diameter of the piston rod is very small as compared to the diameter of the piston, then it can be neglected and discharge of pump per second,

$$Q = \left(\frac{\pi}{4}D^2 + \frac{\pi}{4}D^2\right) \times \frac{L \times N}{60} = 2 \times \frac{\pi}{4}D^2 \times \frac{L \times N}{60} = \frac{2ALN}{60} \quad \text{...(20.5)}$$

Work done by double-acting reciprocating pump

Work done per second = Weight of water delivered × Total height

= ρg × Discharge per second × Total height

$$= \rho g \times \frac{2ALN}{60} \times (h_s + h_d) = 2\rho g \times \frac{ALN}{60} \times (h_s + h_d)$$

∴ Power required to drive the double-acting pump in kW,

$$P = \frac{\text{Work done per second}}{1000} = 2\rho g \times \frac{ALN}{60} \times \frac{(h_s + h_d)}{1000}$$

$$= \frac{2\rho g \times ALN \times (h_s + h_d)}{60{,}000}$$

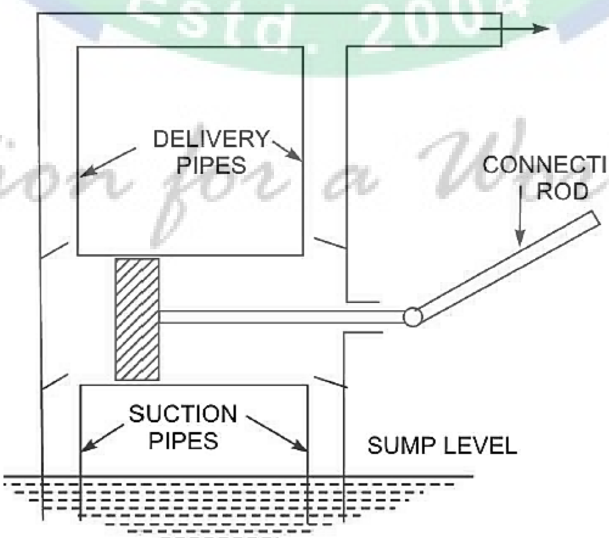


Fig.5

SLIP OF RECIPROCATING PUMP

The actual discharge of the pump is always less than theoretical discharge. The difference between theoretical discharge and actual discharge is known as Slip of the reciprocating pump

$$\text{Slip} = Q_{th} - Q_{act}$$

But slip is mostly expressed as percentage slip which is given by,

$$\text{Percentage slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 = \left(1 - \frac{Q_{act}}{Q_{th}}\right) \times 100$$

$$= (1 - C_d) \times 100 \qquad \left(\because \frac{Q_{act}}{Q_{th}} = C_d\right)$$

where C_d = Co-efficient of discharge.

Negative Slip of the Reciprocating Pump.

Negative Slip of the Reciprocating Pump. Slip is equal to the difference of theoretical discharge and actual discharge. If actual discharge is more than the theoretical discharge, the slip of the pump will become –ve. In that case, the slip of the pump is known as negative slip.

Negative slip occurs when delivery pipe is short, suction pipe is long and pump is running at high speed.

Example *A single acting reciprocating pump, running at 50 rpm, delivers 0.01m3/s of water. The diameter of the piston is 200 mm and stroke length 400 m. Determine: i) theoretical discharge of the pump ii) Co – efficient of discharge and iii) Slip and the percentage of slip of the pump.*

Given:

Solution. Given :

Speed of the pump, $N = 50$ r.p.m.

Actual discharge, $Q_{act} = .01\ m^3/s$

Dia. of piston, $D = 200$ mm = .20 m

∴ Area, $A = \frac{\pi}{4}(.2)^2 = .031416\ m^2$

Stroke, $L = 400$ mm = 0.40 m.

(*i*) Theoretical discharge for single-acting reciprocating pump is given by equation (20.1) as

$$Q_{th} = \frac{A \times L \times N}{60} = \frac{.031416 \times .40 \times 50}{60} = \mathbf{0.01047\ m^3/s.\ Ans.}$$

(*ii*) Co-efficient of discharge is given by

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.01}{.01047} = \mathbf{0.955.\ Ans.}$$

(*iii*) Using equation (20.8), we get

$$\text{Slip} = Q_{th} - Q_{act} = .01047 - .01 = \mathbf{0.00047\ m^3/s.\ Ans.}$$

And percentage slip

$$= \frac{(Q_{th} - Q_{act})}{Q_{th}} \times 100 = \frac{(.01047 - .01)}{.01047} \times 100$$

$$= \frac{.00047}{.01047} \times 100 = \mathbf{4.489\%.\ Ans.}$$

Example *A double-acting reciprocating pump, running at 40 r.p.m., is discharging 1.0 m³ of water per minute. The pump has a stroke of 400 mm. The diameter of the piston is 200 mm. The delivery and suction head are 20 m and 5 m respectively. Find the slip of the pump and power required to drive the pump.*

Speed of pump, $N = 40$ r.p.m.

Actual discharge, $Q_{act} = 1.0 \text{ m}^3/\text{min} = \frac{1.0}{60} \text{ m}^3/\text{s} = 0.01666 \text{ m}^3/\text{s}$

Stroke, $L = 400 \text{ mm} = 0.40 \text{ m}$

Diameter of piston, $D = 200 \text{ mm} = 0.20 \text{ m}$

$\therefore$ Area, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (.2)^2 = 0.031416 \text{ m}^2$

Suction head, $h_s = 5$ m

Delivery head, $h_d = 20$ m.

Theoretical discharge for double-acting pump is given by equation (20.5) as,

$$Q_{th} = \frac{2ALN}{60} = \frac{2 \times .031416 \times 0.4 \times 40}{60} = .01675 \text{ m}^3/\text{s}.$$

Using equation (20.8), Slip $= Q_{th} - Q_{act} = .01675 - .01666 =$ **.00009 m³/s. Ans.**

Power required to drive the double-acting pump is given by equation (20.7) as,

$$P = \frac{2 \times \rho g \times ALN \times (h_s + h_d)}{60{,}000} = \frac{2 \times 1000 \times 9.81 \times .031416 \times .4 \times 40 \times (5 + 20)}{60{,}000}$$

$$= \mathbf{4.109 \text{ kW. Ans.}}$$

INDICATOR DIAGRAM

indicator diagram is a graph between pressure head and stroke length of the piston for one complete revolution. The pressure head is taken as ordinate and stroke length as abscissa.

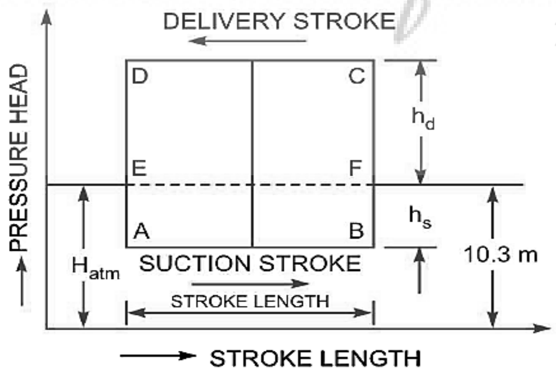


Fig. 6 *Ideal indicator diagram.*

Let H_{atm} = Atmospheric pressure head = 10.3 m of water,

L = Length of the stroke,

h_s = Suction head, and

h_d = Delivery head.

we know that the work done by the pump per second

$$= \frac{\rho \times g \times ALN}{60} \times (h_s + h_d)$$

$$= K \times L(h_s + h_d) \qquad \left(\text{where } K = \frac{\rho g A N}{60} = \text{Constant}\right)$$

$$\propto L \times (h_s + h_d) \qquad \ldots(i)$$

Work done by pump $\propto$ Area of indicator diagram.

SEPARATION OF LIQUID

If the pressure in the cylinder is below the vapour pressure, dissolved gasses will be liberated from the liquid and cavitation will takes place. The continuous flow of liquid will not exist which means separation of liquid takes place. The pressure at which separation takes place is called separation pressure and head corresponding to the separation pressure is called separation pressure head.

The ways to avoid cavitation in reciprocating pumps:

1. **Design:** Ensure that there are no sharp corners or curvatures of flow in the system while designing the pump.
2. **Material:** Cavitation resistant materials like Bronze or Nickel can be used.
3. **Model Testing:** Before manufacturing, a scaled down model should be tested.
4. **Admission of air:** High pressure air can be injected into the low pressure zones of flowing liquid to prevent bubble formation.

AIR VESSELS

An air vessel is a closed chamber containing compressed air in the top portion and liquid (or water) at the bottom of the chamber. At the base of the chamber there is an opening through which the liquid (or water) may flow into the vessel or out from the vessel. When the liquid enters the air vessel, the air gets compressed further and when the liquid flows out the vessel, the air will expand in the chamber.

An air vessel is fitted to the suction pipe and to the delivery pipe at a point close to the cylinder of a single-acting reciprocating pump :

(*i*) to obtain a continuous supply of liquid at a uniform rate,

(*ii*) to save a considerable amount of work in overcoming **friction in suction and delivery pipes**

(*iii*) to run the pump at a high speed without separation.

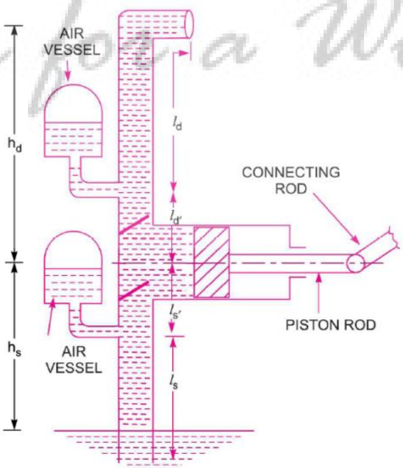


Fig.7

COMPARISON BETWEEN CENTRIFUGAL PUMPS AND RECIPROCATING PUMPS

Centrifugal pumps	*Reciprocating pumps*
1. The discharge is continuous and smooth.	1. The discharge is fluctuating and pulsating.
2. It can handle large quantity of liquid.	2. It handles small quantity of liquid only.
3. It can be used for lifting highly viscous liquids.	3. It is used only for lifting pure water or less viscous liquids.
4. It is used for large discharge through smaller heads.	4. It is meant for small discharge and high heads.
5. Cost of centrifugal pump is less as compared to reciprocating pump.	5. Cost of reciprocating pump is approximately four times the cost of centrifugal pump.
6. Centrifugal pump runs at high speed. They can be coupled to electric motor.	6. Reciprocating pump runs at low speed. Speed is limited due to consideration of separation and cavitation.
7. The operation of centrifugal pump is smooth and without much noise. The maintenance cost is low.	7. The operation of reciprocating pump is complicated and with much noise. The maintenance cost is high.
8. Centrifugal pump needs smaller floor area and installation cost is low.	8. Reciprocating pump requires large floor area and installation cost is high.
9. Efficiency is high.	9. Efficiency is low.

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TURBINES

Hydraulic machines are defined as those machines which convert either hydraulic energy (energy possessed by water) into mechanical energy (which is further converted into electrical energy) or mechanical energy into hydraulic energy. The hydraulic machines, which convert the hydraulic energy into mechanical energy, are called turbines while the hydraulic machines which convert the mechanical energy into hydraulic energy. The study of hydraulic machines consists of turbines and pumps.

Turbines are defined as the hydraulic machines which convert hydraulic energy into mechanical energy. This, mechanical energy is used in running an electric generator which is directly coupled to the shaft of the turbine. Thus the mechanical energy is converted into electrical energy. The electric power which is obtained from the hydraulic energy (energy of water) is known as Hydroelectric power. At present the generation of hydroelectric power is the cheapest as compared by the power generated by other sources such as oil, coal etc.

General Layout of a Hydroelectric Power Plant

1. A dam constructed across a river to store water.
2. Pipes of large diameters called penstocks, which carry water under pressure from the storage reservoir to the turbines. These pipes are made of steel or reinforced concrete.
3. Turbines having different types of vanes fitted to the wheels.
4. Tail race, which is a channel which carries water away from the turbines after the water has worked on the turbines. The surface of water in the tail race channel is also known as tail race.

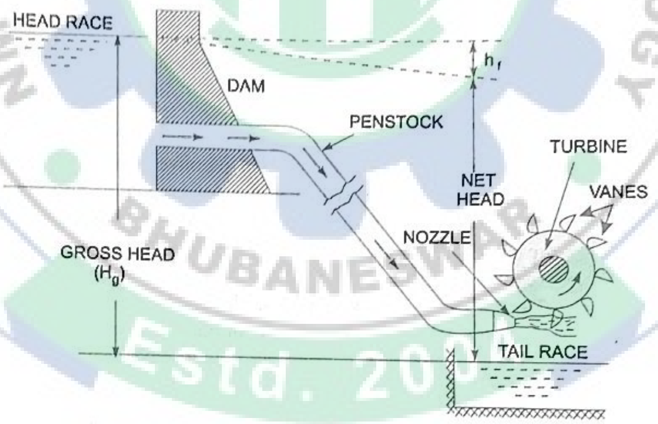


Fig. Layout of hydroelectric power plant

Definitions of Heads and Efficiencies of a Turbine

1. Gross Head. The difference between the head race level and tail race level when no water is flowing is known as Gross Head. It is denoted by 'H_g".
2. Net Head. It is also called effective head and is defined as the head available at the inlet of the turbine, when water is flowing from head race to the turbine, a loss of head due to friction between water and penstock occurs. Though there are other losses also such as loss due to bend, Pipes, fittings, loss at the entrance of penstock etc., yet they are having small magnitude as compared to head loss due to friction. In 'h_f' is the head loss due to friction between penstocks and water then net heat on turbine is given by

$$H = H_g - h_f$$

where H_g = Gross head, $h_f = \frac{4 \times f \times L \times V^2}{D \times 2g}$,

in which

V = Velocity of flow in penstock,

L = Length of penstock,

D = Diameter of penstock.

Efficiencies of a Turbine.

(*a*) **Hydraulic Efficiency (η_h).**

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{\text{R.P.}}{\text{W.P.}}$$

Power supplied at the inlet of turbine in S.I.units is known as water power. It is given by

$$\text{W.P.} = \frac{\rho \times g \times Q \times H}{1000} \text{ kW}$$

R.P. = Power delivered to runner *i.e.*, runner power

$$= \frac{W}{g} \frac{\left[V_{w_1} \pm V_{w_2}\right] \times u}{1000} \text{ kW} \qquad \text{...for Pelton Turbine}$$

$$= \frac{W}{g} \frac{\left[V_{w_1} u_1 \pm V_{w_2} u_2\right]}{1000} \text{ kW} \qquad \text{...for a radial flow turbine}$$

(*b*) **Mechanical Efficiency (η_m).**

$$\eta_m = \frac{\text{Power at the shaft of the turbine}}{\text{Power delivered by water to the runner}} = \frac{\text{S.P.}}{\text{R.P.}}$$

(*c*) **Volumetric Efficiency (η_v).**

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$$

(*d*) **Overall Efficiency (η_o)**

$$\eta_o = \frac{\text{Volume available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}} = \frac{\text{Shaft power}}{\text{Water power}}$$

$$= \frac{\text{S.P.}}{\text{W.P.}}$$

$$= \eta_m \times \eta_h$$

CLASSIFICATION OF HYDRAULIC TURBINES

1. According to the type of energy at inlet :
   (*a*) Impulse turbine, and (*b*) Reaction turbine.
2. According to the direction of flow through runner :
   (*a*) Tangential flow turbine, (*b*) Radial flow turbine,
   (*c*) Axial flow turbine, and (*d*) Mixed flow turbine.
3. According to the head at the inlet of turbine :
   (*a*) High head turbine, (*b*) Medium head turbine, and
   (*c*) Low head turbine.
4. According to the specific speed of the turbine :
   (*a*) Low specific speed turbine, (*b*) Medium specific speed turbine, and
   (*c*) High specific speed turbine.

Impulse Turbine	Reaction Turbine
1. All the available energy of the fluid is converted into kinetic energy by an efficient nozzle that forms a free jet.	1. Only a portion of the fluid energy is transformed into kinetic energy before the fluid enters the turbine runner.
2. The jet is unconfined and at atmospheric pressure throughout the action of water on the runner, and during its subsequent flow to the tail race.	2. Water enters the runner with an excess pressure, and then both the velocity and pressure change as water passes through the runner.
3. Blades are only in action when they are in front of the nozzle.	3. Blades are in action all the time.
4. Water may be allowed to enter a part or whole of the wheel circumference.	4. Water is admitted over the circumference of the wheel.
5. The wheel does not run full and air has free access to the buckets.	5. Water completely fills the vane passages throughout the operation of the turbine.
6. Casing has no hydraulic function to perform; it only serves to prevent splashing and to guide the water to the tail race.	6. Pressure at inlet to the turbine is much higher than the pressure at outlet ; unit has to be sealed from atmospheric conditions and, therefore, casing is absolutely essential.
7. Unit is installed above the tail race.	7. Unit is kept entirely submerged in water below the tail race.
8. Flow regulation is possible without loss.	8. Flow regulation is always accompanied by loss.
9. When water glides over the moving blades, its relative velocity either remains constant or reduces slightly due to friction.	9. Since there is continuous drop in pressure during flow through the blade passages, the relative velocity does increase.

PELTON WHEEL (OR TURBINE)

The Pelton wheel or Pelton turbine is a tangential flow impulse turbine. The water strikes the bucket along the tangent of the runner. The energy available at the inlet of the turbine is only kinetic energy. The pressure at the inlet and outlet of the turbine is atmospheric. This turbine is used for high heads and is named after L.A. Pelton, an American Engineer.

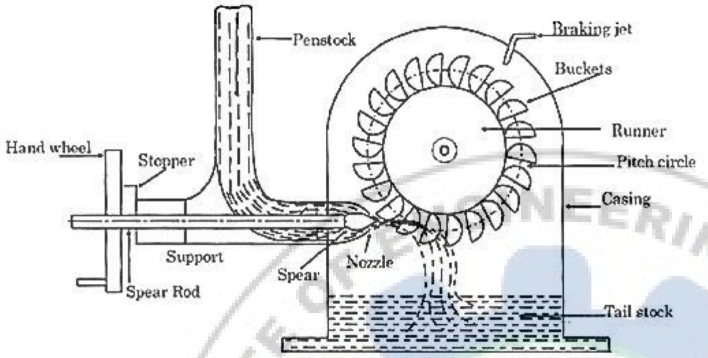
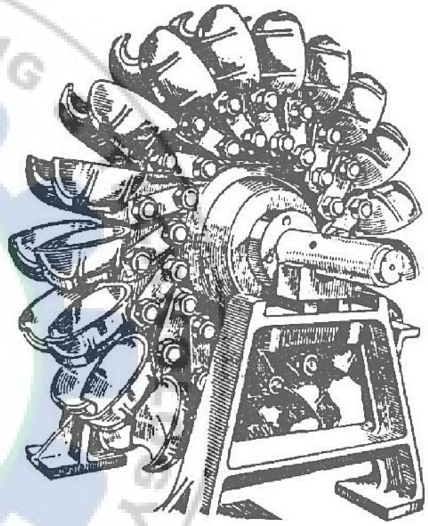


Fig. : Pelton Turbine



Main parts of Pelton Wheel

1. Nozzle and Flow Regulating Arrangement. The amount of water striking the buckets (vanes) of the runner is controlled by providing a spear in the nozzle as shown in Fig. 18.2. The spear is a conical needle which is operated either by a hand wheel or automatically in an axial direction depending upon the size of the unit. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced. On the other hand, if the spear is pushed back, the amount of water striking the runner increases.

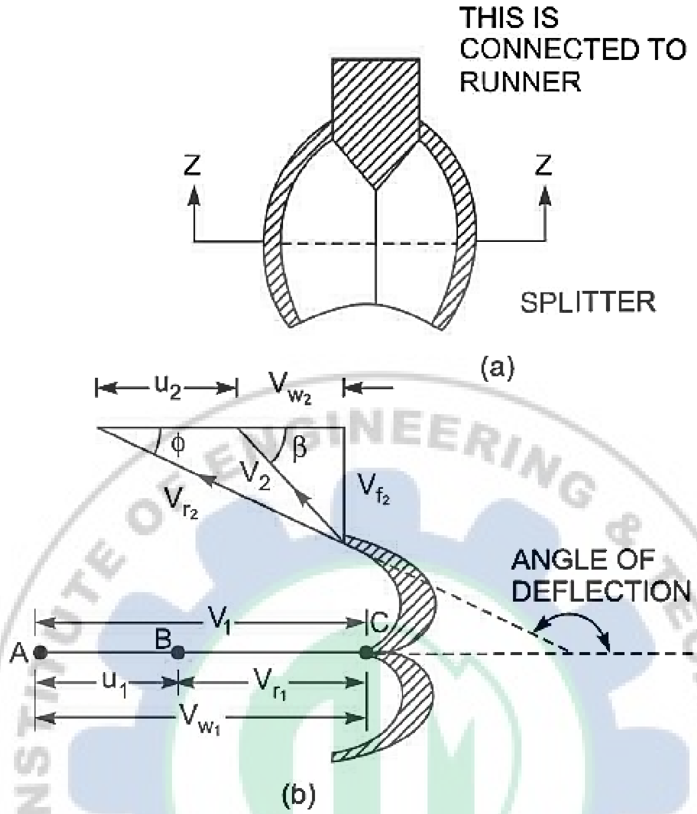
2. Runner with Buckets. Fig. 18.3 shows the runner of a Pelton wheel. It consists of a circular disc on the periphery of which a number of buckets evenly spaced are fixed. The shape of the buckets is of a double hemispherical cup or bowl. Each bucket is divided into two symmetrical parts by a dividing wall which is known as splitter.

The jet of water strikes on the splitter. The splitter divides the jet into two equal parts and the jet comes out at the outer edge of the bucket. The buckets are shaped in such a way that the jet gets deflected through 160° or 170°. The buckets are made of cast iron, cast steel bronze or stainless steel depending upon the head at the inlet of the turbine.

3. Casing. Fig. 18.4 shows a Pelton turbine with a casing. The function of the casing is to prevent the splashing of the water and to discharge water to tail race. It also acts as safeguard against accidents. It is made of cast iron or fabricated steel plates. The casing of the Pelton wheel does not perform any hydraulic function.

4. Breaking Jet.

Velocity Triangles and Work done for Pelton Wheel.



Let H = Net head acting on the Pelton wheel

$$= H_g - h_f$$

where H_g = Gross head and $h_f = \dfrac{4fLV^2}{D^* \times 2g}$

where D^* = Dia. of Penstock, N = Speed of the wheel in r.p.m.,
D = Diameter of the wheel, d = Diameter of the jet.

Then V_1 = Velocity of jet at inlet = $\sqrt{2gH}$

$$u = u_1 = u_2 = \frac{\pi DN}{60}.$$

The velocity triangle at inlet will be a straight line where

$$V_{r_1} = V_1 - u_1 = V_1 - u$$

$$V_{w_1} = V_1$$

$$\alpha = 0° \text{ and } \theta = 0°$$

From the velocity triangle at outlet, we have

$$V_{r_2} = V_{r_1} \text{ and } V_{w_2} = V_{r_2} \cos\phi - u_2.$$

The force exerted by the jet of water in the direction of motion is given by equation

$$F_x = \rho a V_1 [V_{w_1} + V_{w_2}]$$

As the angle β is an acute angle, +ve sign should be taken.

$$a = \text{Area of jet} = \frac{\pi}{4}d^2.$$

Now work done by the jet on the runner per second

$$= F_x \times u = \rho a V_1 \, [V_{w_1} + V_{w_2}] \times u \text{ Nm/s}$$

Power given to the runner by the jet

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{1000} \text{ kW}$$

Work done/s per unit weight of water striking/s

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\text{Weight of water striking/s}}$$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\rho a V_1 \times g} = \frac{1}{g}\left[V_{w_1} + V_{w_2}\right] \times u$$

The energy supplied to the jet at inlet is in the form of kinetic energy and is equal to $\frac{1}{2}mV^2$

$\therefore$ K.E. of jet per second $= \frac{1}{2}(\rho a V_1) \times V_1^2$

$\therefore$ **Hydraulic efficiency**, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}}$

$$= \frac{\rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u}{\frac{1}{2}(\rho a V_1) \times V_1^2} = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2}$$

Now $V_{w_1} = V_1,\; V_{r_1} = V_1 - u_1 = (V_1 - u)$

$\therefore$ $V_{r_2} = (V_1 - u)$

and $V_{w_2} = V_{r_2} \cos\phi - u_2 = V_{r_2} \cos\phi - u = (V_1 - u)\cos\phi - u$

Substituting the values of V_{w_1} and V_{w_2} in equation

$$\eta_h = \frac{2\left[V_1 + (V_1 - u)\cos\phi - u\right] \times u}{V_1^2}$$

$$= \frac{2\left[V_1 - u + (V_1 - u)\cos\phi\right] \times u}{V_1^2} = \frac{2(V_1 - u)\,[1 + \cos\phi]\,u}{V_1^2}.$$

The efficiency will be maximum for a given value of V_1 when

$$\frac{d}{du}(\eta_h) = 0 \quad \text{or} \quad \frac{d}{du}\left[\frac{2u(V_1 - u)(1 + \cos\phi)}{V_1^2}\right] = 0$$

$$\text{or} \quad \frac{(1 + \cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0 \quad \text{or} \quad \frac{d}{du}[2uV_1 - 2u^2] = 0 \quad \left(\because \frac{1 + \cos\phi}{V_1^2} \neq 0\right)$$

$$2V_1 - 4u = 0 \quad \text{or} \quad u = \frac{V_1}{2}$$

substituting the value of $u = \dfrac{V_1}{2}$

$$\text{Max. } \eta_h = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + \cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2 \times \frac{V_1}{2}(1 + \cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{(1 + \cos\phi)}{2}.$$

Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \text{ (= 12 for most cases)}$$

(*vi*) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5\text{ m}$$

where m = Jet ratio

(*vii*) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

Example *A Pelton wheel has a mean bucket speed of 10 metres per second with a jet of water flowing at the rate of 700 litres/s under a head of 30 metres. The buckets deflect the jet through an angle of 160°. Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume co-efficient of velocity as 0.98.*

Speed of bucket, $u = u_1 = u_2 = 10$ m/s

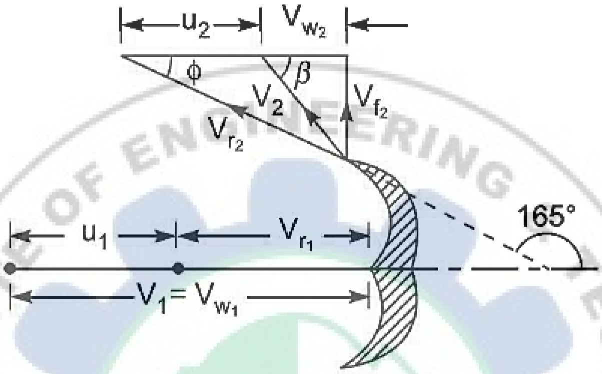
Discharge, $Q = 700$ litres/s $= 0.7\ m^3/s$, Head of water, $H = 30$ m

Angle of deflection $= 160°$

$\therefore$ Angle, $\phi = 180° - 160° = 20°$

Co-efficient of velocity, $C_v = 0.98$.

The velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 30} = 23.77$ m/s



$$\therefore \quad V_{r_1} = V_1 - u_1 = 23.77 - 10$$

$$= 13.77 \text{ m/s}$$

$$V_{w_1} = V_1 = 23.77 \text{ m/s}$$

From outlet velocity triangle,

$$V_{r_2} = V_{r_1} = 13.77 \text{ m/s}$$

$$V_{w_2} = V_{r_2} \cos \phi - u_2$$

$$= 13.77 \cos 20° - 10.0 = 2.94 \text{ m/s}$$

Work done by the jet per second on the runner is given by equation (18.9) as

$$= \rho a V_1 \left[V_{w_1} + V_{w_2}\right] \times u$$

$$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10 \qquad (\because \quad aV_1 = Q = 0.7\ m^3/s)$$

$$= 186970 \text{ Nm/s}$$

$\therefore$ Power given to turbine $= \dfrac{186970}{1000} =$ **186.97 kW. Ans.**

The hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

= 0.9454 or 94.54%. Ans.

Example *A Pelton wheel is to be designed for the following specifications : Shaft power = 11,772 kW ; Head = 380 metres ; Speed = 750 r.p.m. ; Overall efficiency = 86% ; Jet diameter is not to exceed one-sixth of the wheel diameter. Determine :*

(i) The wheel diameter, *(ii) The number of jets required, and*

(iii) Diameter of the jet.

Take $K_{v_1} = 0.985$ *and* $K_{u_1} = 0.45$

Shaft power, S.P. = 11,772 kW

Head , $H = 380$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 86\%$ or 0.86

Ratio of jet dia. to wheel dia. $= \dfrac{d}{D} = \dfrac{1}{6}$

Co-efficient of velocity, $K_{v_1} = C_v = 0.985$

Speed ratio, $K_{u_1} = 0.45$

The velocity of wheel, $u = u_1 = u_2$

$$= \text{Speed ratio} \times \sqrt{2gH} = 0.45 \times \sqrt{2 \times 9.81 \times 380} = 38.85 \text{ m/s}$$

$$u = \frac{\pi DN}{60} \quad \therefore \quad 38.85 = \frac{\pi DN}{60}$$

$$D = \frac{60 \times 38.85}{\pi \times N} = \frac{60 \times 38.85}{\pi \times 750} = \mathbf{0.989\ m.}$$

But $\dfrac{d}{D} = \dfrac{1}{6}$

$\therefore$ Dia. of jet, $d = \dfrac{1}{6} \times D = \dfrac{0.989}{6} = \mathbf{0.165\ m.\ Ans.}$

Discharge of one jet, q = Area of jet × Velocity of jet

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4}(.165) \times 85.05 \text{ m}^3\text{/s} = 1.818 \text{ m}^3\text{/s}$$

Now $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}} = \dfrac{11772}{\dfrac{\rho g \times Q \times H}{1000}}$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}, \text{ where } Q = \text{Total discharge}$$

$\therefore$ Total discharge, $Q = \dfrac{11772 \times 1000}{1000 \times 9.81 \times 380 \times 0.86} = 3.672 \text{ m}^3\text{/s}$

$\therefore$ Number of jets $= \dfrac{\text{Total discharge}}{\text{Discharge of one jet}} = \dfrac{Q}{q} = \dfrac{3.672}{1.818} = \mathbf{2\ jets.\ Ans.}$

Example *The penstock supplies water from a reservoir to the Pelton wheel with a gross head of 500 m. One third of the gross head is lost in friction in the penstock. The rate of flow of water through the nozzle fitted at the end of the penstock is 2.0 m^3/s. The angle of deflection of the jet is 165°. Determine the power given by the water to the runner and also hydraulic efficiency of the Pelton wheel. Take speed ratio = 0.45 and C_v = 1.0.*

Solution. Given :

Gross head, $H_g = 500$ m

Head lost in friction, $h_f = \dfrac{H_g}{3} = \dfrac{500}{3} = 166.7$ m

∴ Net head, $H = H_g - h_f = 500 - 166.7 = 333.30$ m

Discharge, $Q = 2.0\ \text{m}^3/\text{s}$

Angle of deflection $= 165°$

∴ Angle, $\phi = 180° - 165° = 15°$

Speed ratio $= 0.45$

Co-efficient of velocity, $C_v = 1.0$

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0 \times \sqrt{2 \times 9.81 \times 333.3} = 80.86$ m/s

Velocity of wheel, $u = \text{Speed ratio} \times \sqrt{2gH}$

or $u = u_1 = u_2 = 0.45 \times \sqrt{2 \times 9.81 \times 333.3} = 36.387$ m/s

∴ $V_{r_1} = V_1 - u_1 = 80.86 - 36.387$

$= 44.473$ m/s

Also $V_{w_1} = V_1 = 80.86$ m/s

From outlet velocity triangle, we have

$$V_{r_2} = V_{r_1} = 44.473$$

$$V_{r_2} \cos\phi = u_2 + V_{w_2}$$

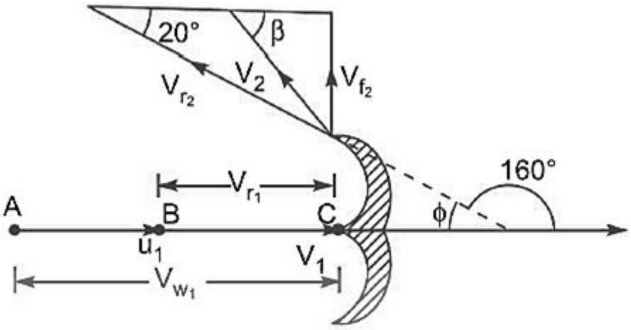
$$44.473 \cos 15° = 36.387 + V_{w_2}$$

or $V_{w_2} = 44.473 \cos 15° - 36.387 = 6.57$ m/s.

Work done by the jet on the runner per second is given by equation (18.9) as

$$\rho a V_1[V_{w_1} + V_{w_2}] \times u = \rho Q[V_{w_1} + V_{w_2}] \times u \qquad (\because \; aV_1 = Q)$$

$$= 1000 \times 2.0 \times [80.86 + 6.57] \times 36.387 = 6362630 \text{ Nm/s}$$



∴ Power given by the water to the runner in kW

$$= \frac{\text{Work done per second}}{1000} = \frac{6362630}{1000} = \mathbf{6362.63\ kW}$$

Hydraulic efficiency of the turbine is given by equation (18.12) as

$$\eta_h = \frac{2\left[V_{w_1} + V_{w_2}\right] \times u}{V_1^2} = \frac{2[80.86 + 6.57] \times 36.387}{80.86 \times 80.86}$$

$$= \mathbf{0.9731} \text{ or } \mathbf{97.31\%}\text{. } \mathbf{Ans.}$$

Example *A Pelton wheel is to be designed for a head of 60 m when running at 200 r.p.m. The Pelton wheel develops 95.6475 kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet, overall efficiency = 0.85 and co-efficient of the velocity is equal to 0.98.*

Head,	$H = 60$ m
Speed	$N = 200$ r.p.m
Shaft power,	S.P. = 95.6475 kW
Velocity of bucket,	$u = 0.45 \times$ Velocity of jet
Overall efficiency,	$\eta_o = 0.85$
Co-efficient of velocity,	$C_v = 0.98$

(*i*) *Velocity of jet*, $V_1 = C_v \times \sqrt{2gH} = 0.98 \times \sqrt{2 \times 9.81 \times 60} = 33.62$ m/s

∴ Bucket velocity, $u = u_1 = u_2 = 0.45 \times V_1 = 0.45 \times 33.62 = 15.13$ m/s

But $u = \frac{\pi DN}{60}$, where D = Diameter of wheel

∴ $15.13 = \frac{\pi \times D \times 200}{60}$ or $D = \frac{60 \times 15.13}{\pi \times 200} = \mathbf{1.44\ m.\ Ans.}$

(*ii*) *Diameter of the jet* (*d*)

Overall efficiency $\eta_o = 0.85$

But $\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{95.6475}{\left(\frac{\text{W.P.}}{1000}\right)} = \frac{95.6475 \times 1000}{\rho \times g \times Q \times H}$ $(\because\ \text{W.P.} = \rho g Q H)$

$$= \frac{95.6475 \times 1000}{1000 \times 9.81 \times Q \times 60}$$

$$Q = \frac{95.6475 \times 1000}{\eta_o \times 1000 \times 9.81 \times 60} = \frac{95.6475 \times 1000}{0.85 \times 1000 \times 9.81 \times 60} = 0.1912 \text{ m}^3\text{/s}$$

But the discharge, Q = Area of jet × Velocity of jet

$$\therefore \quad 0.1912 = \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} d^2 \times 33.62$$

$$\therefore \quad d = \sqrt{\frac{4 \times 0.1912}{\pi \times 33.62}} = 0.085 \text{ m} = \mathbf{85 \text{ mm.}}$$

(*iii*) *Size of buckets*

Width of buckets $= 5 \times d = 5 \times 85 = 425$ mm

Depth of buckets $= 1.2 \times d = 1.2 \times 85 =$ **102 mm.**

(*iv*) *Number of buckets on the wheel* is given by equation (18.17) as

$$Z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times .085} = 15 + 8.5 = \mathbf{23.5 \text{ say } 24.}$$

FRANCIS TURBINE

The Francis turbine is a mixed flow reaction turbine. This turbine is used for medium heads with medium discharge. Water enters the runner and flows towards the center of the wheel in the radial direction and leaves parallel to the axis of the turbine.

Turbines are subdivided into impulse and reaction machines. In the impulse turbines, the total head available is converted into the kinetic energy. In the reaction turbines, only some part of the available total head of the fluid is converted into kinetic energy so that the fluid entering the runner has pressure energy as well as kinetic energy. The pressure energy is then converted into kinetic energy in the runner.

The Francis turbine is a type of reaction turbine that was developed by James B. Francis. Francis turbines are the most common water turbine in use today. They operate in a water head from 40 to 600 m and are primarily used for electrical power production. The electric generators which most often use this type of turbine have a power output which generally ranges just a few kilowatts up to 800 MW.

Main components of Francis turbine

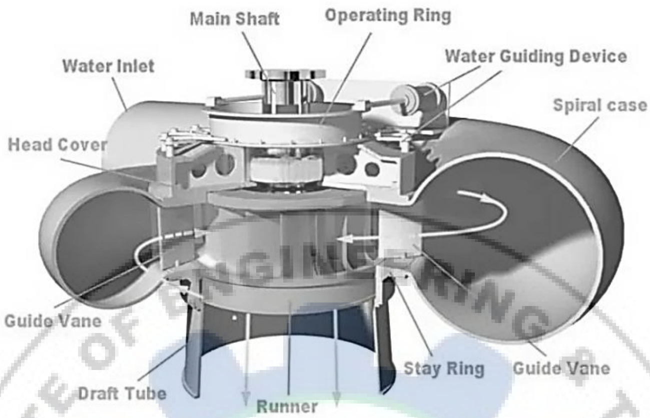
1. Spiral Casing

The water flowing from the reservoir or dam is made to pass through this pipe with high pressure. The blades of the turbines are circularly placed, which means the water striking the blades of the turbine should flow in the circular axis for efficient striking. So, the spiral casing is used, but due to the circular movement of the water, it loses its pressure.

To maintain the same pressure, the diameter of the casing is gradually reduced, to maintain the pressure uniformly, thus uniform momentum or velocity striking the runner blades.

2. Stay Vanes

This guides the water to the runner blades. Stay vanes remain stationary at their position and reduces the swirling of water due to radial flow and as it enters the runner blades. Hence, makes the turbine more efficient.



Francis Turbine

3. Guide Vanes

Guide vanes are also known as wicket gates. The main function or usages of the guide vanes are to guide the water towards the runner and it also regulates the quantity of water supplied to runner. It also guides the water to flow at an angle and that is appropriate for the design.

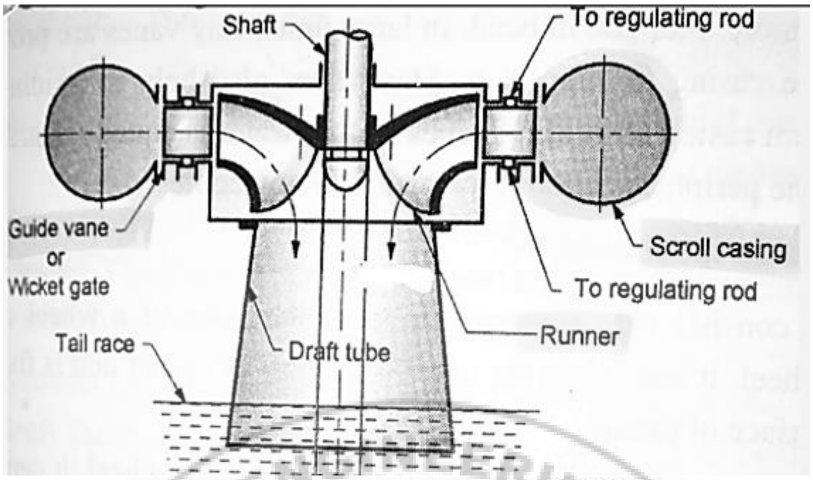
4. Runner Blades:

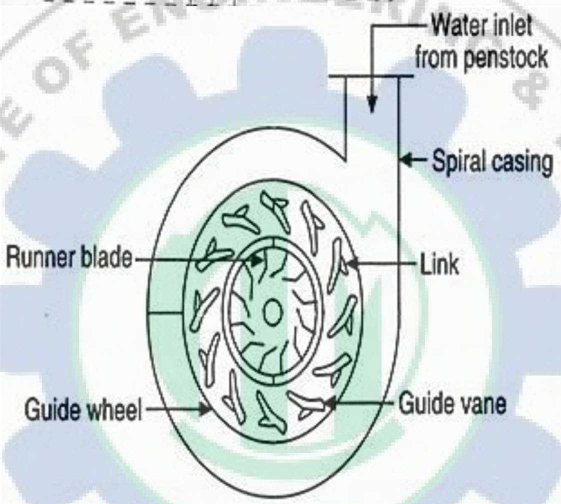
Absorbs the energy from the water and converts it to rotational motion of the main shaft. The runner blades design decides how effectively a turbine is going to perform. The runner blades are divided into two parts. The lower half is made in the shape of a small bucket so that it uses the impulse action of water to rotate the turbine.

The upper part of the blades uses the reaction force of water flowing through it. These two forces together make the runner rotate.

Draft Tube

The draft tube is an expanding tube which is used to discharge the water through the runner and next to the tailrace. The main function of the **draft tube** is to reduce the water velocity at the time of discharge. Its cross-section area increases along its length, as the water coming out of runner blades, is at considerably low pressure, so its expanding cross-section area helps it to recover the pressure as it flows towards the tailrace.





Working principles of Francis turbine

- The water is admitted to the runner through guide vanes or wicket gates. The opening between the vanes can be adjusted to vary the quantity of water admitted to the turbine. This is done to suit the load conditions.
- The water enters the runner with a low velocity but with a considerable pressure. As the water flows over the vanes the pressure head is gradually converted into velocity head.
- This kinetic energy is utilized in rotating the wheel Thus the hydraulic energy is converted into mechanical energy.
- The outgoing water enters the tailrace after passing through the draft tube. The draft tube enlarges gradually and the enlarged end is submerged deeply in the tailrace water.
- Due to this arrangement a suction head is created at the exit of the runner.

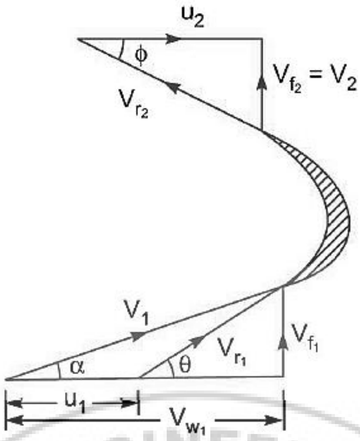
Velocity Triangle

velocity of whirl at outlet (*i.e.*, V_{w_2}) will be zero. Hence the work done by water on the runner per second will be

$$= \rho Q[V_{w_1} u_1]$$

And work done per second per unit weight of water striking/s = $\frac{1}{g}\left[V_{w_1} u_1\right]$

Hydraulic efficiency will be given by, $\eta_h = \frac{V_{w_1} u_1}{gH}$.



Example *A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.62 m. The peripheral velocity = 0.26 $\sqrt{2gH}$ and the radial velocity of flow at inlet is 0.96$\sqrt{2gH}$. The wheel runs at 150 r.p.m. and the hydraulic losses in the turbine are 22% of the available energy. Assuming radial discharge, determine :*

(i) The guide blade angle, *(ii) The wheel vane angle at inlet,*
(iii) Diameter of the wheel at inlet, and *(iv) Width of the wheel at inlet.*

Overall efficiency $\eta_o = 75\% = 0.75$

Power produced, S.P. = 148.25 kW

Head, $H = 7.62$ m

Peripheral velocity, $u_1 = 0.26\sqrt{2gH} = 0.26 \times \sqrt{2 \times 9.81 \times 7.62} = 3.179$ m/s

Velocity of flow at inlet, $V_{f_1} = 0.96\sqrt{2gH} = 0.96 \times \sqrt{2 \times 9.81 \times 7.62} = 11.738$ m/s.

Speed, $N = 150$ r.p.m.

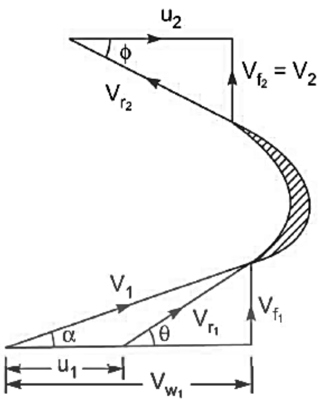
Hydraulic losses = 22% of available energy

Discharge at outlet = Radial

$$V_{w_2} = 0 \text{ and } V_{f_2} = V_2$$

Hydraulic efficiency is given as

$$\eta_h = \frac{\text{Total head at inlet} - \text{Hydraulic loss}}{\text{Head at inlet}}$$



$$= \frac{H - .22\,H}{H} = \frac{0.78\,H}{H} = 0.78$$

But $$\eta_h = \frac{V_{w_1} u_1}{gH}$$

$\therefore$ $$\frac{V_{w_1} u_1}{gH} = 0.78$$

$\therefore$ $$V_{w_1} = \frac{0.78 \times g \times H}{u_1}$$

$$= \frac{0.78 \times 9.81 \times 7.62}{3.179} = 18.34 \text{ m/s.}$$

(*i*) *The guide blade angle*, *i.e.*, α. From inlet velocity triangle,

$$\tan \alpha = \frac{V_{f_1}}{V_{w_1}} = \frac{11.738}{18.34} = 0.64$$

$\therefore$ $\alpha = \tan^{-1} 0.64 =$ **32.619°** or **32° 37′. Ans.**

(*ii*) *The wheel vane angle at inlet*, *i.e.*, θ

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{11.738}{18.34 - 3.179} = 0.774$$

$\therefore$ $\theta = \tan^{-1} .774 =$ **37.74** or **37° 44.4′. Ans.**

(*iii*) *Diameter of wheel at inlet* (D_1).

Using the relation, $$u_1 = \frac{\pi D_1 N}{60}$$

$$D_1 = \frac{60 \times u_1}{\pi \times N} = \frac{60 \times 3.179}{\pi \times 50} = \mathbf{0.4047\ m.\ Ans.}$$

(*iv*) *Width of the wheel at inlet* (B_1)

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{148.25}{\text{W.P.}}$$

But $$\text{W.P.} = \frac{\text{WH}}{1000} = \frac{\rho \times g \times Q \times H}{1000} = \frac{1000 \times 9.81 \times Q \times 7.62}{1000}$$

$\therefore$ $$\eta_o = \frac{148.25}{\dfrac{1000 \times 9.81 \times Q \times 7.62}{1000}} = \frac{148.25 \times 1000}{1000 \times 9.81 \times Q \times 7.62}$$

$$Q = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times \eta_o} = \frac{148.25 \times 1000}{1000 \times 9.81 \times 7.62 \times 0.75} = 2.644 \text{ m}^3\text{/s}$$

$$Q = \pi D_1 \times B_1 \times V_{f_1}$$

$$2.644 = \pi \times .4047 \times B_1 \times 11.738$$

$$B_1 = \frac{2.644}{\pi \times .4047 \times 11.738} = \mathbf{0.177\ m.}$$

Example *The following data is given for a Francis Turbine. Net head H = 60 m ; Speed N = 700 r.p.m.; shaft power = 294.3 kW ;* η_o *= 84% ;* η_h *= 93%; flow ratio = 0.20 ; breadth ratio n = 0.1; Outer diameter of the runner = 2 × inner diameter of runner. The thickness of vanes occupy 5% of circumferential area of the runner, velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine :*

(i) Guide blade angle,
(ii) Runner vane angles at inlet and outlet,
(iii) Diameters of runner at inlet and outlet, and
(iv) Width of wheel at inlet.

Net head, $H = 60$ m

Speed, $N = 700$ r.p.m.

Shaft power $= 294.3$ kW

Overall efficiency, $\eta_o = 84\% = 0.84$

Hydraulic efficiency, $\eta_h = 93\% = 0.93$

Flow ratio, $\dfrac{V_{f_1}}{\sqrt{2gH}} = 0.20$

$\therefore$ $V_{f_1} = 0.20 \times \sqrt{2gH}$

$= 0.20 \times \sqrt{2 \times 9.81 \times 60} = 6.862$ m/s

Breadth ratio, $\dfrac{B_1}{D_1} = 0.1$

Outer diameter, $D_1 = 2 \times$ Inner diameter $= 2 \times D_2$

Velocity of flow, $V_{f_1} = V_{f_2} = 6.862$ m/s.

Thickness of vanes = 5% of circumferential area of runner

$\therefore$ Actual area of flow $= 0.95\, \pi D_1 \times B_1$

Discharge at outlet = Radial

$\therefore$ $V_{w_2} = 0$ and $V_{f_2} = V_2$

Using relation, $\eta_o = \dfrac{\text{S.P.}}{\text{W.P.}}$

$$0.84 = \frac{294.3}{\text{W.P.}}$$

$\therefore$ $\text{W.P.} = \dfrac{294.3}{0.84} = 350.357$ kW.

But $\text{W.P.} = \dfrac{WH}{1000} = \dfrac{\rho \times g \times Q \times H}{1000} = \dfrac{1000 \times 9.81 \times Q \times 60}{1000}$

$\therefore$ $\dfrac{1000 \times 9.81 \times Q \times 60}{1000} = 350.357$

$\therefore$ $Q = \dfrac{350.357 \times 1000}{60 \times 1000 \times 9.81} = 0.5952\ \text{m}^3/\text{s}.$

Using equation (18.21), Q = Actual area of flow × Velocity of flow

$= 0.95\, \pi D_1 \times B_1 \times V_{f_1}$

$= 0.95 \times \pi \times D_1 \times 0.1 D_1 \times V_{f_2}$ $\quad (\because \quad B_1 = 0.1\, D_1)$

or $$0.5952 = 0.95 \times \pi \times D_1 \times 0.1 \times D_1 \times 6.862 = 2.048\ D_1^2$$

$\therefore$ $$D_1 = \sqrt{\frac{0.5952}{2.048}} = 0.54 \text{ m}$$

But $$\frac{B_1}{D_1} = 0.1$$

$\therefore$ $$B_1 = 0.1 \times D_1 = 0.1 \times .54 = .054 \text{ m} = 54 \text{ mm}$$

Tangential speed of the runner at inlet,

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.54 \times 700}{60} = 19.79 \text{ m/s.}$$

Using relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \text{ or } 0.93 = \frac{V_{w_1} \times 19.79}{9.81 \times 60}$$

$\therefore$ $$V_{w_1} = \frac{0.93 \times 9.81 \times 60}{19.79} = 27.66 \text{ m/s.}$$

(*i*) *Guide blade angle* (α)

From inlet velocity triangle, $\tan \alpha = \dfrac{V_{f_1}}{V_{w_1}} = \dfrac{6.862}{27.66} = 0.248$

$\therefore$ $$\alpha = \tan^{-1} 0.248 = 13.928° \text{ or } \mathbf{13° \ 55.7'}. \textbf{ Ans.}$$

(*ii*) *Runner vane angles at inlet and outlet* (θ *and* ϕ)

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{6.862}{27.66 - 19.79} = 0.872$$

$\therefore$ $$\theta = \tan^{-1} 0.872 = 41.09° \text{ or } \mathbf{41° \ 5.4'}. \textbf{ Ans.}$$

From outlet velocity triangle, $\tan \phi = \dfrac{V_{f_2}}{u_2} = \dfrac{V_{f_1}}{u_2} = \dfrac{6.862}{u_2}$ ...(*i*)

But $$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times D_1}{2} \times \frac{N}{60} \qquad \left(\because \ D_2 = \frac{D_1}{2} \text{ given}\right)$$

$$= \pi \times \frac{.54}{2} \times \frac{700}{60} = 9.896 \text{ m/s.}$$

Substituting the value of u_2 in equation (*i*),

$$\tan \phi = \frac{6.862}{9.896} = 0.6934$$

$\therefore$ $$\phi = \tan^{-1} .6934° = 34.74 \text{ or } \mathbf{34° \ 44.4'}. \textbf{ Ans.}$$

(*iii*) *Diameters of runner at inlet and outlet*

$$D_1 = 0.54 \text{ m}, D_2 = \mathbf{0.27 \ m}. \textbf{ Ans.}$$

(*iv*) *Width of wheel at inlet*

$$B_1 = \mathbf{54 \ mm}. \textbf{ Ans.}$$

AXIAL FLOW REACTION TURBINE

If the water flows parallel to the axis of the rotation of the shaft, the turbine is known as axial flow turbine. And if the head at the inlet of the turbine is the sum of pressure energy and kinetic energy and during the flow of water through runner a part of pressure energy is converted into kinetic energy, the turbine is known as reaction turbine.

For the axial flow reaction turbine, the shaft of the turbine is vertical. The lower end of the shaft is made larger which is known as 'hub' or 'boss'. The vanes are fixed on the hub and hence hub acts as a runner for axial flow reaction turbine. The following are the important type of axial flow reaction turbines :

1. Propeller Turbine, and 2. Kaplan Turbine.

When the vanes are fixed to the hub and they are not adjustable, the turbine is known as propeller turbine. But if the vanes on the hub are adjustable, the turbine is known as a *Kaplan Turbine*, after the name of V Kaplan, an Austrian Engineer. This turbine is suitable where a large quantity of water at low head is available. Fig. 18.25 shows the runner of a Kaplan turbine, which consists of a hub fixed to the shaft. On the hub, the adjustable vanes are fixed as shown in Fig. 18.25.

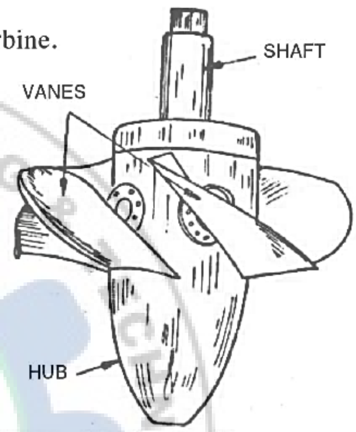
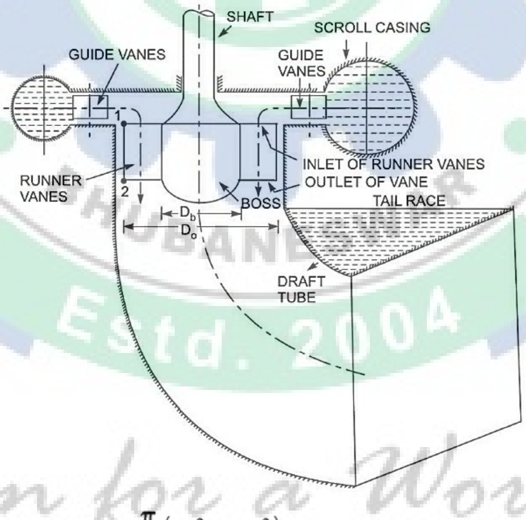


Fig. 18.25 *Kaplan turbine runner.*

The main parts of a Kaplan turbine are :

1. Scroll casing,
2. Guide vanes mechanism,
3. Hub with vanes or runner of the turbine, and
4. Draft tube.

Main components of Kaplan turbine



$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^3\right) \times V_{f_1}$$

where D_o = Outer diameter of the runner,

D_b = Diameter of hub, and

V_{f_1} = Velocity of flow at inlet.

Some Important Point for Propeller (Kaplan Turbine)

1. The peripheral velocity at inlet and outlet are equal

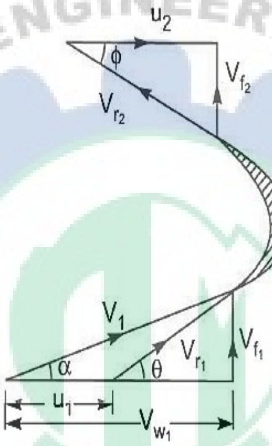
$\therefore \quad u_1 = u_2 = \frac{\pi D_o N}{60}$, where D_o = Outer dia. of runner

2. Velocity of flow at inlet and outlet are equal

$\therefore \quad V_{f_1} = V_{f_2}$.

3. Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}\left(D_o^2 - D_b^2\right).$$



Example *A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of whirl is zero at outlet, determine :*

(i) Runner vane angles at inlet and outlet at the extreme edge of the runner, and

(ii) Speed of the turbine.

Solution. Given :

Head, $H = 20$ m

Shaft power, S.P. = 11772 kW

Outer dia. of runner, $D_o = 3.5$ m

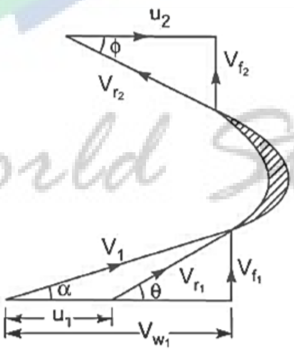
Hub diameter, $D_b = 1.75$ m

Guide blade angle, $\alpha = 35°$

Hydraulic efficiency, $\eta_h = 88\%$

Overall efficiency, $\eta_o = 84\%$

Velocity of whirl at outlet = 0.



Using the relation, $\eta_o = \frac{\text{S.P.}}{\text{W.P.}}$

where $\quad \text{W.P.} = \frac{\text{W.P.}}{1000} = \frac{\rho \times g \times Q \times H}{1000}$, we get

$$0.84 = \frac{11772}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$= \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 20} \qquad (\because \rho = 1000)$$

$$\therefore \quad Q = \frac{11772 \times 1000}{0.84 \times 1000 \times 9.81 \times 20} = 71.428 \text{ m}^3\text{/s.}$$

Using equation (18.25), $Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$

or $71.428 = \frac{\pi}{4}(3.5^2 - 1.75^2) \times V_{f_1} = \frac{\pi}{4}(12.25 - 3.0625)\, V_{f_1}$

$$= 7.216\, V_{f_1}$$

$$\therefore \quad V_{f_1} = \frac{71.428}{7.216} = 9.9 \text{ m/s.}$$

From inlet velocity triangle, $\tan \alpha = \frac{V_{f_2}}{V_{w_1}}$

$$\therefore \quad V_{w_1} = \frac{V_{f_1}}{\tan \alpha} = \frac{9.9}{\tan 35^\circ} = \frac{9.9}{.7} = 14.14 \text{ m/s}$$

Using the relation for hydraulic efficiency,

$$\eta_h = \frac{V_{w_1} u_1}{gH} \qquad (\because V_{w_2} = 0)$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

(*i*) Runner vane angles at inlet and outlet at the extreme edge of the runner are given as :

$$\tan \theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{(14.14 - 12.21)} = 5.13$$

$$\therefore \quad \theta = \tan^{-1} 5.13 = \mathbf{78.97^\circ} \text{ or } \mathbf{78^\circ\, 58'}. \textbf{ Ans.}$$

For Kaplan turbine, $u_1 = u_2 = 12.21$ m/s and $V_{f_1} = V_{f_2} = 9.9$ m/s

$\therefore$ From outlet velocity triangle, $\tan \phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$

$$\therefore \quad \phi = \tan^{-1} .811 = \mathbf{39.035^\circ} \text{ or } \mathbf{39^\circ\, 2'}. \textbf{ Ans.}$$

(*ii*) Speed of turbine is given by $u_1 = u_2 = \frac{\pi D_o N}{60}$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$\therefore \quad N = \frac{60 \times 12.21}{\pi \times 3.50} = \mathbf{66.63 \text{ r.p.m.}} \textbf{ Ans.}$$

Example *A Kaplan turbine runner is to be designed to develop 9100 kW. The net available head is 5.6 m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency = 86% and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and the specific speed of the turbine.*

Power, $P = 9100$ kW

Net head, $H = 5.6$ m

Speed ratio $= 2.09$

Flow ratio $= 0.68$

Overall efficiency, $\eta_o = 86\% = 0.86$

Diameter of boss $= \frac{1}{3}$ of diameter of runner

or $$D_b = \frac{1}{3} D_o$$

Now, speed ratio $= \dfrac{u_1}{\sqrt{2gH}}$

$$\therefore \quad u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.6} = 21.95 \text{ m/s}$$

Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}}$

$$\therefore \quad V_{f_1} = 0.68 \times \sqrt{2 \times 9.81 \times 5.6} = 7.12 \text{ m/s}$$

The overall efficiency is given by, $\eta_o = \dfrac{P}{\left(\dfrac{\rho \times g.Q.H}{1000}\right)}$

or $$Q = \frac{P \times 1000}{\rho \times g \times H \times \eta_o} = \frac{9100 \times 1000}{1000 \times 9.81 \times 5.6 \times 0.86} \qquad (\because \ \rho g = 1000 \times 9.81 \text{ N/m}^3)$$

$$= 192.5 \text{ m}^3\text{/s.}$$

The discharge through a Kaplan turbine is given by

$$Q = \frac{\pi}{4}[D_o^2 - D_b^2] \times V_{f_1}$$

or $$192.5 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.12 \qquad \left(\because \ D_b = \frac{D_o}{3}\right)$$

$$= \frac{\pi}{4}\left[1 - \frac{1}{9}\right] D_o^2 \times 7.12$$

$$\therefore \quad D_o = \sqrt{\frac{4 \times 192.5 \times 9}{\pi \times 8 \times 7.12}} = \textbf{6.21 m. Ans.}$$

The speed of turbine is given by, $u_1 = \dfrac{\pi DN}{60}$

$$\therefore \quad N = \frac{60 \times u_1}{\pi \times D} = \frac{60 \times 21.95}{\pi \times 6.21} = \textbf{67.5 r.p.m. Ans.}$$

The specific speed is given by, $N_s = \dfrac{N\sqrt{P}}{H^{5/4}} = \dfrac{67.5 \times \sqrt{9100}}{5.6^{5/4}} = \textbf{746. Ans.}$

Example *A Kaplan turbine runner is to be designed to develop 7357.5 kW shaft power. The net available head is 5.50 m. Assume that the speed ratio is 2.09 and flow ratio is 0.68, and the overall efficiency is 60%. The diameter of the boss is $\frac{1}{3}$rd of the diameter of the runner. Find the diameter of the runner, its speed and its specific speed.*

Shaft power, $P = 7357.5$ kW

Head, $H = 5.50$ m

Speed ratio $= \dfrac{u_1}{\sqrt{2gH}} = 2.09$

$\therefore$ $u_1 = 2.09 \times \sqrt{2 \times 9.81 \times 5.50} = 21.71$ m/s

Flow ratio $= \dfrac{V_{f_1}}{\sqrt{2gH}} = 0.68$

$\therefore$ $V_{f_1} = 2.68 \times \sqrt{2 \times 9.81 \times 5.50} = 7.064$ m/s

Overall efficiency, $\eta_o = 60\% = 0.60$

Diameter of boss, $D_b = \dfrac{1}{3} \times D_o$

Using relation,

$$\eta_o = \frac{\text{Shaft power}}{\text{Water power}} = \frac{7357.5}{\dfrac{\rho \times g \times Q \times H}{1000}}$$

or

$$0.60 = \frac{7357.5 \times 1000}{\rho \times g \times Q \times H} = \frac{7357.5 \times 1000}{1000 \times 9.81 \times Q \times 5.5}$$

$\therefore$

$$Q = \frac{7357.5 \times 1000}{1000 \times 9.81 \times 5.5 \times 0.60} = 227.27 \text{ m}^3\text{/s.}$$

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

$$227.27 = \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.064 \qquad \left(\because D_b = \frac{D_o}{3}\right)$$

$$= \frac{\pi}{4} \times \frac{8}{9} D_o^2 \times 7.064 = 4.9316\ D_o^2$$

$\therefore$

$$D_o = \sqrt{\frac{227.27}{4.9316}} = \textbf{6.788 m. Ans.}$$

And

$$D_b = \frac{1}{3} \times 6.788 = \textbf{2.262 m. Ans.}$$

Using the relation,

$$u_1 = \frac{\pi D_o \times N}{60}$$

$\therefore$

$$N = \frac{60 \times u_1}{\pi D_o} = \frac{60 \times 21.71}{\pi \times 6.788} = \textbf{61.08 r.p.m. Ans.}$$

The specific speed (N_s) is given by,

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{61.08 \times \sqrt{7357.5}}{5.50^{5/4}} = \textbf{622 r.p.m. Ans.}$$

Dimensional analysis

Dimensional analysis is a method of dimensions. It is a mathematical technique used in research work for design and for conducting model tests. It deals with the dimensions of the physical quantities involved in the phenomenon. All physical quantities are measured by comparison, which is made with respect to an arbitrarily fixed value. Length L, mass M and time T are three fixed dimensions which are of importance in Fluid Mechanics. If in any problem of fluid mechanics, heat is involved then temperature is also taken as fixed dimension. These fixed dimensions are called fundamental dimensions or fundamental quantity.

Secondary or derived quantities are those quantities which possess more than one fundamental dimension. For example, velocity is denoted by distance per unit time (L/T), density by mass per unit volume| (M/L^3) and acceleration distance per second Square (L/T^2). Then velocity, density, deceleration become as secondary or derived quantities. The expressions (L/T), (M/L^3) and (L/T^2) are called the dimensions of velocity, density and acceleration respectively. The dimensions of mostly used physical quantities in Fluid Mechanics.

Dimensional Homogeneity

If an equation truly expresses a proper relationship among variables in a physical process, then it will be dimensionally homogeneous. The equations are correct for any system of units and consequently each group of terms in the equation must have the same dimensional representation. This is also known as the law of dimensional homogeneity.

Dimensional variables

These are the quantities, which actually vary during a given case and can be plotted against each other. Dimensional constants: These are normally held constant during a given run. But, they may vary from case to case.

Pure constants

They have no dimensions, but, while performing the mathematical manipulation, they can arise.

Buckingham pi Theorem

The dimensional analysis for the experimental data of unknown flow problems leads to some non-dimensional parameters. These dimensionless products are frequently referred as *pi terms*. Based on the concept of *dimensional homogeneity*, these dimensionless parameters may be grouped and expressed in functional forms. This idea was explored by the famous scientist Edgar Buckingham (1867-1940) and the theorem is named accordingly.

Buckingham pi theorem, states that if an equation involving k variables is dimensionally homogeneous, then it can be reduced to a relationship among $(k-r)$ independent dimensionless products, where r is the minimum number of reference dimensions required to describe the variable. For a physical system, involving k variables, the functional relation of variables can be written mathematically as,

$$y = f\left(x_1, x_2 ...., x_k\right)$$

It should be ensured that the dimensions of the variables on the left side of the equation are equal to the dimensions of any term on the right side of equation. Now, it is possible to rearrange the above equation into a set of dimensionless products (*pi terms*), so that

$$\Pi_1 = \varphi\left(\Pi_2, \Pi_3 ...., \Pi_{k-r}\right)$$

Here, $\varphi\left(\Pi_2, \Pi_3 ...., \Pi_{k-r}\right)$ is a function of Π_2 through Π_{k-r}. The required number of *pi terms* is less than the number of original reference variables by r. These reference dimensions are usually the basic dimensions M, L and T (Mass, Length and Time).

Determination of pi Terms

Several methods can be used to form dimensionless products or *pi terms* that arise in dimensional analysis. But, there is a systematic procedure called *method of repeating variables* that allows in deciding the dimensionless and independent *pi terms*. For a given problem, following distinct steps are followed.

Step I: List out all the variables that are involved in the problem. The 'variable' is any quantity including dimensional and non-dimensional constants in a physical situation under investigation. Typically, these variables are those that are necessary to describe the "geometry" of the system (diameter, length etc.), to define fluid properties (density, viscosity etc.) and to indicate the external effects influencing the system (force, pressure etc.). All the variables must be independent in nature so as to minimize the number of variables required to describe the complete system.

Step II: Express each variable in terms of basic dimensions. Typically, for fluid mechanics problems, the basic dimensions will be either M, L and T or F, L and T. Dimensionally, these two sets are related through Newton's second law $(F = m.a)$ so that $F = MLT^{-2}$ e.g. $\rho = ML^{-3}$ or $\rho = FL^{-4}T^{2}$. It should be noted that these basic dimensions should not be mixed.

Step III: Decide the required number of *pi terms*. It can be determined by using *Buckingham pi theorem* which indicates that the number of *pi terms* is equal to $(k-r)$, where k is the number of variables in the problem (determined from Step I) and r is the number of reference dimensions required to describe these variables (determined from Step II).

Step IV: Amongst the original list of variables, select those variables that can be combined to form *pi terms*. These are called as *repeating variables*. The required number of *repeating variables is equal to the number of reference dimensions*. Each *repeating variable* must be dimensionally independent of the others, i.e. they cannot be combined themselves to form any dimensionless product. Since there is a possibility of repeating variables to appear in more than one *pi term*, so dependent variables should not be chosen as one of the repeating variable.

Step V: Essentially, the *pi terms* are formed by multiplying one of the non-repeating variables by the product of the repeating variables each raised to an exponent that will make the combination dimensionless. It usually takes the form of $x_i \; x_1^a \, x_2^b \, x_3^c$ where the exponents a, b and c are determined so that the combination is dimensionless.

Step VI: Repeat the 'Step V' for each of the remaining non-repeating variables. The resulting set of *pi terms* will correspond to the required number obtained from Step III.

III.

Step VII: After obtaining the required number of *pi terms*, make sure that all the *pi terms* are dimensionless. It can be checked by simply substituting the basic dimension $(M, L \text{ and } T)$ of the variables into the *pi terms*.

Step VIII: Typically, the final form of relationship among the *pi terms* can be written in the form of Eq. (6.1.2) where, Π_1 would contain the dependent variable in the numerator. The actual functional relationship among *pi terms* is determined from experiment.

Non Dimensional numbers in Fluid Dynamics

Forces encountered in flowing fluids include those due to inertia, viscosity, pressure, gravity, surface tension and compressibility. These forces can be written as follows;

$$\text{Inertia force: } m.a = \rho V \frac{dV}{dt} \propto \rho V^2 L^2$$

$$\text{Viscous force: } \tau A = \mu A \frac{du}{dy} \propto \mu V L$$

$$\text{Pressure force: } (\Delta p) A \propto (\Delta p) L^2$$

$$\text{Gravity force: } m g \propto g \rho L^3$$

$$\text{Surface tension force: } \sigma L$$

$$\text{Compressibility force: } E_v A \propto E_v L^2$$

The notations used in Eq. (6.2.1) are given in subsequent paragraph of this section. It may be noted that the ratio of any two forces will be dimensionless. Since, inertia forces are very important in fluid mechanics problems, the ratio of the inertia force to each of the other forces listed above leads to fundamental dimensionless groups. Some of them are defined as given below;

Reynolds number (Re): It is defined as the ratio of inertia force to viscous force. Mathematically,

$$\text{Re} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

where V is the velocity of the flow, L is the characteristics length, ρ, μ and ν are the density, dynamic viscosity and kinematic viscosity of the fluid respectively. If Re is very small, there is an indication that the viscous forces are dominant compared to inertia forces. Such types of flows are commonly referred to as "creeping/viscous flows". Conversely, for large Re, viscous forces are small compared to inertial effects and such flow problems are characterized as inviscid analysis. This number is also used to study the transition between the laminar and turbulent flow regimes.

Euler number (E_u): In most of the aerodynamic model testing, the pressure data are usually expressed mathematically as,

$$E_u = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

where Δp is the difference in local pressure and free stream pressure, V is the velocity of the flow, ρ is the density of the fluid. The denominator in Eq. (6.2.3) is called "dynamic pressure". E_u is the ratio of pressure force to inertia force and many a times the pressure coefficient (c_p) is a also common name which is defined by same manner. In the study of cavitations phenomena, similar expressions are used where, Δp is the difference in liquid stream pressure and liquid-vapour pressure. This dimensional parameter is then called as "cavitation number".

Froude number (F_r): It is interpreted as the ratio of inertia force to gravity force. Mathematically, it is written as,

$$F_r = \frac{V}{\sqrt{g.L}}$$

where V is the velocity of the flow, L is the characteristics length descriptive of the flow field and g is the acceleration due to gravity. This number is very much significant for flows with free surface effects such as in case of open-channel flow. In such types of flows, the characteristics length is the depth of water. F_r less than unity indicates sub-critical flow and values greater than unity indicate super-critical flow. It is also used to study the flow of water around ships with resulting wave motion.

Weber number (W_e): It is defined as the ratio of the inertia force to surface tension force. Mathematically,

$$W_e = \frac{\rho V^2 L}{\sigma}$$

where V is the velocity of the flow, L is the characteristics length descriptive of the flow field, ρ is the density of the fluid and σ is the surface tension force. This number is taken as an index of droplet formation and flow of thin film liquids in which there is an interface between two fluids. The inertia force is dominant compared to surface tension force when, $W_e \gg 1$ (e.g. flow of water in a river).

Mach number (M): It is the key parameter that characterizes the compressibility effects in a fluid flow and is defined as the ratio of inertia force to compressibility force. Mathematically,

$$M = \frac{V}{c} = \frac{V}{\sqrt{\frac{dp}{d\rho}}} = \frac{V}{\sqrt{\frac{E_v}{\rho}}}$$

where V is the velocity of the flow, c is the local sonic speed, ρ is the density of the fluid and E_v is the bulk modulus. Sometimes, the square of the Mach number is called "Cauchy number" (C_a) i.e.

$$C_a = M^2 = \frac{\rho V^2}{E_v}$$

Both the numbers are predominantly used in problems in which fluid compressibility is important. When, M_a is relatively small (say, less than 0.3), the inertial forces induced by fluid motion are sufficiently small to cause significant change in fluid density. So, the compressibility of the fluid can be neglected. However, this number is most commonly used parameter in compressible fluid flow problems, particularly in the field of gas dynamics and aerodynamics.

Strouhal number (S_t): It is a dimensionless parameter that is likely to be important in unsteady, oscillating flow problems in which the frequency of oscillation is ω and is defined as,

$$S_t = \frac{\omega L}{V}$$

where V is the velocity of the flow and L is the characteristics length descriptive of the flow field. This number is the measure of the ratio of the inertial forces due to unsteadiness of the flow (local acceleration) to inertia forces due to changes in

Parameter	Mathematical expression	Qualitative definition	Importance
Prandtl number	$P_r = \frac{\mu c_p}{k}$	$\frac{\text{Dissipation}}{\text{Conduction}}$	Heat convection
Eckert number	$E_c = \frac{V^2}{c_p T_0}$	$\frac{\text{Kinetic energy}}{\text{Enthalpy}}$	Dissipation
Specific heat ratio	$\gamma = \frac{c_p}{c_v}$	$\frac{\text{Enthalpy}}{\text{Internal energy}}$	Compressible flow
Roughness ratio	$\frac{\varepsilon}{L}$	$\frac{\text{Wall roughness}}{\text{Body length}}$	Turbulent rough walls
Grashof number	$G_r = \frac{\beta(\Delta T) g L^3 \rho^2}{\mu^2}$	$\frac{\text{Buoyancy}}{\text{Viscosity}}$	Natural onvection
Temperature ratio	$\frac{T_w}{T_0}$	$\frac{\text{Wall temperature}}{\text{Stream temperature}}$	Heat transfer
Pressure coefficient	$C_p = \frac{p - p_\infty}{(1/2)\rho V^2}$	$\frac{\text{Static pressure}}{\text{Dynamic pressure}}$	Hydrodynamics, Aerodynamics
Lift coefficient	$C_L = \frac{L}{(1/2) A \rho V^2}$	$\frac{\text{Lift force}}{\text{Dynamic force}}$	Hydrodynamics,Aero dynamics
Drag coefficient	$C_D = \frac{D}{(1/2) A \rho V^2}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$	Hydrodynamics, Aero dynamics

Flow Similarity

In order to achieve similarity between model and prototype behavior, all the corresponding pi terms must be equated to satisfy the following conditions.

Geometric similarity

A model and prototype are geometric similar if and only if all body dimensions in all three coordinates have the same linear-scale ratio. In order to have geometric similarity between the model and prototype, the model and the prototype should be of the same shape, all the linear dimensions of the model can be related to corresponding dimensions of the prototype by a constant scale factor. Usually, one or more of these *pi terms* will involve ratios of important lengths, which are purely geometrical in nature.

Kinematic similarity

The motions of two systems are kinematically similar if homogeneous particles lie at same points at same times. In a specific sense, the velocities at corresponding points are in the same direction (i.e. same streamline patterns) and are related in magnitude by a constant scale factor.

Dynamic similarity

When two flows have force distributions such that identical types of forces are parallel and are related in magnitude by a constant scale factor at all corresponding points, then the flows are dynamic similar. For a model and prototype, the dynamic similarity exists, when both of them have same length-scale ratio, timescale ratio and force-scale (or mass-scale ratio).

In order to have complete similarity between the model and prototype, all the similarity flow conditions must be maintained. This will automatically follow if all the important variables are included in the dimensional analysis and if all the similarity requirements based on the resulting pi terms are satisfied. For example, in compressible flows, the model and prototype should have same Reynolds number, Mach number and specific heat ratio etc. If the flow is incompressible (without free surface), then same Reynolds numbers for model and prototype can satisfy the complete similarity.

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